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EXACT TRAVELING WAVE SOLUTIONS FOR SOME NONLINEAR EVOLUTION EQUATINS BY USING THE EXP$(-\varphi(\xi))$-EXPANSION METHOD

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AUTHORS' CONTRIBUTIONS

This work was carried out in collaboration between all authors. Author LR designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript and managed literature searches. Authors LC, DS, MM managed the analysis of the study and literature searches. All authors read and approved the final manuscript.

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ABSTRACT

In this paper, we employ the $\exp(-\varphi(\xi))$-expansion method to find the exact traveling wave solutions involving parameters of nonlinear evolution equations. When these parameters are taken to be special values, the solitary wave solutions are derived from the exact traveling wave solutions. It is shown that the proposed method provides a more powerful mathematical tool for constructing exact traveling wave solutions for many other nonlinear evolution equations.

Keywords: The $\exp(-\varphi(\xi))$-expansion method; nonlinear evolution equations; traveling wave solutions; solitary wave solutions; kink-anti kink shaped .

1 Introduction

Many models in mathematics and physics are described by nonlinear differential equations. Nowadays, research in physics devotes much attention to nonlinear partial differential evolution model equations, appearing in various fields of science, especially fluid mechanics, solid-state physics, plasma physics, and nonlinear optics. Large varieties of physical, chemical, and biological phenomena are governed by nonlinear partial differential equations. One of the most exciting advances of nonlinear science and theoretical physics has been the development of methods to look for exact solutions of nonlinear partial differential equations. Exact solutions to nonlinear partial differential equations play an important role in nonlinear science, especially in nonlinear physical science since they can provide much physical information and more insight into the physical aspects of the problem and thus lead

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to further applications. Nonlinear wave phenomena of dispersion, dissipation, diffusion, reaction and convection are very important in nonlinear wave equations. In recent years, quite a few methods for obtaining explicit traveling and solitary wave solutions of nonlinear evolution equations have been proposed. Such methods are tanh - sech method [1]-[3], extended tanh - method [4]-[6], sine - cosine method [7]-[9], homogeneous balance method [10, 11],F-expansion method [12]-[14], exp-function method [15, 16], trigonometric function series method [17], $(\frac{G'}{G})-$ expansion method [18]-[21], Jacobi elliptic function method [22]-[25], The $\exp(-\varphi(\xi))$-expansion method[26]-[28] and so on.

The objective of this article is to apply The $\exp(-\varphi(\xi))$-expansion method for finding the exact traveling wave solution of Nonlinear dynamics of microtubules- A new model and The Kundu-Eckhaus equation which play an important role in biology and mathematical physics.

The rest of this paper is organized as follows: In Section 2, we give the description of The $\exp(-\varphi(\xi))$-expansion method In Section 3, we use this method to find the exact solutions of the nonlinear evolution equations pointed out above. In Section 5, conclusions are given.

2 Description of Method

Consider the following nonlinear evolution equation

$$F(u, u_t, u_x, u_{tt}, u_{xx}, ...) = 0, \tag{2.1}$$

where F is a polynomial in $u(x, t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following,we give the main steps of this method

Step 1. We use the wave transformation

$$u(x, t) = u(\xi), \qquad \xi = x - ct, \tag{2.2}$$

where c is a positive constant, to reduce Eq.(2.1)to the following ODE:

$$P(u, u', u'', u''', ...) = 0, \tag{2.3}$$

where P is a polynomial in $u(\xi)$ and its total derivatives,while $' = \frac{d}{d\xi}'$.

Step 2. Suppose that the solution of ODE (2.3) can be expressed by a polynomial in $exp(-\varphi(\xi))$ as follows

$$u(\xi) = \sum_{i=0}^{n} a_m \left(exp\left(-\varphi\left(\xi\right)\right)\right)^m, \tag{2.4}$$

Since a_m $(0 \leq m \leq n)$ are constants to be determined, such that $a_m \neq 0$.. the positive integer m can be determined by considering the homogenous balance between the highest order derivatives and nonlinear terms appearing in Eq.(2.3). Moreover precisely, we define the degree of $u\left(\xi\right)$ as $D\left(u\left(\xi\right)\right) = m$, which gives rise to degree of other expression as follows:

$$D\left(\frac{d^q u}{d\xi^q}\right) = n + q, \; D\left(u^p\left(\frac{d^q u}{d\xi^q}\right)^s\right) = np + s\left(n + q\right).$$

Therefore, we can find the value of m in Eq.(2.3), where $\varphi = \varphi(\xi)$ satisfies the ODE in the form

$$\varphi'(\xi) = exp\left(-\varphi\left(\xi\right)\right) + \mu exp\left(\varphi\left(\xi\right)\right) + \lambda, \tag{2.5}$$

the solutions of ODE (2.3) are
when $\lambda^2 - 4\mu > 0, \mu \neq 0,$

$$\varphi(\xi) = ln\left(\frac{-\sqrt{\lambda^2 - 4\mu}\ tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\xi + C_1\right)\right) - \lambda}{2\mu}\right), \tag{2.6}$$

and

$$\varphi(\xi) = ln\left(\frac{-\sqrt{\lambda^2 - 4\mu}\ coth\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\xi + C_1\right)\right) - \lambda}{2\mu}\right), \tag{2.7}$$

when $\lambda^2 - 4\mu > 0, \mu = 0,$

$$\varphi(\xi) = -ln\left(\frac{\lambda}{exp\left(\lambda\left(\xi + C_1\right)\right) - 1}\right), \tag{2.8}$$

when $\lambda^2 - 4\mu = 0, \mu \neq 0, \lambda \neq 0,$

$$\varphi(\xi) = ln\left(-\frac{2\left(\lambda\left(\xi + C_1\right) + 2\right)}{\lambda^2\left(\xi + C_1\right)}\right), \tag{2.9}$$

when $\lambda^2 - 4\mu = 0, \mu = 0, \lambda = 0,$

$$\varphi(\xi) = ln\left(\xi + C_1\right), \tag{2.10}$$

when $\lambda^2 - 4\mu < 0,$

$$\varphi(\xi) = ln\left(\frac{\sqrt{4\mu - \lambda^2}\ tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\ \left(\xi + C_1\right)\right) - \lambda}{2\mu}\right), \tag{2.11}$$

and

$$\varphi(\xi) = ln\left(\frac{\sqrt{4\mu - \lambda^2}\ cot\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\ \left(\xi + C_1\right)\right) - \lambda}{2\mu}\right), \tag{2.12}$$

where $a_m, \ldots\ldots, \lambda, \mu$ are constants to be determined later,

Step 3. After we determine the index parameter m, we substitute Eq.(2.4) along Eq.(2.5) into Eq.(2.3) and collecting all the terms of the same power $exp\left(-m\varphi(\xi)\right)$, $m = 0, 1, 2, 3, ...$ and equating them to zero, we obtain a system of algebraic equations, which can be solved by Maple or Mathematica to get the values of a_i.

Step 4. substituting these values and the solutions of Eq.(2.5) into Eq.(2.3) we obtain the exact solutions of Eq.(2.3).

It is to be noted here that the construction of the exp $\left(-\varphi\left(\xi\right)\right)$ is similar to the construction of the $\left(\frac{G'}{G}\right)$-expansion. For better understanding of the duality of both methods we cite [29], [30] and [31].

3 Application

Here, we will apply the exp($-\varphi(\xi)$)-expansion method described in sec.2 to find the exact traveling wave solutions and then the solitary wave solutions for the following nonlinear systems of evolution equations.

3.1 Example 1: Nonlinear Dynamics of Microtubules- A New Model [32]

The starting point of the present modelling is the fact that the bonds between dimers within the same PF are significantly stronger than the soft bonds between neighbouring (parallel protofilaments)PFs . This implies that the longitudinal displacements of pertaining dimers in a single PF should cause the longitudinal wave propagating along PF. The averaged impact of soft bonds with collateral PFs is taken to be described by the nonlinear double-well potential.

The present model assumes only one degree of freedom per dimer. This is z_n, a longitudinal displacement of a dimer at a position n.

The Hamiltonian for one PF is represented as

$$H = \sum_{n} \left[\frac{m}{2} \dot{z}_n^2 + \frac{k}{2} \left(z_{n+1} - z_2 \right)^2 + V \left(z_n \right) \right], \tag{3.1}$$

where dot means the first derivative with respect to time, is a mass of the dimer and is a harmonic constant describing the nearestneighbour interaction between the dimers belonging to the same PF. The first term represents a kinetic energy of the dimer, the second one, which we call harmonic energy, is a potential energy of the chemical interaction between the neighbouring dimers belonging to the same PF and the last term is the combined potential

$$V \left(z_n \right) = -C z_n - \frac{1}{2} A z_n^2 + \frac{1}{4} B z_n^4, \;\; C = qE, \tag{3.2}$$

where E is the magnitude of the intrinsic electric field and q represents the excess charge within the dipole. It is assumed that $q > 0$ and $E > 0$. One can recognize an energy of the dimer in the intrinsic electric field E at the site n and the well known double-well potential with positive parameters A and B that should be estimated The Hamiltonian given by before equations is rather common in physics. The first attempt to use it in nonlinear dynamics of (microtubules) MTs was done almost 20 years ago. To be more precise, the Hamiltonian in [33] would be obtained from before equations if z_n were replaced by u_n Hence, we refer to these two models as u-model and z-model. However, the meanings of u_n in [33] and in the present paper are completely different. The u-model assumes an angular degree of freedom, while the coordinate u_n is a projection of the top of the dimer on the direction of PF. On the other hand, the coordinate z_n is a real displacement of the dimer along x axis. This will be further elaborated later.

Using generalized coordinates z_n and $m\dot{z}_n$ and assuming a continuum approximation $z_n(t) \to z(x,t)$, we straightforwardly obtain the following nonlinear dynamical equation of motion

$$m \frac{\partial^2 z}{\partial t^2} - k l^2 \frac{\partial^2 z}{\partial x^2} - qE - Az + Bz^3 + \gamma \frac{\partial z}{\partial t} = 0 \tag{3.3}$$

The last term represents a viscosity force with γ being a viscosity coefficient. It is well known that, for a given wave equation, a traveling wave $z(\xi)$ is a solution which depends upon x and t only through a unified variable $x = \kappa x - \omega t$, where κ and ω are constants. This allows us to obtain the final dimensionless ordinary differential equation

$$\alpha u'' - \rho u' - u + u^3 - \sigma = 0, \tag{3.4}$$

where

$$u' = \frac{du}{d\xi},\ \alpha = \frac{m\omega^2 - kl^2\kappa^2}{A},\ z = \sqrt{\frac{A}{B}}u\,,\ \rho = \frac{\gamma\omega}{A}\ and\,\sigma = \frac{qE}{A\sqrt{\frac{A}{B}}}.$$

Balancing between u'' and u^3, we get $(n + 2 = 2n) \Rightarrow (n = 1)$.So that, we assume the solution of Eq.(3.4) by using (2.4), we get:

$$u = a_0 + a_1 exp\left(-\varphi\left(\xi\right)\right). \tag{3.5}$$

Substituting Eq.(3.4) and its derivative into Eq.(3.4) and collecting the coefficients of $exp\left(-N\varphi\left(\xi\right)\right)$, where $N = 0, 1, 2, \ldots$ and set it to zero we obtain the system

$$2\,\alpha\,a_1 + {a_1}^3 = 0, \tag{3.6}$$

$$3\,\alpha\,a_1\lambda + \rho\,a_1 + 3\,a_0{a_1}^2 = 0, \tag{3.7}$$

$$\alpha\,\left(2\,a_1\mu + a_1\lambda^2\right) + \rho\,a_1\lambda - a_1 + 3\,{a_0}^2a_1 = 0, \tag{3.8}$$

$$\alpha\,a_1\lambda\,\mu + \rho\,a_1\mu - a_0 + {a_0}^3 - \sigma = 0. \tag{3.9}$$

Solving above system by using maple 16, we get:

$$\rho = \frac{1}{2}\sqrt{12\,{a_1}^2\mu - 3\,{a_1}^2\lambda^2 + 12},\ \alpha = \frac{-1}{2}\,{a_1}^2,$$
$$\sigma = \frac{1}{9}\,\left(4\,{a_1}^2\mu + 1 - {a_1}^2\lambda^2\right)\sqrt{12\,{a_1}^2\mu - 3\,{a_1}^2\lambda^2 + 12},\ a_1 = a_1$$
$$a_0 = \frac{1}{2}\,a_1\lambda - \frac{1}{6}\sqrt{12\,{a_1}^2\mu - 3\,{a_1}^2\lambda^2 + 12}$$

Thus the solution is

$$u\left(\xi\right) = \frac{1}{2}\,a_1\lambda - \frac{1}{6}\sqrt{12\,{a_1}^2\mu - 3\,{a_1}^2\lambda^2 + 12} + a_1\,exp\left(-\varphi\left(\xi\right)\right). \tag{3.10}$$

Let us now discuss the following cases:

When $\lambda^2 - 4\mu > 0, \mu \neq 0,$

$$u_1 = \frac{1}{2}\,a_1\lambda - \frac{1}{6}\sqrt{12\,{a_1}^2\mu - 3\,{a_1}^2\lambda^2 + 12} + a_1\frac{2\mu}{-\sqrt{\lambda^2 - 4\mu}\ tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\xi + C_1\right)\right) - \lambda}, \tag{3.11}$$

and

$$u_2 = \frac{1}{2}\,a_1\lambda - \frac{1}{6}\,\sqrt{12\,{a_1}^2\mu - 3\,{a_1}^2\lambda^2 + 12} + a_1\frac{2\mu}{-\sqrt{\lambda^2 - 4\mu}\;coth\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\xi + C_1\right)\right) - \lambda}. \quad (3.12)$$

When $\lambda^2 - 4\mu > 0, \mu = 0$,

$$u_3 = \frac{1}{2}\,a_1\lambda - \frac{1}{6}\,\sqrt{12\,{a_1}^2\mu - 3\,{a_1}^2\lambda^2 + 12} + a_1\frac{\lambda}{exp\left(\lambda\left(\xi + C_1\right)\right) - 1}. \quad (3.13)$$

When $\lambda^2 - 4\mu = 0, \mu \neq 0, \lambda \neq 0$,

$$u_4 = \frac{1}{2}\,a_1\lambda - \frac{1}{6}\,\sqrt{12\,{a_1}^2\mu - 3\,{a_1}^2\lambda^2 + 12} - a_1\frac{2\left(\lambda\left(\xi + C_1\right) + 2\right)}{\lambda^2\left(\xi + C_1\right)}. \quad (3.14)$$

When $\lambda^2 - 4\mu = 0, \mu = 0, \lambda = 0$,

$$u_5 = \frac{1}{2}\,a_1\lambda - \frac{1}{6}\,\sqrt{12\,{a_1}^2\mu - 3\,{a_1}^2\lambda^2 + 12} + a_1\frac{1}{\xi + C_1}. \quad (3.15)$$

When $\lambda^2 - 4\mu < 0$,

$$u_6 = \frac{1}{2}\,a_1\lambda - \frac{1}{6}\,\sqrt{12\,{a_1}^2\mu - 3\,{a_1}^2\lambda^2 + 12} + a_1\frac{2\mu}{\sqrt{4\mu - \lambda^2}\;tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\xi + C_1\right)\right) - \lambda}. \quad (3.16)$$

and

$$u_7 = \frac{1}{2}\,a_1\lambda - \frac{1}{6}\,\sqrt{12\,{a_1}^2\mu - 3\,{a_1}^2\lambda^2 + 12} + a_1\frac{2\mu}{\sqrt{4\mu - \lambda^2}\;cot\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\xi + C_1\right)\right) - \lambda}. \quad (3.17)$$

3.2 Example 2: The Kundu- Eckhaus Equation

Let us consider the Kundu- Eckhaus equation [34]

$$iQ_t + Q_{xx} - 2\sigma\left|Q\right|^2 Q + \delta^2\left|Q\right|^4 Q + 2i\delta\left(\left|Q\right|^2\right)_x Q = 0. \quad (3.18)$$

We may choose the following traveling wave transformation:

$$Q\left(x,t\right) = e^{i\left(\alpha x + \beta t\right)}u\left(\xi\right),\;\; \xi = ik\left(x - 2\alpha t\right),$$

where k, α and β are constants to be determined later.

Substituting these into Eq.(3.18) we have

$$-\left(\beta + \alpha^2\right)u - k^2 u'' - 2\sigma u^3 + \delta^2 u^5 - 4k\delta u^2 u' = 0. \quad (3.19)$$

balancing between u'' and $u^5 \Rightarrow N + 2 = 5N \Rightarrow N = \frac{1}{2}$.
using the transformation

$$u\left(\xi\right) = \left(v\left(\xi\right)\right)^{\frac{1}{2}}, \quad (3.20)$$

into (3.19) we get

$$-4\left(\beta+\alpha^2\right)v^2+k^2\left(v'\right)^2-2k^2vv''-8\sigma v^3+4\delta^2v^4-8k\delta v^2v'=0. \qquad (3.21)$$

balancing between vv'' with $v^4 \Rightarrow N+N+2=4N \Rightarrow N=1$.

So that we have the same formal solution of Eq.(3.4).

Substituting Eq.(3.5) and its derivative into Eq.(3.21) and collecting the coefficients of $exp\left(-N\varphi\left(\xi\right)\right)$, where $N=0,1,2,\ldots$ and set it to zero we obtain the system

$$-3\,k^2{a_1}^2+4\,\delta^2{a_1}^4+8\,k\delta\,{a_1}^3=0, \qquad (3.22)$$

$$2\,k^2{a_1}^2\lambda-2\,k^2\left(2\,a_0a_1+3\,{a_1}^2\lambda\right)-8\,\sigma\,{a_1}^3+16\,\delta^2a_0{a_1}^3-8\,k\delta\,\left(-2\,a_0{a_1}^2-{a_1}^3\lambda\right)=0, \qquad (3.23)$$

$$\begin{gathered}-4\,\left(\beta+\alpha^2\right){a_1}^2+k^2\left(2\,{a_1}^2\mu+{a_1}^2\lambda^2\right)-2\,k^2\left(3\,a_0a_1\lambda+2\,{a_1}^2\mu+{a_1}^2\lambda^2\right)\\-24\,\sigma\,a_0{a_1}^2+24\,\delta^2{a_0}^2{a_1}^2-8\,k\delta\,\left(-{a_0}^2a_1-2\,a_0{a_1}^2\lambda-{a_1}^3\mu\right)=0,\end{gathered} \qquad (3.24)$$

$$\begin{gathered}-2\,\left(4\,\beta+4\,\alpha^2\right)a_0a_1+2\,k^2{a_1}^2\mu\,\lambda-2\,k^2\left(2\,a_0a_1\mu+a_0a_1\lambda^2+{a_1}^2\mu\,\lambda\right)-24\,\sigma\,{a_0}^2a_1\\+16\,\delta^2{a_0}^3a_1-8\,k\delta\,\left(-{a_0}^2a_1\lambda-2\,a_0{a_1}^2\mu\right)=0,\end{gathered} \qquad (3.25)$$

$$-4\,\left(\beta+\alpha^2\right){a_0}^2+k^2{a_1}^2\mu^2-2\,k^2a_0a_1\lambda\,\mu-8\,\sigma\,{a_0}^3+4\,\delta^2{a_0}^4+8\,k\delta\,{a_0}^2a_1\mu=0. \qquad (3.26)$$

Solving above system by using maple 16, we get:

$$\begin{gathered}k=k,\;\alpha=\alpha,\;\beta=-\alpha^2+k^2\mu-\frac{1}{4}\,k^2\lambda^2,\;\delta=\frac{k\left(-2\pm\sqrt{2}\right)}{2a_1},\\ \sigma=\frac{-k^2}{2a_1}\left(-3\pm\sqrt{7}\right)\left(\pm\sqrt{\lambda^2-4\mu}\right),\;a_1=a_1,\\ a_0=\frac{a_1}{2}\left(\lambda\pm\sqrt{\lambda^2-4\mu}\right).\end{gathered}$$

Thus the solution is

$$u\left(\xi\right)=\frac{a_1}{2}\left(\lambda\pm\sqrt{\lambda^2-4\mu}\right)+a_1\,exp\left(-\varphi\left(\xi\right)\right). \qquad (3.27)$$

Let us now discuss the following cases:

When $\lambda^2-4\mu>0,\mu\neq0$,

$$v_1=\frac{a_1}{2}\left(\lambda\pm\sqrt{\lambda^2-4\mu}\right)+a_1\frac{2\mu}{-\sqrt{\lambda^2-4\mu}\;tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\xi+C_1\right)\right)-\lambda}. \qquad (3.28)$$

So that

$$u_1=\left(v_1\right)^{\left(\frac{1}{2}\right)}. \qquad (3.29)$$

and

$$v_2 = \frac{a_1}{2}\left(\lambda \pm \sqrt{\lambda^2 - 4\mu}\right) + a_1 \frac{2\mu}{-\sqrt{\lambda^2 - 4\mu}\, coth\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\xi + C_1\right)\right) - \lambda}. \tag{3.30}$$

So that

$$u_2 = (v_2)^{\left(\frac{1}{2}\right)}. \tag{3.31}$$

When $\lambda^2 - 4\mu > 0, \mu = 0,$

$$v_3 = \frac{a_1}{2}\left(\lambda \pm \sqrt{\lambda^2 - 4\mu}\right) + a_1 \frac{\lambda}{exp\left(\lambda\left(\xi + C_1\right)\right) - 1}. \tag{3.32}$$

So that

$$u_3 = (v_3)^{\left(\frac{1}{2}\right)}. \tag{3.33}$$

When $\lambda^2 - 4\mu = 0, \mu \neq 0, \lambda \neq 0,$

$$v_4 = \frac{a_1}{2}\left(\lambda \pm \sqrt{\lambda^2 - 4\mu}\right) - a_1 \frac{2\left(\lambda\left(\xi + C_1\right) + 2\right)}{\lambda^2\left(\xi + C_1\right)}. \tag{3.34}$$

So that

$$u_4 = (v_4)^{\left(\frac{1}{2}\right)}. \tag{3.35}$$

When $\lambda^2 - 4\mu = 0, \mu = 0, \lambda = 0,$

$$v_5 = \frac{a_1}{2}\left(\lambda \pm \sqrt{\lambda^2 - 4\mu}\right) + a_1 \frac{1}{\xi + C_1}. \tag{3.36}$$

So that

$$u_5 = (v_5)^{\left(\frac{1}{2}\right)}. \tag{3.37}$$

When $\lambda^2 - 4\mu < 0,$

$$v_6 = \frac{a_1}{2}\left(\lambda \pm \sqrt{\lambda^2 - 4\mu}\right) + a_1 \frac{2\mu}{\sqrt{4\mu - \lambda^2}\, tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\xi + C_1\right)\right) - \lambda}. \tag{3.38}$$

So that

$$u_6 = (v_6)^{\left(\frac{1}{2}\right)}. \tag{3.39}$$

and

$$v_7 = \frac{a_1}{2}\left(\lambda \pm \sqrt{\lambda^2 - 4\mu}\right) + a_1 \frac{2\mu}{\sqrt{4\mu - \lambda^2}\, cot\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\xi + C_1\right)\right) - \lambda}. \tag{3.40}$$

So that

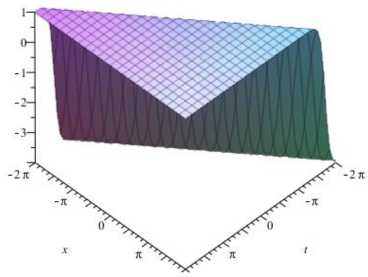
$$u_7 = (v_7)^{\left(\frac{1}{2}\right)}. \tag{3.41}$$

- **Remark**: *All the obtained results have been checked with Maple 16 by putting them back into the original equation and found correct.*

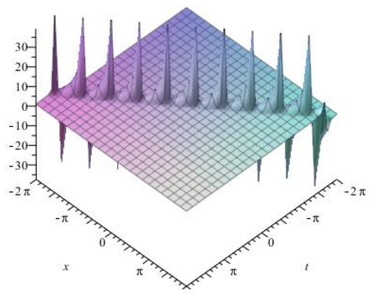
4 Figures

Physical explanations for some obtained solutions.
In this section, we describe all figure for the exact traveling wave solutions by selecting some special values pf parameters in the exact solutions using the mathematical software Maple 18, which can be shown below in the Figs.1, 2 and 3. From these explicit solutions, we see that:

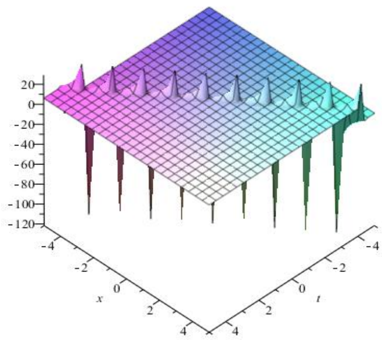
Eqs.(3.11) and (3.28) are kink shaped soliton solutions, Eqs.(3.12) and (3.29) are singular kink shaped soliton solutions while, Eqs.(3.13), (3.14), (3.15), (3.16), (3.17), (3.32), (3.34), (3.36), (3.38) and (3.40) are singular soliton solutions. The graphical representation of these solutions are shown in the following figures.



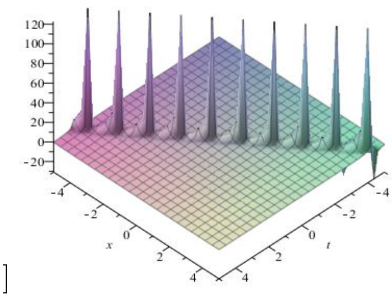
[Eq.(3.11)]



[Eq.(3.12)]

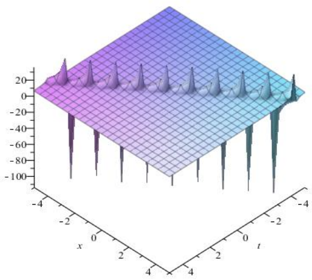


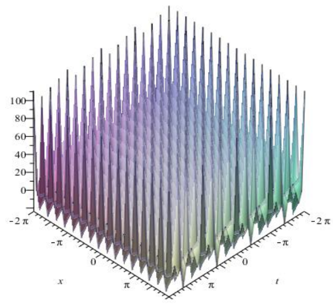
[Eq.(3.13)]



[Eq.(3.14)]

Figure 1: The Solitary wave solution of Eqs.(3.11), (3.12),(3.13) and (3.14)


[Eq.(3.15)]


[Eq.(3.16)]

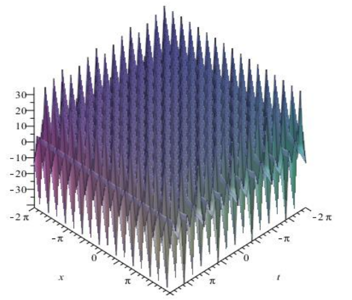
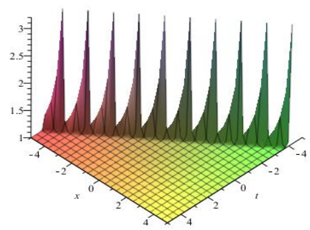
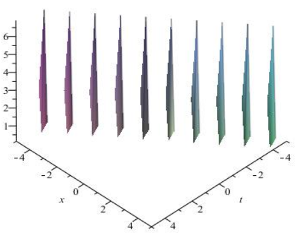

[Eq.(3.17)]

Figure 2: The Solitary wave solution of Eqs.(3.15) and (3.16), (3.17)


[Eq.(3.33)]


[Eq.(3.35)]

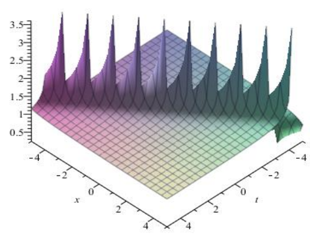

[Eq.(3.37)]

Figure 3: The Solitary wave solution of Eqs.(3.33), (3.35) and (3.37)

5 Conclusion

The $\exp(-\varphi(\xi))$-expansion method has been applied in this paper to find the exact traveling wave solutions and then the solitary wave solutions of two nonlinear evolution equations, namely, Nonlinear dynamics of microtubules - A new model and The Kundu- Eckhaus equation . Let us compare between our results obtained in the present article with the well-known results obtained by other authors using different methods as follows: Our results of Nonlinear dynamics of microtubules - A new model and The Kundu- Eckhaus equation are new and different from those obtained in [32], [33] and [34] and Figs. 1, 2 and 3, show the solitary traveling wave solution of Nonlinear dynamics of microtubules - A new model and The Kundu- Eckhaus equation. We can conclude that the $\exp(-\varphi(\xi))$-expansion method is is a very powerful and efficient technique in finding exact solutions for wide classes of nonlinear problems and can be applied to many other nonlinear evolution equations in mathematical physics. Another possible merit is that the reliability of the method and the reduction in the size of computational domain give this method a wider applicability.

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Competing Interests

The authors declare that no competing interests exist.

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