Port Said Engineering Research Journal
Faculty of Engineering
Suez Canal University
Volume 1, No. 1, March 2003, pp., 39-51

A proposed local dry zenith delay model for GPS Measurements in Egypt, derived from surface pressure data

Ashraf El-Kutb Mousa*, Gamal S. El-Fiky*, Mostafa Rabah*, Ahmed Elhattab**

ABSTRACT

The principal limiting error in modern space geodesy techniques is the mis-modeling of the atmospheric delay. This delay, usually separated and referred as hydrostatic (dry) and wet, is described as the product of its value at the zenith and a function that scales its elevation dependence. Accordingly there are proposed models for the prediction of the zenith delay as well as for scaling this zenith value at other elevations. Most of the available dry zenith delay models are global ones. Such models do not satisfy the requirements of precise geodetic application for Egypt, so a new model is very needed for the Egyptian local meteorological conditions.

Current paper introduces a local dry zenith model using surface meteorological data. The new model is based on empirical least squares fitting of the expected delays as calculated using ray tracing. Meteorological data from nine locations evenly distributed over Egypt were used for this model. The results indicated that the zenith dry delay values could be calculated accurately using surface pressure. To test the designed model, additional two sites were used. The tests of the new model showed that it is accurate to about 1.0 mm and its performance meets the needed accuracy for the present precise geodetic application such as crustal deformation studies in Egypt.

Keywords: Hydrostatic delay, Local model for Egypt, surface pressure data, geodetic application

---

* *National Research Institute of Astronomy and Geophysics, Helwan, Cairo, Egypt.*

***Faculty of Engineering, Suez Canal University, Port-Said, Egypt.*

INTRODUCTION

The Global Positioning System (GPS) is a satellite-based positioning system now widely used for navigation and relative positioning. It operates in two microwave frequency bands and measures the time and phase differences of signals transmitted from Earth orbiting satellites and a ground receiver. The use of dual-band measurements allows the dispersive effects of the ionosphere to be effectively eliminated from the measurements, but the non-depressive delay due to the Earth's neutral atmosphere remains as one of the limiting error sources for the GPS system [1]. Since the refractivity along the ray path is not easily or economically measured, the delay must in general be calculated or estimated [2]. The delay is usually divided into two components, which have been designated the hydrostatic (dry) and wet components of the atmospheric delay [3]. The line of sight delay for each component is modeled as the product of the zenith delay and a mapping function that describes the elevation angle dependence of the delay. Currently, for space geodesy, the zenith dry delay is calculated a priori from models using the surface pressures [4, 5, 6]. However, most of the available zenith dry delay fails to satisfy the needed accuracy for precise geodetic applications in Egypt such as the vertical crustal deformation studies [7].

Davis [8] showed that the vertical component of the baseline is most seriously affected by mis-modeling error of the tropospheric delay. The primary reason for this is that an error in the vertical component induces an error in the model of the group delay, which is proportional to the sine of the elevation angle. Thus, it correlates highly with all forms of errors in the model for the tropospheric delay. Herring [9] have shown that the error in the estimated vertical coordinates is approximately one third of the error in the maximum path length correction due to tropospheric delay.

Mousa [7] used Ray-Tracing method for testing the available hydrostatic delay models of Saastamoinen, Davis, Hopfield and Baby. He used nine sites representing the different conditions in Egypt. His results showed that the Hopfield model is the best model among the available models for Egypt. However, all of the tested models, including Hopfield do not satisfy the requirements of precise geodetic application such as crustal deformation studies.

The aim of the present study is to develop a local zenith dry delay model for Egypt to meet the accuracy requirements of precise geodetic applications such as vertical crustal movement studies. The delays produced by ray tracing one year of atmospheric profiles for nine stations evenly distributed allover Egypt, were the input data for the model design. The new model is tested using two sites in Egypt. The error of the new introduced local model is always less that 1.0 mm.

2. Theoretical Analysis

A radio wave propagating through the earth's atmosphere experiences variation in the atmospheric refractivity along its path. This reduces the propagation speed and bends the ray path (Figure 1). The atmospheric delay **(ΔL)**, along the actual path traveled by the radio wave is given by [e.g. Mendes, 10]

$$\Delta L = \int_{S} n ds - L_G \qquad \textbf{(1)}$$

Where: *n* is the refractive index of the moist air, *S* is the actual path traveled, and *G* is the straight-line path from the satellite to the receiver point. It is more numerically convenient to express the above integral using refractivity [8]. Refractivity **(N)** is given as

$$\mathbf{N = (n-1) * 10^{-6}} \qquad \textbf{(2)}$$

Using equation (2), the atmospheric delay ΔL can be written as:

$$\Delta L = 10^{-6} \int_{S} N ds + (L_S - L_G) \qquad \textbf{(3)}$$

Where: the integration is along ***S*** (i.e., the actual path traveled). The first term in the right hand side of equation (3) is the refractivity error, while the last term represents the geometric range error, the difference in length between the straight line ***G*** and the curved path ***S***.

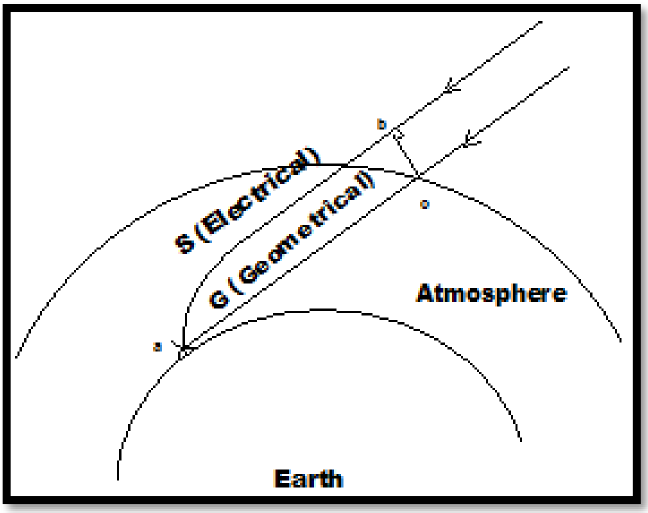

Figure (1): Schematic ray path of satellite signal for space geodetic measurement.

Anywhere in the troposphere, the refractivity, ***N***, can be expressed as [11]:

$$N = K1\frac{Pd}{T} + k2\frac{Pw}{T} + k3\frac{Pw}{T^2} \quad , \qquad \textbf{(4)}$$

Where: ***Pd***, and ***Pw*** are the partial pressures of the dry gases and water vapor respectively, in millibar and ***T*** is the absolute temperature in Kelvin. Several attempts have been made to calculate the constants ***k1***, ***K2*** and ***K3*** [12].

The first term of the right hand side of equation (4) represents the dry part of refractivity, while the last two terms are the wet part. Using the equation of state of air, Davis et al. [3] presented a more compact form for refractivity

$$N = k1R_d\rho + k2'\frac{Pw}{T} + k3\frac{Pw}{T^2} \qquad \textbf{(5)}$$

Where ρ is the total mass density of the air, and ***k2'*** is a modified version of ***k2***. The first term of the right hand side of equation (5) is called the hydrostatic refractivity, while the other two form the wet refractivity. It is important here to understand the difference between equation (4) and (5). Since in equation (5), a part of the water vapor influence has been absorbed in the hydrostatic delay. Throughout the current study, the two terms hydrostatic and dry will be used interchangeably, but the meaning refer to the hydrostatic definition.

As the hydrostatic delay **(ΔL_h)** is our main concern here, it can be expressed by:

$$\Delta L_h = 10^{-6} \int k1 R_d \rho ds \qquad \textbf{(6)}$$

2.1 Hydrostatic zenith delay

Using equation (6), the hydrostatic delay in the zenith direction **(ΔL^z_h)** is obtained by replacing the path elements ***ds*** with the vertical elements ***dz***

$$\Delta L_h^z = 10^{-6} \int k1 R_d \rho dz \qquad \textbf{(7)}$$

Equetion (7) can be integrated using the condition that hydrostatic equilibrium is satisfied, i.e.

$$dp/dz = - \rho (z) g(z) \qquad \textbf{(8)}$$

where **g(z)** is the acceleration due to gravity at the vertical coordinate ***z***, ***P(z)*** is the total pressure.

$$\Delta L_h^z = 10^{-6} K1 R_d \frac{Ps}{gm} \qquad \textbf{(9)}$$

Where ***gm*** is the mean gravity acceleration and ***Ps*** is the surface pressure.

Based on this theoretical approach, several models were developed by Saatamoinen [13], Davis et al. [3], and Baby et al. [6]. On the other hand, Hopfield developed her model for zenith dry delay based on the quartic refractivity profile. However, the evaluation of these models showed that they are not accurate for precise geodetic application in Egypt [7].

3. Data Analysis

Nine station distributed allover Egypt are used for designing of the new model. These stations are chosen to represent different climate conditions. They are also mostly distributed evenly to cover Egypt as seen in Figure (2). Moreover, their locations are chosen in active areas where precise geodetic networks are established for crustal movement studies. The station coordinates are listed in Table (1). For every station, the monthly average values of the meteorological parameters are used. This results in 108 point for the model design.

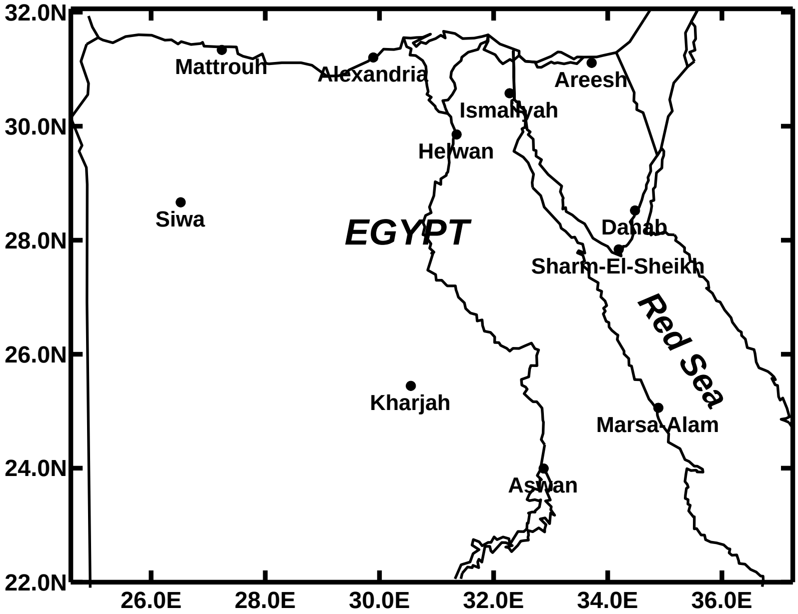

Figure (2):Geographical map for Egypt showing the distribution of the sites used for designing and testing the new model in this study.

The MSIS model [14] is used to provide the vertical profiles of temperature and pressure that is needed for the ray tracing analysis. The average monthly value of the temperature and pressure are used. Ray tracing is the process of determining the path of an electromagnetic signal, based on geometric optics theory applied over a series of thin spherical shells, concentric with the earth, and within which a constant refractivity is assumed [15]. For the current analysis, layer thickness of 100 m is used.

Table (1) Geodetic Coordinates in WGS 84 of the used station for the Design and Testing of the New Model

Coordinates City	**Latitude** (deg.)	**Longitude** (deg.)	**Height** (m.)
Areesh	31.11802	33.71026	35.479
Alex	31.21264	29.88461	29.343
Mattrouh	31.34558	27.23088	57.961
Sharm	27.84643	34.18375	257.090
Helwan	29.86191	31.34436	146.170
Siwa	28.67216	26.51038	18.600
Marsa Alam	25.06681	34.87807	43.827
Aswan	24.00198	32.86900	186.950
Kharjah	25.45169	30.54236	96.300
Ismailyah	30.58515	32.27210	24.511
Dahab	28.52881	34.46997	142.356

First, the dry refractivity is calculated from the temperature and pressure values at each layer using the first part of equation (5). Then the ray tracing is carried out numerically to integrate equation (3) in the vertical direction. Here, we have to note that the geometrical delay, the term (LS – LG) in equation (3), is zero in the zenith direction. The results of the ray tracing are considered the true values of the dry delay.

Figure (3) clearly indicates that the ray traced zenith delays show very high correlation with surface pressure. The correlation coefficient was found to be 0.9995. Thus a linear relation exists between the dry delay and surface pressure. Therefore, the new model for the Egyptian meteorological conditions takes the form:

$$\Delta L^{z}_{h} = a + b * P, \qquad (10)$$

With *a* and *b* as constant parameters. These two constant are estimated from the data fitting to be a = 35.482674 (± 0.00009) mm and b = 2.2460948 (± 0.008) mm/mbar, respectively.

3.1 Model Testing and discussion

Although the refractivity at any point in a dry atmosphere depends on the pressure and the temperature (the ratio *P/T* in Eq. 4), the dry delay is a linear function of the pressure only. This is theoretically correct since the ration (*P/T*) is equivalent to density, and the integral of density with height yields surface pressure [16]. Thus the introduced local model in this study is a semi-empirical model as it has the above theoretical basis but produced with least squares fitting of data.

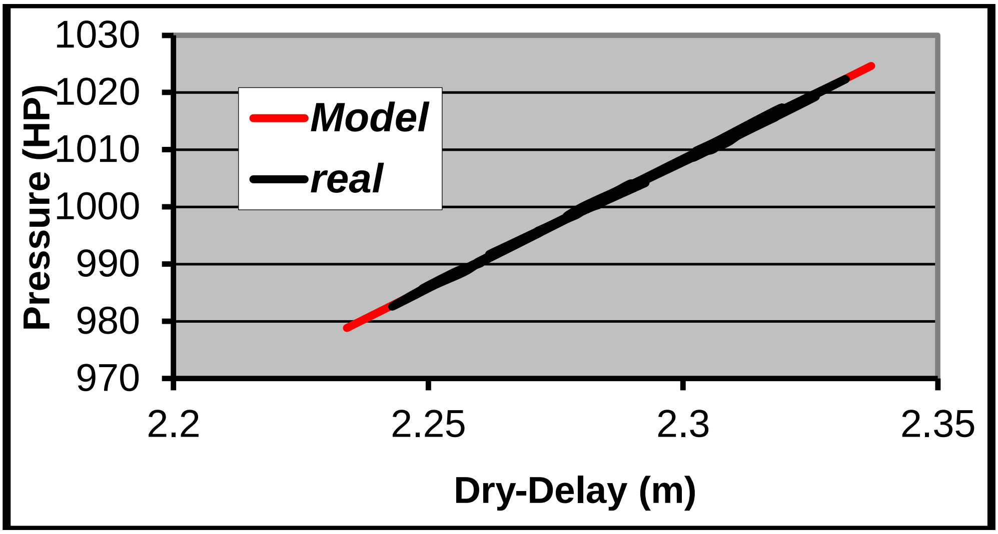

Figure (3). Correlation between Zenith dry delay and surface pressure.

It is clear from Figure (3) that the error of new local model is very small for the nine sites used in the design due to high correlation. An addition testing was preformed using another two sites at Ismailyah and Dahab to check the error of the estimated model. The new sites location and geodetic coordinates are given in Figure (2) and Table (1) respectively. The results of this testing are showed in Figures (4) and (5). Figure (4) represents the errors of the new model against the ray tracing data at the Ismailyah site while Figure (5) shows it against the ray tracing data at the Dahab site. Both figures show clearly that the errors are always less than the 1.0 mm level needed for precise geodetic applications. On the other side, the new model performance is checked against Hopfield model that is considered to be the best available model for Egypt as given by Mousa, [7]. It is very clear that the new local model developed in this study is much better than that of Hopfield. The errors of Hopfield model reach about 3.0 mm for the Ismailyah and about 2.0 mm for the Dahab sites.

The capability of GPS surveying to estimate the crustal deformation and other precise geodetic application such as geoid determination has open up a new era for the precise geodetic applications [e.g., 17, 18]. No longer are accurate geodetic measurements limited to scalar strain rates or narrow zones of deformation. The challenge ahead is to use this technology in an optimal way in the study of interesting problems in geodesy or geophysics. This is not an easy task, since both GPS hardware and software are developing so rapidly. The receiver, and analysis algorithms used in last year's campaign are not necessarily appropriate for this year's.

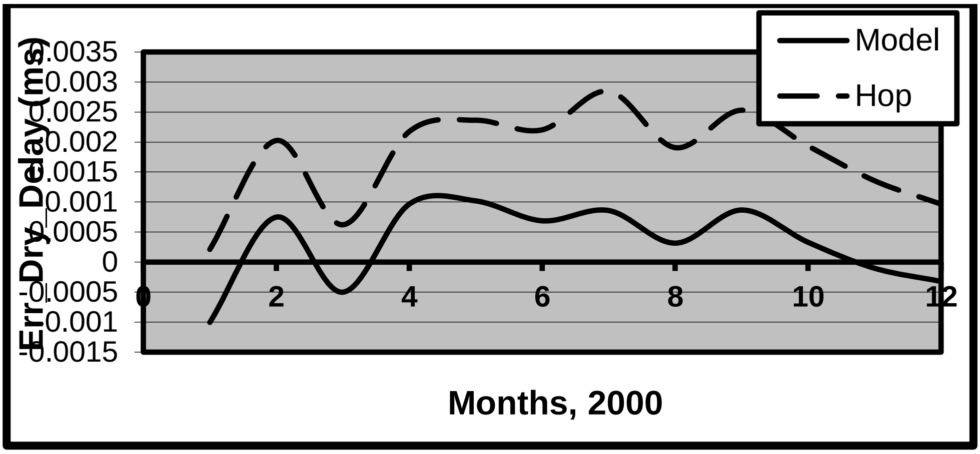

Figure (4) The new Zenith dry delay models errors in mm for one year at the Ismailyah site against the errors of Hopfield model. Straight line indicates the new models and the dashed one indicates the Hopfield one.

On the other hand, the accuracy obtainable from GPS depends strongly on the capabilities and reliability of the receiver and antennas, as well as on the sophistication of the analysis software [19]. As it is known, receiver should be dual frequency and track at least six satellites simultaneously. Antennas should have well-defined phase centers at both frequencies and be as unaffected as possible to signal multipathing. In the analysis software, there are several reasonable approaches to structure, but the most important five are: (1) accurate modeling of satellite motions and the relative positions of the satellite and receiver antennas, (2) the ability of estimating corrections to site coordinates and orbital parameters from phase observations that are free of clock errors, (3) both automatic and interactive data editing, (4) algorithms for resolution of phase ambiguities, and (5) modeling of atmospheric fluctuations. In the present study, we have introduced a new model for the Egyptian meteorological conditions. By which, we can get the required accuracy for our precise geodetic applications. On the other hand the previous global studies have shown that incorporating such developed model in this study into the geodetic parameter estimation improves the RMS scatters of the parameter estimates at sites which are able to make measurements at elevation at least as low as 10°. This will encourage the Egyptian geodesists to use the GPS measurements in the precise application such as vertical crustal movement studies and geoid determination using GPS/ Leveling nets.

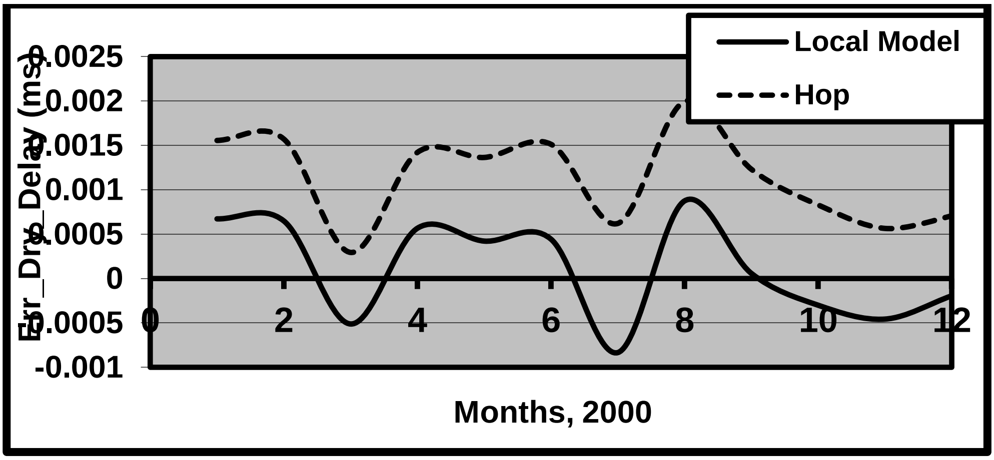

Figure (5): The new Zenith dry delay models errors in mm at the Dahab site against the errors of Hopfield model. Straight line indicates the new models and the dashed one indicates the Hopfield one.

4. Conclusions and Recommendations

Neutral atmospheric delay is the main source of error for GPS and other space geodetic techniques. The available hydrostatic delay models did not meet the precision limit needed for precise applications such as vertical crustal movements in Egypt. A local model is designed in the present study using the ray tracing of meteorological data at well-distributed nine sites all over Egypt. The new model is found to be a linear function of the surface pressure.

Testing of the new model using two sites, other than the used in the design, shows that the new model errors are always less than the 1.0 mm level that are required for precise application. On the other hand, performance of new model developed in this study found to be much better than that of Hopfield model. Such a good performance is very encouraging and thus we recommend using the new introduced model in Egypt when working with precise applications such as vertical crustal movements and geoid determination using GPS/ Leveling nets.

Acknowledgments

We thank Prof. Ali Tealeb, the president of National Research Institute of Astronomy and Geophysics, Helwan, Cairo, for his kind help and valuable discussions.

Reference

[1] Niell, A. E. (1996). " Global Mapping Function for the Atmospheric Delay at Radio Wavelengths." Journal of Geophysical Research, Vol. 101, No. B2 pp. 3227-3246.

[2] Mousa, A. K. (1997). "Characteristics of Wet Tropospheric Delay Deduced From Water Vapor Radiometer Data and Their Implications for GPS Baseline Solution Accuracy. " Ph.D Thesis, Graduate School of Science, Kyoto University, Kyoto, Japan.

[3] Davis, J. L., T.A. Herring, I. I. Shapiro, A.E.E. Rogers, and G. Elgered (1985). "Geodesy by Radio Interferometry: Effects of Atmospheric Modeling Errors on Estimates of Baseline Length." Radio Science, Vol. 96, No. B1, pp. 643-650.

[4] Hopfield, H. S. (1969). "Two-Quadratic Tropospheric Refractivity Profile For Correcting Satellite Data." Journal of Geophysical Research, Vol. 74, No. 18, pp. 4487- 4499.

[5] Saastamoinen, J. (1972). " Atmospheric Correction for the Troposphere and Stratosphere in Radio Ranging of Satellites." in The Use of Artificial Satellites for Geodesy (S. W. Henriksen et al, eds.), Geophysics Monograph Series, Vol. 15, pp. 274-251, AGU.

[6] Baby, H.B., P. Gole, and J. Laverbnat (1988). “A Model for the Tropospheric Excess Path Length of Radio Waves From Surface Meteorological Measurements”. Radio Science, Vol. 23, pp. 1023- 1038.

[7] Mousa, A. K. (2002). “Assessment of Zenith Hydrostatic Delay Models for the Egyptian Climate.” Paper submitted to Alexandria Engineering Journal, Faculty of Eng., Alex. Univ., Egypt.

[8] Davis, J.L. (1986) “Atmospheric Propagation Effects on Radio Interferometry” Ph. D. Dissertation. Air Force Geophysics Laboratory Technical Report AFGL- TR-86-0243, Hanscom AFB, MA.

[9] Herring, T. A. (1992). “Modeling Atmospheric Delays in the Analysis of Space Geodetic Data.” Symposium of Transatmospheric Signals in Geodesy, Netherlands Geod. Commis. Ser. 36, 1992, pp. 157-164, Ned. Comm. Voor Geod., Delft.

[10] Mendes, V.B. (1999). “Modeling The Neutral-Atmospheric Propagation Delay in Radiometric Space Techniques.” Ph. D. Dissertation, Department of Geodesy and Geomatics Engineering Technical Report No. 199, University of New Brunswick, Fredericton, New Brunswick, Canada.

[11] Smith, E. K. and S. Weintraub (1953). “The Constants in the Equation of Atmospheric Refractive Index at Radio Frequencies.” Proceedings of the Institute of Radio Engineers, Vol. 41, No. 8, pp. 1035-1037.

[12] Bean, B.R. and E.J. Dutton (1966). “Radio Meteorology.” National Bureau of Standards Monograph 92, U.S. Government Printing Office, Washington, D.C.

[13] Saastamoinen, J. (1973). “ Contributions to the Theory of Atmospheric Refraction.” In three parts. Bulletin Geodesique, No. 105, pp 278-298; No. 106, pp. 383-397; No. 107, pp 13-34.

[14] Hedin, A.E. (1991). “Extension of the MSIS Thermosphereic Model into Middle and Lower Atmosphere.” JGR, Vol. 96, No. A2, pp. 1159-1172

[15] Kerr, D.E. (Ed.) (1953). “Propagation of Short Radiowaves.” McGraw-Hill, New York.

[16] Hopfield, H.S. (1971). “Tropospheric Effect on Electro-magnetically Measured Range: Prediction From Surface Weather Data.” Radio Science, vol. 6, No. 3, pp. 357- 367.

[17] Segall, P. and J. L. Davis (1997). " GPS Applications for Geodynamic and Earthquake Studies." Annu. Rev. Earth Planet. Sci., 25, 301-336.

[18] El-Fiky, G. S., T. Kato, and N. Oware, (1999). "Crustal Deformation and Interplate Coupling in the Shikoku District, Japan, as Seen from Continuous GPS Observation". Tectonophs., 314, 387-399.

[19] Bradford, H. H., R. W. King., and M. H. Murray (1991). "Measurements of Crustal Deformation Using the Global Positioning System." Annu, Rev. Earth Planet. Sci., 19, 351-382.