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Research articles

Magnetohydrodynamic Ferro-Nano fluid flow in a semi-porous curved tube under the effect of hall current and nonlinear thermal radiative

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ABSTRACT

Investigation of hall current as well as joule heating effects on the flow and heat transfer characteristics of Ferro-nanofluid flow through a curved tube studied in this work. This work meets a lot of engineering applications such as heat exchangers at low velocity conditions, collectors of solar energy, cooling devices of electronic and micro-electronic equipments and geophysical transport in electrically conducting. The modeling based on the momentum equations by incorporating the hall current effect as well as the energy equation by incorporating the joule heating and thermal radiative effects. Similarity transformation technique has been used to obtain non-linear ordinary differential system which solved analytically as a series solution using Differential Transformation Method (DTM) with associated of Padé approximation technique. The validation of the obtained results has been examined against a previous published works. The results confirm that the hall current has a direct and strong effect on the flow behavior, such that the presence of hall current reduces the flow pressure within the tube as well as increase the heat flux from the inner surface of the tube. Moreover a third component of velocity generates as well as a normal force called Lorentz force due to the presence of the hall current.

1. Introduction

In the last few years, nanofluids have a great attention of the researches due to its importance in the engineering and bio fields. Term of Nano-fluid refer to a fluid containing Nano size particles which may be metals, oxides or carbon nanotubes. Mixing the nanoparticles with the fluids produce a new fluid has a modified thermal conductivity and heat transfer coefficient. Choi and Estman [1] they investigated experimentally the influence of Nano particles on the thermal conductivity of the fluids. By this date, the nanofluids became the target for many researches. Elbashbeshy et al. [2] they concerned their study on the effect of thermal radiation on the nanofluid boundary layer over a moving plate with variable thickness. Abdel-wahed et al. [3] extended the work of Elbashbeshy to study the effect of the Brownian motion of the nanoparticles as well as the thermophoresis force on the boundary layer behavior over the same plate. Abdel-wahed *et al.* [4] discussed the influence of Nonlinear thermal radiation on the MHD boundary layer flow of nanofluid over a moving plate. Abdel-wahed [5] developed the previous publications by taking the effect of the microrotation of the suspended microelements into unsteady nanofluid boundary layer over a moving surface. On the other hand, mixed convection flow through a vertical or curved channel is one of the simulation techniques for many engineering applications, for example, the cooling devices of electronic equipment, collector of solar energy, heat exchangers, and so on. This topic has been studied by Muthuraj et al. [6] they discussed the influence of viscous dissipation and the thermal diffusion on the micropolar fluid past a vertical channel, after that Kaladhar [7] extended their work to study the effect of joule heating and hall current on the same problem. Lourdu et al. [8] presented a semi-analytic solution for the MHD mixed convection flow in vertical channel under the effect of thermophoresis force. Fakour et al. [9] studied in their research the mixed convection flow of nanofluid past a vertical channel under a variable wall temperature. Prasad et al. [10] investigated the mixed convection flow with temperature dependent transport properties. Radiation effect on the fully developed mixed convection flow studied by Patra et al. [11] they considered the flow was flowing through a vertical channel with asymmetric heating of its sides. Also Das et al. [12] discussed the influence of thermal radiation on the transient natural convection flow of nanofluid in a vertical channel. Mixed convection flow through a vertical or curved channel is not stop at the engineering applications; it has many applications in the biomedical field such as the blood flow through veins and Arteries. Abdel-wahed [13] has discussed the motion of the blood flow through a vertical porous conduit under the effect of Lorentz force as an application of the blood filtration process.

The term of Ferro-fluid refer to the fluids which have a strong

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Nomenclatures

a,b	constants parameter (t^{-1})
u, v, w	linear velocities (m/s)
T	temperature (K)
f	dimensionless velocity
K	thermal conductivity (W/mK)
J	current density (Ampere/m^2)
ℓ	width of the tube (m)
B_0	magnetic field strength
q_r	thermal radiative flux (kW/m^2)
C_p	specific heat (J/kgK)
C_f	skin friction coefficient
p	Pressure (N/m^2)
e	electron charge
Greek symbols	
ϕ	nanoparticles volume fraction
θ	dimensionless temperature
ρ	fluid density (kg/m^3)
μ	dynamic viscosity (Ns/m^2)
ν	kinematic viscosity (m^2/s)
σ	electrical conductivity ($1.3806488 \times 10^{-23}$ m^2 kg/s^2K)
τ_e	electron collusion time
η_e	electron density number
Dimensionless parameters	
$m = \frac{e\tau_e B_0}{m_e}$	hall current parameter
$k = \frac{R}{\ell}$	curvature parameter
$M = \frac{B_0{}^2\ell\sigma_f}{u\rho_f}$	hartman number
$E_C = \frac{\rho_f (U^2 s^2)}{\ell^2(\rho C_p)_f T_H}$	Eckert number
$Pr = \frac{\nu_f (C_p)_f}{k_f}$	Prandtl number
$Rd = \frac{16\sigma^* T_\infty^3}{3k_f k^*}$	radiation parameter
Subscripts	
f	fluid phase
nf	nano-fluid
s	solid particles
w	condition of the wall
∞	ambient condition

magnetism in the presence of magnetic field. The ferrofluids are made from Ferro Nano magnetic particles suspended in a water or solvent and the particles are thoroughly coated with a surfactant to inhibit clumping. So the magnetic attraction of nanoparticles is weak enough that they surfactant's Van der Waals force is sufficient to prevent magnetic clumping or agglomeration. These types of fluids have strong application in the engineering and bio fields

Such as semi-active dampers, loudspeakers and magnetic resonance imaging moreover, it has also been proposed in a form of Nano surgery to separate a tumor from the tissue in which it has grown.

The applications of the ferrofluids in the thermodynamics engineering have been discussed by a many researchers. Aaiza et al. [14] they studied the time-dependent motion of MHD mixed convection flow of a Ferro-fluid in a vertical channel, under the effect of thermal radiative. Selimefendigil et al. [15] focused their work on the effect of the magnetic dipole on the flow and heat transfer characteristics of a Ferro-nanofluid over rotating cylinder. Malvandi et al. [16] studied the effect of magnetic field on nanoparticles migration and heat transfer of water/alumina nanofluid in a channel. Abdel-wahed [17] studied the effect of hall current and joule heating on the boundary layer containing Ferro-nanoparticles flowing over rotating disk. Das et al. [18] investigated the mixed convection hydrodynamic flow of nanofluid in vertical channel taking the effect of induced magnetic field with large magnetic Reynold number. Abbas et al. [19] investigated the effects of thermal radiative and the hall current on a viscous fluid moving inside a curved channel. Noreen e. al. [20] investigated the flow of copper nanoparticles-water within a curved peristalsis channel. Naveed e. al. [21] studied the flow of the magnetohydrodynamic unsteady fluid over a curved surface. Abbas e. al. [22] examined the effect of heat generation and thermal radiation on slip MHD nanofluid over a curved surface. Sajid e. al. [23] studied the effect of joule heating on MHD Ferro-nanofluid moving through a semi-porous curved channel.

According to a literature survey, turns out that no study regarding to the hall current with joule heating effects on the Ferro nanofluid through a curved tube has done before. So this study deals to investigate the effect of nonlinear thermal radiative flux as well as the hall current on the flow and heat transfer characteristics of a Ferro Nano-fluid (Fe_4O_3) moving inside a curved tube and the next section deals the physical model considered for this.

2. Modeling of the problem

The problem simulated by considering a three dimensional steady-laminar-incompressible flow of an electrically conducting Ferro-nano-fluid (Fe_2O_3-water) with hall current and joule heating effects runs through a semi porous curved tube of width(ℓ). The flow has components of velocities (u, v, w)in the directions of increasing(s, r, z), respectively. It is worth mentioning that, the third dimension of the flow generated due to the effect of the hall current but there is no effect on the flow or heat transfer characteristics in that direction. Assume that the tube subjected to uniform radial magnetic field of strengthB_0 and thermal radiative flux q_r. Moreover, the upper side of tube is porous and has a temperature (T_∞), the lower side is impermeable and has a temperature(T_w) (see Fig. 1).

The effect of hall current without the effect of the electric field $\mathbb{E}$ according to Ohm's law defined as [19]:

$$J = \frac{\sigma_{nf}}{1+m^2}\left[\underset{-}{V} \times \underset{-}{B} - \frac{J \times B}{e\eta_e}\right]$$

Such that V is the velocity vector, B is the magnetic induction vector, J is the current density, m is the hall current.

According to the above assumptions, The governing system of the problem is [19,23]:

$$\frac{\partial}{\partial r}[(r+R)v] + R\frac{\partial u}{\partial s} = 0 \tag{1}$$

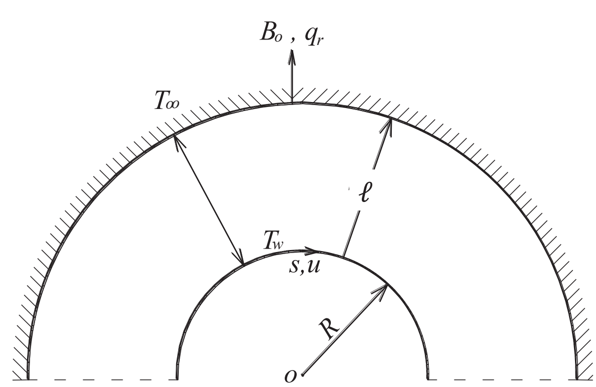

Fig. 1. Physical model and coordinate system.

$$\frac{u^2}{r+R}+\frac{1}{\rho_{nf}}\frac{\partial p}{\partial r}=0 \tag{2}$$

$$v\frac{\partial u}{\partial r}+\left(\frac{Ru}{r+R}\right)\frac{\partial u}{\partial s}+\frac{uv}{r+R}=-\frac{1}{\rho_{nf}}\left(\frac{R}{r+R}\right)\frac{\partial p}{\partial s}+\left(\frac{\mu_{nf}}{\rho_{nf}}\right)\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r+R}\frac{\partial u}{\partial r}-\frac{u}{(r+R)^2}\right)-\frac{\sigma_{nf}B_0{}^2}{\rho_{nf}(1+m^2)}(u+mw) \tag{3}$$

$$v\frac{\partial w}{\partial r}+\left(\frac{Ru}{r+R}\right)\frac{\partial w}{\partial s}=\left(\frac{\mu_{nf}}{\rho_{nf}}\right)\left(\frac{\partial^2 w}{\partial r^2}+\left(\frac{1}{r+R}\right)\frac{\partial w}{\partial r}\right)+\frac{\sigma_{nf}B_0{}^2}{\rho_{nf}(1+m^2)}(mu-w) \tag{4}$$

$$(\rho C_p)_{nf}\left(v\frac{\partial T}{\partial r}+\left(\frac{Ru}{r+R}\right)\frac{\partial T}{\partial s}\right)=k_{nf}\left(\frac{\partial^2 T}{\partial r^2}+\left(\frac{1}{r+R}\right)\frac{\partial T}{\partial r}\right)+\sigma_{nf}B_0^2(u^2+w^2)-\left(\frac{1}{r+R}\right)\frac{\partial}{\partial r}(r+R)q_r \tag{5}$$

With the following boundary conditions

$$at\ r=0;\ u=U,\ v=w=0,\ T=T_w \tag{6}$$

$$at\ r=\ell;\ u=w=0,\ v=-\frac{\mathscr{V}R}{R+r},\ T=T_\infty \tag{7}$$

Such that: $\mathscr{V}$ is suction($\mathscr{V}>0$)/injection($\mathscr{V}<0$) parameter.

The properties of Nanofluid defined as follows [17]

$$\left.\begin{aligned}&\mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}},\ \rho_{nf}=(1-\phi)\rho_f+\phi\rho_s,\\&(\rho C_p)_{nf}=(\rho C_p)_f(1-\phi)+\phi(\rho C_p)_s\\&\frac{\sigma_{nf}}{\sigma_f}=1+\frac{3\phi\left(\frac{\sigma_s}{\sigma_f}-1\right)}{\left(\frac{\sigma_s}{\sigma_f}+2\right)-\phi\left(\frac{\sigma_s}{\sigma_f}-1\right)}\\&\frac{k_{nf}}{k_f}=\frac{(k_s+2k_f)-2\phi(k_s-k_f)}{(k_s+2k_f)+\phi(k_s-k_f)}\end{aligned}\right\} \tag{8}$$

where ϕ is the nanoparticles concentration, subscript s used for Nano-solid-particles, and subscript f used for base fluid.

The thermo-physical properties of water and the elements Fe_4O_3 shown in Table 1.

According to Rosseland approximation, the radiative heat flux simplified as [4,18]:

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial r} \tag{9}$$

where $\sigma^*$and k^* are the Stefen-Boltzman constant and the mean absorption coefficient respectively. The term T^4 can be obtained as a linear function of temperature. Thereby, Taylor series used to expanding T^4 about T_∞ and neglecting higher order terms, one can get

$$T^4\cong 4TT_\infty^3-3T_\infty^4 \tag{10}$$

Introducing the non-dimensional variables

$$\eta=\frac{r}{\ell},\ u=\frac{Us}{\ell}f^{'}(\eta),\ v=-\frac{\mathscr{V}R}{R+r}f(\eta),\ w=\frac{Us}{\ell}g(\eta),\\ p=-\frac{u^2s^2\rho_f}{\ell^2}p(\eta),\ \theta(\eta)=-\left(\frac{T-T_\infty}{T_w-T_\infty}\right) \tag{11}$$

where η is the similarity variable. Substitute Eqs. (9)–(11)into Eqs. (1)–(5), the following system of non-linear ordinary differential equations obtained in the following form:

$$f^{iv}+\left(\frac{2}{\eta+k}\right)f^{'''}-\frac{f^{''}}{(\eta+k)^2}+\frac{f^{'}}{(\eta+k)^3}+A_1R_e\left(\frac{k}{\eta+k}\right)[ff^{'''}-f^{'}f^{''}]+A_1R_e\frac{k}{(\eta+k)^2}[ff^{''}-f^{'2}]\\-A_1R_e\frac{k}{(\eta+k)^3}ff^{'}-\frac{A_1A_5R_eM}{A_2(1+m^2)}[f^{''}+mg^{'}]-\frac{A_1A_5R_eM}{A_2(1+m^2)(\eta+k)}[f^{'}+mg]=0 \tag{12}$$

$$g^{''}+\left(\frac{g^{'}}{\eta+k}\right)+A_1R_e\frac{k}{(\eta+k)}[fg^{'}-f^{'}g]+\frac{A_1A_5R_eM}{A_2(1+m^2)}[mf^{'}-g]=0 \tag{13}$$

$$\frac{A_6}{A_3}\left[\theta^{''}+\frac{\theta^{'}}{\eta+k}\right]+Rd[1+(T_r-1)\theta]^3\theta^{''}+\left(\frac{Rd}{\eta+k}\right)[1+(T_r-1)\theta]^3\theta^{'}+PrR_e\left(\frac{k}{\eta+k}\right)f\theta^{'}\\+3Rd(T_r-1)[1+(T_r-1)\theta]^2\theta^{'2}+\frac{A_5PrR_eMEc}{A_3}[f^{'2}+g^2]=0 \tag{14}$$

With the following boundary conditions:

$$at\ \eta=0;\ f(0)=0,\ f^{'}(0)=1,\ g(0)=0\ and\ \theta(0)=1 \tag{15}$$

$$at\ \eta=1;\ f(1)=1,\ f^{'}(1)=0,\ g(1)=0\ and\ \theta(1)=0 \tag{16}$$

Such that $k=\frac{R}{\ell}$ is the curvature parameter, $P_r=\frac{\nu_f(\rho C_p)_f}{k_f}$ is Prandtl number, $M=\frac{\sigma_f B_0{}^2\ell}{U\rho_f}$ is Hartman number, $E_C=\frac{\rho_f(U^2s^2)}{\ell^2(\rho C_p)_f T_H}$ is the Eckert number, $R_e=\frac{\ell U}{\nu_f}$ is Reynold number, $T_r=\frac{T_w}{T_\infty}$ is the temperature ratio and $Rd=\frac{16\sigma^* T_\infty^3}{3k_f k^*}$ is Radiation parameter.

And

$$A_1=(1-\phi)^{2.5}\left[1-\phi+\phi\frac{\rho_s}{\rho_f}\right],\ A_2=\left[1-\phi+\phi\frac{\rho_s}{\rho_f}\right],\ A_3=\frac{(\rho C_p)_{nf}}{(\rho C_p)_f},$$
$$A_4=(1-\phi)^{2.5}\frac{(\rho C_p)_{nf}}{(\rho C_p)_f},\ A_5=\frac{\sigma_{nf}}{\sigma_f}\ and\ A_6=\frac{k_{nf}}{k_f}$$

Once the velocity obtained, the pressure $p(\eta)$ can be obtained in the following form:

$$p(\eta)=\left(\frac{\eta+k}{kA_1R_e}\right)\left[f^{'''}+\frac{f^{''}}{(\eta+k)}-\frac{f^{'2}}{(\eta+k)^2}\right]+ff^{''}-f^{'2}+\frac{ff^{'}}{(\eta+k)}-\frac{A_5M}{A_2(1+m^2)}[f^{'}+mg] \tag{17}$$

The physical quantities of interest are the surface shear stress τ_{rs} and the transversal heat fluxq_w, which defined as:

$$\tau_{rs}=\mu_{nf}\left[\frac{du}{dr}-\frac{u}{r+R}\right]_{r=0},\ q_w=q_r-k_{nf}\left[\frac{\partial T}{\partial r}\right]_{r=0} \tag{18}$$

Therefore, the surface skin friction and the Nusselt number defined as:

$$C_f=\frac{\tau_{rs}}{u_w^2\rho_{nf}},\ N_{u_s}=\frac{sq_w}{k_f(T_w-T_\infty)} \tag{19}$$

$$\therefore\ \sqrt{R_{e_s}}C_f=\frac{1}{A_1}\left(f^{''}(0)-\frac{1}{k}\right),\ N_{u_s}=-[A_6+RdT_r{}^3]\theta^{'}(0) \tag{20}$$

Table 1
Thermo-physical properties of water and the elements Fe_3O_4 [17].

properties	fluid (water)	Fe_3O_4
$Cp(j/kgK)$	4179	670
$\rho(kg/m^3)$	997.1	5180
$k(W/mK)$	0.613	9.7
$\sigma(\Omega.\ m^{-1})$	0.05	25,000

3. Methodology of solution using DTM-Padé

Consider a function $f(i)$ which is analytic in a domain D and let $i = i_o$ represent any point in D. The function $f(i)$ is then represented by a power series whose center is located at i_o. The differential transform of the function $f(i)$ is given by

$$F(n) = \frac{1}{n!}\left[\frac{d^n f(i)}{di^n}\right]_{i=i_0} \tag{21}$$

where $f(i)$the original is function and$F(n)$ is the transformed function. The inverse transformation is defined by:

$$f(i) = \sum_{m=0}^{\infty}(i - i_0)^n F(n) \tag{22}$$

From Eq. (21) and (22), one can obtain

$$f(i) = \sum_{n=0}^{\infty}\frac{(i - i_0)^n}{n!}\left[\frac{d^n f(i)}{dx^n}\right]_{i=i_0} \tag{23}$$

It is worth mentioning that, the concept of differential transformation is derived from the Taylor series expansion, so the Eq. (23) can be rewritten as:

$$f(i) \cong \sum_{m=0}^{n}(i - i_0)^n F(i) \tag{24}$$

The Padé approximant technique has been applied, where converting the polynomial approximation into a ratio of two polynomials. It is useful to combine the series solution obtained by the DTM with the Padé approximant to provide an effective solution to handle our boundary-value problems and improve the converging of the results. Rashidi et al. [24] presented the DTM as approach method to solve the MHD stagnation-point flow in porous media problem. Rashidi et al. [25] investigated the nano boundary-layers over stretching surfaces using Padé approximant-DTM technique. El-Zahar et al. [26] used the differential transform method as an approximate analytical solution for singularly perturbed fourth order boundary value problems. Mosayebidorcheh et al. [27] employ the differential transform method to solve the boundary layer equation of the Power-Law Pseudo plastic Fluid. Sepasgozara et al. [28] they solved the problem of heat and mass transfer in a porous channel using differential transformation method. Mosayebidorcheh et al. [29] they used DTM technique to study the behavior of transient thermal on the radial fins of rectangular, triangular and hyperbolic profiles.

In this section, the Differential Transform Method (DTM) is used to obtain a series solution for the coupled system (12)-(14). By using the following transforms:

$$f(\eta) \to F[h],\quad f^{'}(\eta) \to (h+1)F[h+1],\quad f^{''}(\eta) \to (h+1)(h+2)F[h+2],$$
$$f^{'''}(\eta) \to (h+1)(h+2)(h+3)F[h+3],$$
$$f^{iv}(\eta) \to (h+1)(h+2)(h+3)(h+4)F[h+4] \tag{25}$$

$$g(\eta) \to G[h],\quad g^{'}(\eta) \to (h+1)G[h+1],\quad g^{''}(\eta) \to (h+1)(h+2)G[h+2] \tag{26}$$

$$\theta(\eta) \to \Theta[h],\quad \theta^{'}(\eta) \to (h+1)\Theta[h+1],\quad \theta^{''}(\eta) \to (h+1)(h+2)\Theta[h+2] \tag{27}$$

where: $F[h]$, $G[h]$ and $\Theta[h]$ are the transformed functions of $f(\eta)$, $g(\eta)$ And $\theta(\eta)$, respectively, and are given by

$$f(\eta) = \sum_{h=0}^{\infty}F(h)\,\eta^h,\; g(\eta) = \sum_{h=0}^{\infty}G(h)\,\eta^h \; and\; \theta(\eta) = \sum_{h=0}^{\infty}\Theta(h)\,\eta^h \tag{28}$$

With initial conditions:

Table 2
Values of skin friction coefficient and the Nusselt number at.$\phi = 0$, $M = 2.25$, Re = 5, $m = 1$, Rd = 0.3, $T_r = 2.5$, Pr = 7, Ec = 0

k	present results		Abbas [19]	
	$-\sqrt{R_{es}}C_f$	N_u	$-\sqrt{R_{es}}C_f$	N_u
1	3.4029	10.5891	3.4029	10.5892
2	3.1711	10.1545	3.1711	10.1546

$$F(0) = 0,\; F(1) = 1,\; F(2) = a,\; F(3) = b,\; G(0) = 0,\; G(1) = c,\; \Theta(0) = 1,\; \Theta(1) = d$$

Substituting by the above transforms into Eqs. (12)–(14), the solution of the system (28) obtained as a function of the unknown initial conditions a, b, $c$and d with assistance of mathematica program. To get the values of these constants a Padé approximation technique applied with the governing conditions $at\ \eta \to 1;\ F = 1,\ F^{'} = 0,\ G = 0\ and\ \theta = 0$ using FindRoot built-in function in mathematica program the required symbols can be obtained.

In order to check the accuracy of the method used, Table 2 presents a comparison with the numerical method that used in Abbas et al. [19]

4. Results and discussion

Flow and heat transfer of Ferro-nanofluid runs through a curved tube under the influence of hall current and thermal radiative flux examined in this research. The effect of some parameter such as (Reynold number, Hartman number, nanoparticles concentration, etc.) on the velocities, pressure and the temperature evident through a set of figures.

Fig. 2 depicts the influence of Reynold number on the velocity; it is clear that the increasing of Reynold number decreases the velocity just to $\eta = 0.3$ and then return to increase. The same behavior appears in the presence of hall current with shifting of the critical point towards the center. The main reason for dividing the flow into two zones is the upper permeable side of the tube which generates a suction force increases the velocity. The increasing of the Reynold number means increasing of the random motion of the particles which increases the normal velocity as shown in Fig. 3.

Moreover, the presence of the hall current produce a normal force called Lorentz force which increases the normal velocity. it is evident from Fig. 4 that the magnitude of the pressure within the tube decreases due to the increase of the Reynold number, and the presence of the hall current cause more reduction in the pressure. Fig. 5 show that the increasing of Reynold number has a direct effected on the internal temperature of the tube such that the temperature increases as the Reynold

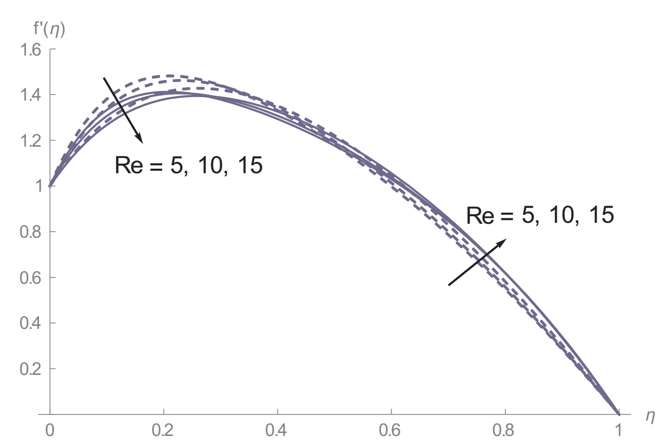

Fig. 2. Transverse velocity profiles with the variation of Reynold number at K = 2, M = 2, Rd = 0.3, T_r = 1.5, Pr = 7, Ec = 0.2.

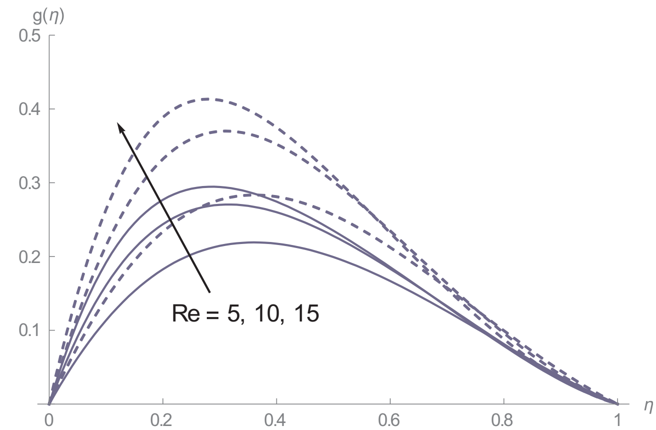

Fig. 3. Normal velocity profiles with the variation of Reynold number at K = 2, M = 2, Rd = 0.3, T_r = 1.5, Pr = 7, Ec = 0.2.

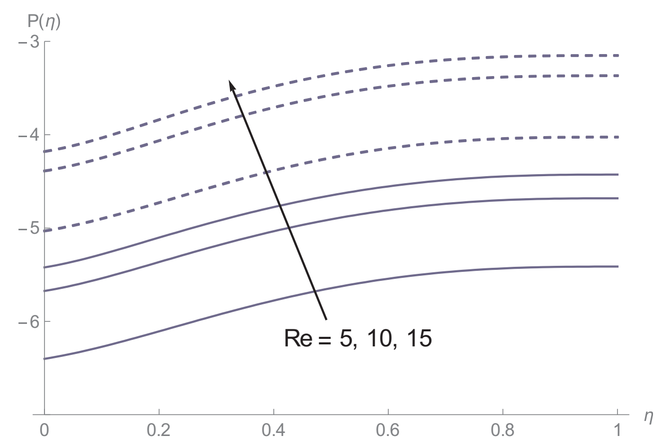

Fig. 4. Pressure profiles with the variation of Reynold number at K = 2, M = 2, Rd = 0.3, T_r = 1.5, Pr = 7, Ec = 0.2.

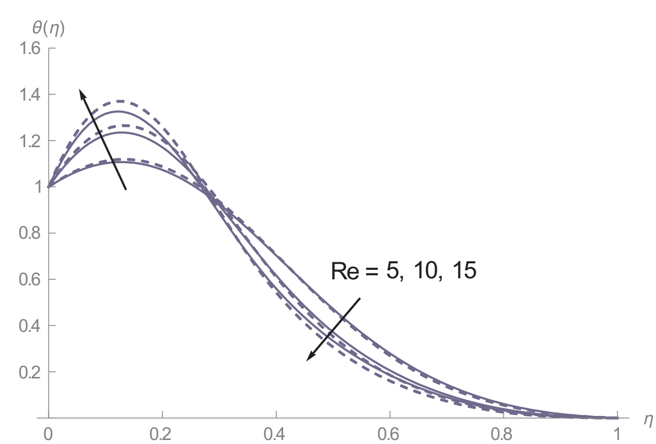

Fig. 5. Temperature profiles with the variation of Reynold number at K = 2, M = 2, Rd = 0.3, T_r = 1.5, Pr = 7, Ec = 0.2.

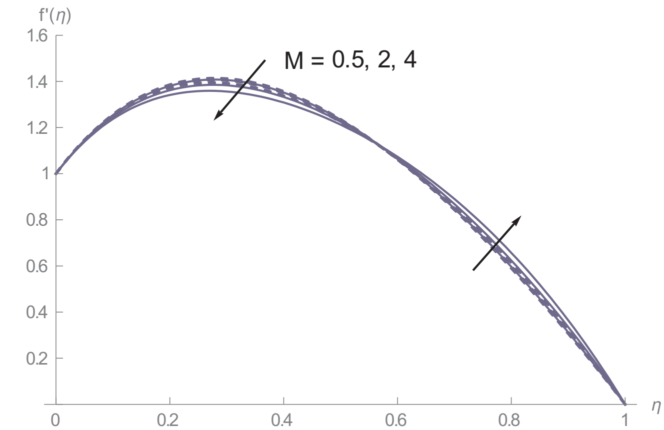

Fig. 6. Transverse velocity profiles with the variation of Hartman number at K = 2, M = 0.5, Re = 3, Rd = 0.3, Tr = 1.5, Pr = 7, Ec = 0.2.

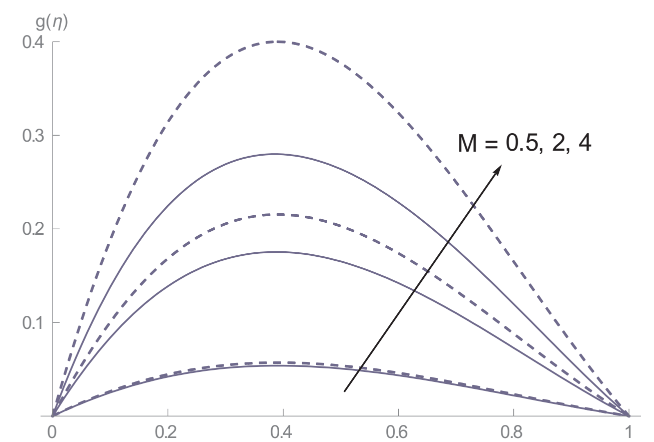

Fig. 7. Normal velocity profiles with the variation of Hartman number at K = 2, M = 0.5, Re = 3, Rd = 0.3, Tr = 1.5, Pr = 7, Ec = 0.2.

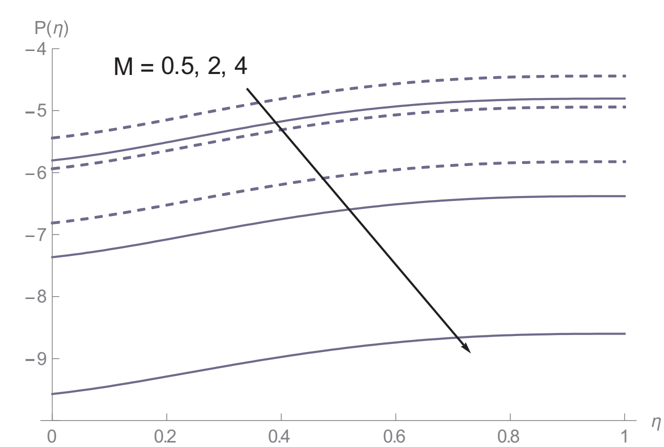

Fig. 8. Pressure profiles with the variation of Hartman number at K = 2, M = 0.5, Re = 3, Rd = 0.3, Tr = 1.5, Pr = 7, Ec = 0.2.

number increases just on the impermeable side of the tube then reverses to decreases near the permeable side.

The influence of Magnetohydrodynamic flow under the presence/absence of hall current on the velocities, pressure and the temperature shown through figures Figs. 6–9 9. The effect of magnetic field is clearer on the normal velocity than the longitudinal velocity especially under the high values and the presence of hall current this is due to the normal forces which produces (Lorentz force) such that Fig. 6 show that the velocity increases by increase the magnetic field and the opposite is true for the normal velocity as depicted in Fig. 7. On the other hand, Figs. 8 and 9 depict that the magnetic field increases the magnitude of the pressure and the temperature, also one can observe that the effect of the hall current (dashed lines) be more clearer at the high values of the magnetic parameter. furthermore, the magnitude of the pressure reduces in the presence of hall current at the same value of magnetic parameter, This may occurs as a result of particles dispersion in the

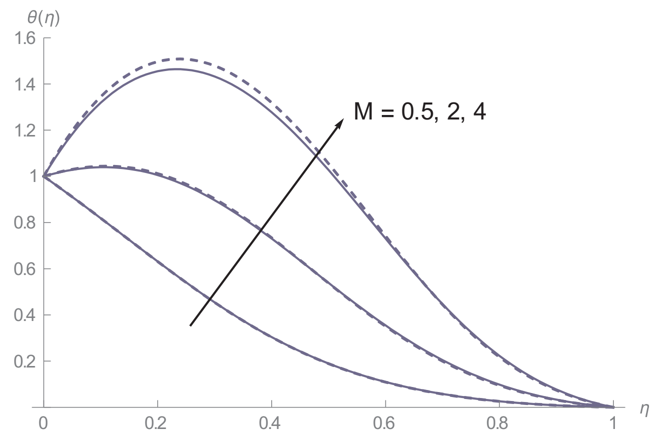

Fig. 9. Temperature profiles with the variation of Hartman number at K = 2, M = 0.5, Re = 3, Rd = 0.3, Tr = 1.5, Pr = 7, Ec = 0.2.

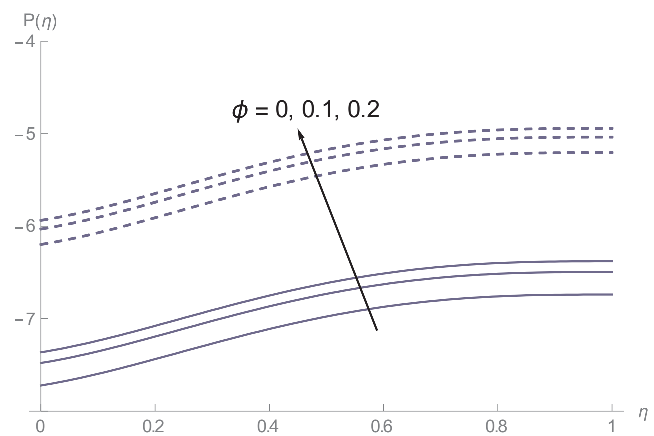

Fig. 10. Pressure profiles with the variation of nanoparticles concentration at K = 2, M = 2, Re = 3, Rd = 0.3, Tr = 1.5, Pr = 7, Ec = 0.2.

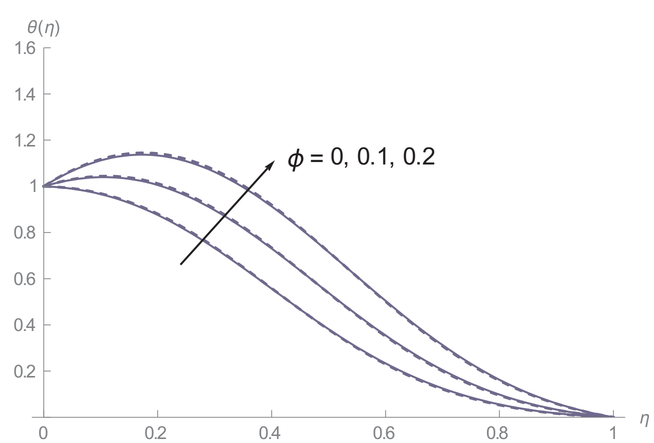

Fig. 11. Temperature profiles with the variation of nanoparticles concentration at K = 2, M = 2, Re = 3, Rd = 0.3, Tr = 1.5, Pr = 7, Ec = 0.2.

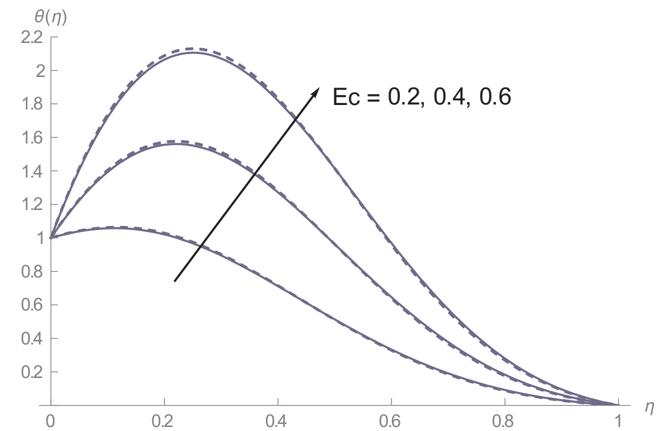

Fig. 12. Temperature profiles with the variation of Eckert number at K = 2, M = 2, Re = 3, m = 0.5, Rd = 0.3, Tr = 1.2, Pr = 7.

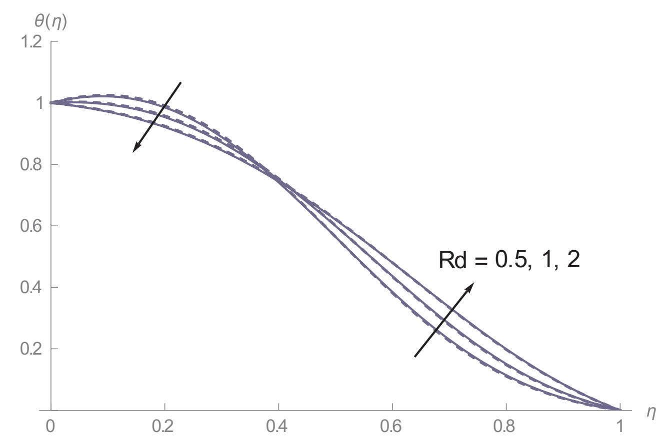

Fig. 13. Temperature profiles with the variation of radiation parameter at K = 2, M = 2, Re = 3, m = 2, Tr = 1.5, Pr = 7, Ec = 0.2.

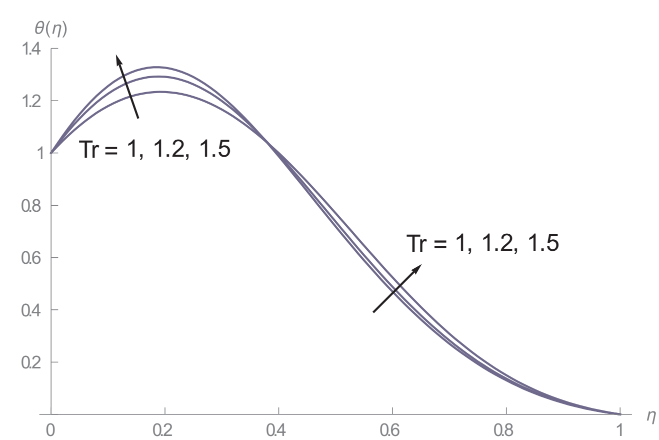

Fig. 14. Temperature profiles with the variation of temperature ratio at K = 2, M = 2, Re = 3, m = 2, Rd = 0.3, Pr = 7, Ec = 0.2.

normal direction.

The influence of nanoparticles concentration on the pressure and temperature is evident through Figs. 10 and 11. It is observed that, the magnitude of the pressure decreases as the nanoparticles concentration increase, this reduction in pressure be more in the presence of hall current. This is due to the increasing of the nanosize particles which increase the density of the nanofluid which leads to decreasing of the kinematic viscosity of the fluid. It is also clear that the value of pressure increases gradually as we get closer to the permeable surface due to the suction force which generated at this side of the tube.

Furthermore, the presence of hall current under strong magnetic field improves the electrical conductivity of fluid which increases as a result of nanoparticle increment. On the other hand, Fig. 11 show that the temperature of the flow increases as the percentage of nanoparticles increase due to the increment of the heat capacity of the mixing flow.

The temperature profiles under the influence of Eckert number, radiation parameter and temperature ratio are depicted through Figs. 12–14. The relationship between flow's kinetic energy and the

Table 3
Values of skin friction coefficient and the Nusselt number for various values of m, k *and* ϕ at.$M = 2$, $\text{Re} = 5$, $Rd = 0.3$, $T_r = 1.2$, $\text{Pr} = 6.2$, $\text{Ec} = 0.2$.

m	k	ϕ	$-\sqrt{R_{es}}C_f$	N_u
0	1	0.00	3.29958	1.02709
		0.05	3.17587	3.96995
		0.10	3.12857	8.57213
	2	0.00	3.02596	0.11143
		0.05	2.90417	2.24289
		0.10	2.85789	6.03751
2	1	0.00	3.36547	1.63181
		0.05	3.24626	4.87230
		0.10	3.20059	9.89685
	2	0.00	3.22334	0.37348
		0.05	3.09793	2.97137
		0.10	3.05001	7.11497

boundary layer enthalpy difference is known as the Eckert number, this ratio is used to characterize heat dissipation. It is clear that the increasing of this ratio increases the temperature due to the improving of the kinetic energy. Furthermore, the temperature decreases under the increasing of the radiation parameter just to $\eta = 0.4$ then return to increase near the permeable side, the opposite behavior occurs on the temperature under the increasing of temperature ratio.

Table 3 presents the values of the skin friction coefficient and the Nusselt number at the inner side of the tube for different values of curvature parameter, nanoparticles concentration and the hall current. Initially, the presence of nanoparticles decreases the value of skin friction; such that at 5% concentration the skin friction decreases by 3.5–4% this may occurs due to the decreasing of the kinematic viscosity of the fluid as a result of increasing of concentration. Actually, the concentration is not the only the factor that controls the skin friction; the curvature parameter has a direct effect on it. Such that one can observe that by increasing the curvature of the tube, the skin friction decreases. Furthermore, the presence of hall current increases the kinetic energy of the flow which produces high skin friction.

Nusselt number is a dimensionless factor refers to the heat flux from the lower side into the tube under the influence of temperature difference between the two sides. Table 3 showed the effect of the hall current and nanoparticles concentration on the heat flux or Nusselt number. By Investigation the numerical values of Nusselt number shown in Table 3, one can observe that the Nusselt number increases in the presence of hall current. Furthermore, it is clear that increasing the nanoparticles concentration increases the heat flux or Nusselt number such that raising the concentration to 5% leads to an increase in the heat flux by 200–250%,

Physically this raising is due to the increasing of thermal conductivity of the flow. Moreover, the curvature of the tube also has a direct effect on the heat flux or Nusselt number such that the analysis of Table 3 refer to decreasing of the Nusselt number by increasing the curvature parameter.

5. Conclusion

This work discussed the effect of hall current and thermal radiation on the flow and heat transfer of a ferrofluid runs through a curved tube. The results obtained confirm that the hall current has a clear effect on the flow behavior such that the Presence of the hall current reduces the pressure of the flow under any condition of the embedded parameters and produce a normal force which generates a velocity in the normal direction to the tube. Moreover, the heat flux shows more than the normal in the presence of hall current. On the other hand, the effect of the nanoparticles concentration and the curvature of the tube on the fluid during its movement inside the tube have been studied. The results have shown that the presence of nanoparticles decrease the skin friction on the tube surface especially for the tubes of high curvature, also the rate of heat transfer is also clearly affected by the presence of nanoparticles and the curvature of the tube such that the heat flux improved in the presence of nanoparticles and decreases by increasing the tube curvature.

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Future works

The author and his team deal to study the flow of the blood through the micro tubes

References

[1] S.U.S. Choi, J.A. Eastman, Enhancing conductivity of fluids with Nanoparticles, ASME Fluid Eng. Div. 231 (1995) 99–105.

[2] E.M.A. Elbashbeshy, T.G. Emam, M.S. Abdel-wahed, Flow and heat transfer over a moving surface with nonlinear velocity and variable thickness in a nanofluid in the presence of thermal radiation, Can. J. Phys. 92 (2014) 124–130.

[3] M.S. Abdel-wahed, E.M.A. Elbashbeshy, T.G. Emam, Flow and heat transfer over a moving surface with non-linear velocity and variable thickness in nanofluids in the presence of Brownian motion, Appl. Math. Comput. 254 (2015) 49–62.

[4] M.S. Abdel-wahed, Nonlinear Rosseland thermal radiation and magnetic field effects on flow and heat transfer over a moving surface with variable thickness in a nanofluid, Can. J. Phys. 95 (2017) 267–273.

[5] M.S. Abdel-wahed, Flow and heat transfer of a weak concentration micropolar-nanofluid over steady/unsteady-moving surface, Appl. Phys. A 123 (2017) 195.

[6] R. Muthuraj, S. Srinivas, A.K. Shukla, T.R. Ramamohan, Effects of thermal-diffusion, diffusion-thermo, and space porosity on MHD mixed convective flow of micropolar fluid in a vertical channel with viscous dissipation, Heat Transfer-Asian Res. 43 (6) (2014).

[7] K. Kaladhar, Natural convection flow of couple stress fluid in a vertical channel with hall and joule heating effects, Procedia Eng. 127 (2015) 1071–1078.

[8] D. Lourdu, R. Muthuraj, R.K. Selvi, S. Srinivas, A.K. Shukla, The influence of thermophoretic particle deposition on fully developed MHD mixed convective flow in a vertical channel with thermal-diffusion and diffusion-thermo effects, Ain Shams Eng. J. 6 (2015) 671–681.

[9] M. Fakour, A. Vahabzadeh, D. Ganji, Scrutiny of mixed convection flow of a nanofluid in a vertical channel, Case Stud. Therm. Eng. 4 (2015) 15–23.

[10] K.V. Prasad, H. Vaidya, K. Vajravelu, MHD mixed convection heat transfer in a vertical channel with temperature-dependent transport properties, J. Appl. Fluid Mech. 8 (4) (2015) 693–701.

[11] R. Patra, S. Das, J.R. Nath, Radiation effect on MHD fully developed mixed convection in a vertical channel with asymmetric heating, J. Appl. Fluid Mech. 7 (3) (2014) 503–512.

[12] S. Das, R.N. Jana, O.D. Makinde, Transient natural convection in a vertical channel filled with nanofluids in the presence of thermal radiation, Alexandria Eng. J. 55 (2016) 253–262.

[13] M.S. Abdel-wahed, Lorentz force effect on mixed convection micropolar flow in a vertical conduit, Eur. Phys. J. Plus 132 (2017) 195.

[14] G. Aaiza, I. Khan, S. Shafie, A. Khalid, A. Khan, Heat transfer in MHD mixed convection flow of a ferrofluid along a vertical channel, PLoS ONE 10 (11) (2015) e0141213, , https://doi.org/10.1371/journal.pone.0141213.

[15] F. Selimefendigil, H.F. Oztop, Effect of a rotating cylinder in forced convection of ferrofluid over a backward facing step, Int. J. Heat Mass Transfer 71 (2014) 142–148.

[16] A. Malvandi, D.D. Ganji, Magnetic field effect on nanoparticles migration and heat transfer of water/alumina nanofluid in a channel, J. Magn. Magn. Mater. 362 (2014) 172–179.

[17] M.S. Abdel-wahed, Rotating Ferro-nanofluid over stretching plate under the effect of hall current and joule heating, J. Magn. Magn. Mater. 429 (2017) 287–293.

[18] S. Das, R.N. Jana, O.D. Makinde, Mixed convective Magnetohydrodynamic flow in a vertical channel filled with nanofluids, Eng. Sci. Technol., Int. J. 18 (2015) 244–255.

[19] Z. Abbas, M. Naveed, M. Sajid, Nonlinear radiative heat transfer and Hall effects on a viscous fluid in a semi-porous curved channel, AIP Adv. 5 (2015), https://doi.org/10.1063/1.4934582 107124.

[20] S.A. Noreen, E.N. Maraj, A.W. Butt, Copper nanoparticles impinging on a curved channel with compliant walls and peristalsis, Eur. Phys. J. Plus 129 (2014) 183.

[21] M. Naveed, Z. Abbas, M. Sajid, Hydromagnetic flow over an unsteady curved stretching surface, Int. J. Eng. Sci. Technol. 19 (2016) 841–845.

[22] Z. Abbas, M. Naveed, M. Sajid, Hydromagnetic slip flow of nanofluid over a curved stretching surface with heat generation and thermal radiation, J. Mol. Liq. 215 (2016) 756–762.

[23] M. Sajid, S.A. Iqbal, M. Naveed, Z. Abbas, Joule heating and Magnetohydrodynamic effects on ferrofluids (Fe_3O_4) flow in a semi-porous curved channel, J. Mol. Liq. 222

(2016) 1115–1120.
[24] M.M. Rashidi, E. Erfani, a new analytical study of MHD stagnation-point flow in porous media with heat transfer, Comput. Fluids 40 (2011) 172–178.
[25] M.M. Rashidi, E. Erfani, the modified differential transform method for investigating nano boundary-layers over stretching surfaces, Int. J. Numer. Meth. Heat Fluid Flow 21 (7) (2011) 864–883.
[26] E.R. El-Zahar, Approximate analytical solutions of singularly perturbed fourth order boundary value problems using differential transform method, J. King Saud Univ. – Sci. 25 (2013) 257–265.
[27] S. Mosayebidorcheh, Solution of the boundary layer equation of the power-law pseudo plastic fluid using differential transform method, Math. Prob. Eng. 685454 (2013).
[28] S. Sepasgozara, M. Faraji, P. Valipour, Application of differential transformation method (DTM) for heat and mass transfer in a porous channel, Propul. Power Res. 6 (1) (2017) 41–48.
[29] S. Mosayebidorcheh, M.R. Gorji, D.D. Ganji, T. Moayebidorcheh, O. Pourmehran, M. Biglarian, Transient thermal behavior of radial fins of rectangular, triangular and hyperbolic profiles with temperature-dependent properties using DTM-FDM, J. Cent. South Univ. 24 (3) (2017) 675–682.