Fast Simple Image Encryption Technique Based on Chaos Based System

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***Abstract*—In this paper, a novel image encryption technique based on chaotic maps is described. Encrypting an image with chaotic maps ensures that the safe cipher characteristics of confusion and diffusion are met. Moreover, chaos-based image cryptosystems have many features compared to the classical cryptographic techniques such as fast, effective security, minimum computing overheads, extremely sensitive to the control parameters and initial settings. These all features make effective encryption techniques with the best performance. Four chaotic maps are used in the proposed scheme to generate the encryption key, initial vector, and ciphering encryption S-Boxes. The maps used are the Chebyshev map, 1D improved quadratic map, 1D Improved Logistic Map and Altered Sine-Logistic based Tent map ASLT.**

Keywords— Chaotic Map, Encryption, Confusion, Diffusion, Chebyshev map, Logistic Map, and Altered Sine-Logistic based Tent map.

I. INTRODUCTION

The development of encryption algorithms based on chaotic dynamics is one of the most promising fields of current cryptography. The main idea of the image encryption algorithm based on the chaotic system is the ability of these systems to generate numbers sequence with chaotic behavior [1, 2]. Any minor changes in the initial condition or control parameters values used in the system results in a completely different sequence. Because of their sensitivity to beginning conditions, chaotic systems are ideal for encryption

II. CHAOTIC MAPS

The chaotic behavior of the chaotic map can be studied by analyzing the bifurcation diagram and Lyapunov exponent. The bifurcation diagram defines the relation between the control parameters and the system and the maximal Lyapunov exponent value defines the dependence of the dynamical system on the initial values [3, 4]. The Lyapunov exponent for 1-D chaotic map is defined as follows.

$$\lambda = \lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} ln|f'(x_i)| \qquad (1)$$

With MLE > 0, the system is extremely sensitive to initial values, whereas for MLE =0, the system is stable. The higher value of λ the, the better the chaotic behavior of the system.

A. The Chebyshev map

The Chebyshev map is a one-dimensional map having a starting condition, x_0, and a bifurcation parameter μ, and is characterized by the following formula [5].

$$x_{i+1} = \left(\mu cos^{-1}(x_i)\right) \qquad (2)$$

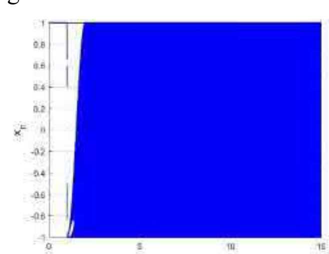
Where the value $\mu \in [2, \infty)$ and $x_i \in [1, -1]$. The dynamic behavior of the Chebyshev mapping is closely related to the control parameter μ. For different value of μ the system will represent different characteristics. The chaotic behavior for Chebyshev map is shown in Figure 1.

B. 1D-IQM

The new modification of the quadratic map is known as 1D improved quadratic map and is defined as follows [6].

$$x_{i+1} = mod\left(\frac{\left((\mu - x_i^2)\times 2^K\right)}{sin(x_i)^R}, 1\right) \qquad (3)$$

Where the initial control parameters values are μ∋[0,10],R∋[1,3],K∋[0,10]. The chaotic behavior of the map is shown in Figure 2.



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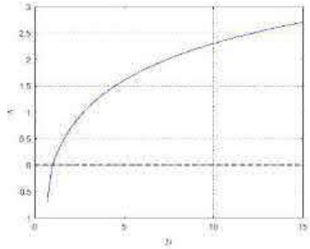
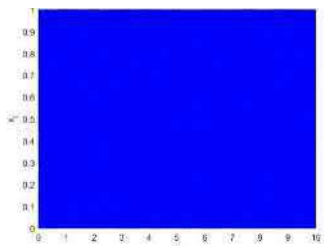


(b)

Figure 1 Bifurcation plot (a) and(b) Lyapunov plot of Chebyshev



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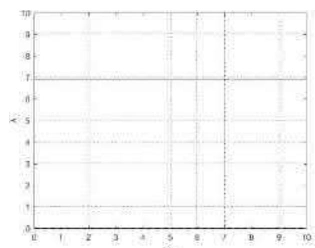


(b)

Figure 2 Bifurcation plot (a) and(b) Lyapunov plot of 1D-IQM

C. Altered Sine-Logistic based Tent map ASLT

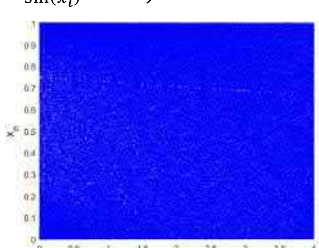
The map is a hybrid combination from the modified versions of the sin and logistic map [7]. The map is defined as below and its chaotic behavior is shown in Figure 3

$$x_{i+1} = \begin{cases} \frac{4-\mu}{4}\sin(\pi x_i) + \frac{\mu}{2}x_i & x_i < 0.5 \\ (4-\mu)x_i(1-x_i) + \frac{r}{2}(1-x_i) & x_i \geq 0.5 \end{cases} \quad (4)$$

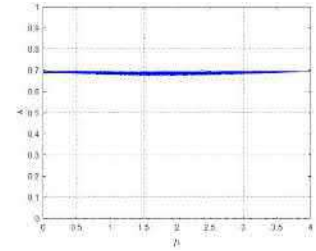
D. 1D Improved Logistic Map

The map is defined as below and its chaotic behavior is shown in Figure 4.

$$x_{i+1} = \text{mod}\left(\frac{(\mu x_i(1-x_i))2^K}{\sin(x_i)^R}, 1\right) \quad (5)$$

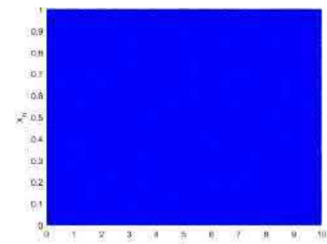


(a)

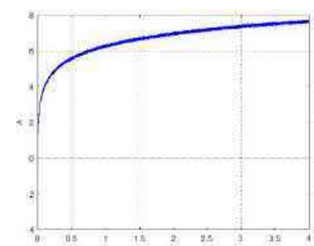


(b)

Figure 3 Bifurcation plot (a) and(b) Lyapunov plot of ASLT



(a)



(b)

Figure 4 Bifurcation plot (a) and(b) Lyapunov plot of Improved Logistic Map

III. BLOCK DIAGRAM OF THE CHAOS-BASED IMAGE CRYPTOSYSTEM

The block diagram consists of two basic phases as shown in Figure 5. The first one is the confusion phase, also known as the pixel permutation, in which the locations of pixels are rearranged over the whole image, but the value of the pixels stay constant, transferring the image into unrecognizable form. The permutation of the pixels is done to reduce the correlation between the values of neighboring elements image. The second phase following the confusion phase is known as the diffusion phase. Since the confusion phase is insufficiently secure and may be readily hacked by an attacker. As a result, when the diffusion phase is executed based on the chaotic map sequence generator, the values of the pixels in the entire image are also modified to increase the level of the system security [8-10].

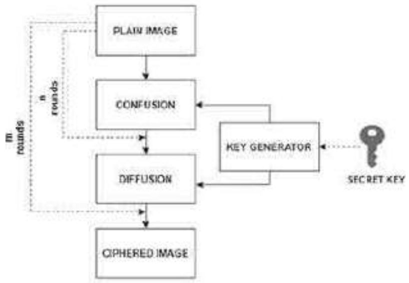


Figure 5 Block diagram of the chaos-based image cryptosystem

IV. The Proposed System

In the proposed system, the permutation stage, the diffusion stage, and key generator of the proposed modified CBC system is shown in Figure 6.

The algorithm used to generate the encryption key and IV is presented in the following steps.

1. Read the size of plain image M, N.
2. Set the initial chaotic control parameter value μ
3. Set the initial state of sequence number x_0
4. Initialize i=1.
5. Do while i<=M*N.
6. Apply Chebyshev equation for encryption key evaluation and ASLT equation for IV evaluation.
7. Evaluate the next state x_(i+1)
8. Adjustment the values
9. i=i+1
10. Go to step 5.

The algorithm used to generate the block cipher encryption boxes Sbox1 and Sbox2 is presented in the following steps.

1. Set the initial chaotic control parameter value k, R, μ
2. Set the initial state of sequence number x_0
3. Initialize i=1
4. Do while i<=N
5. Apply 1D-IQM equation for Sbox1 evaluation and 1D Improved Logistic Map equation for Sbox2.
6. Evaluate the next state x_{i+1}

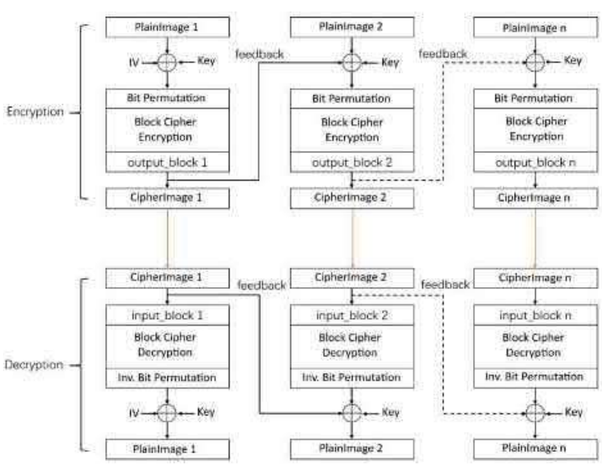


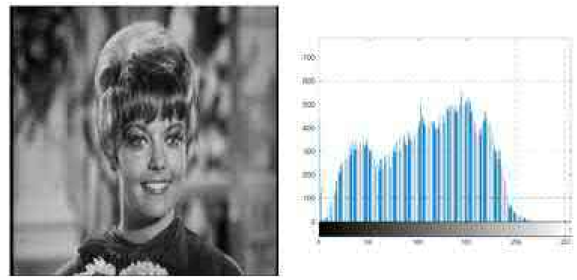
Figure 6 Block diagram of the proposed scheme

V. Statistical Attack Analysis

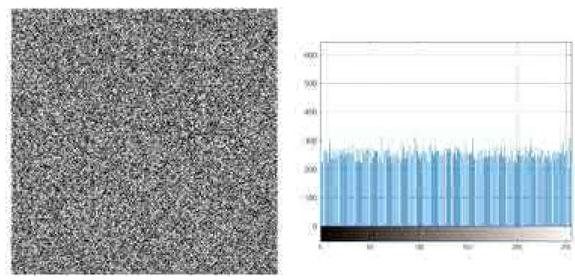
The statistical attack analysis of the plain image and the ciphered image can be defined by the following

A. Histogram

Histogram of the ciphered image is considered the simplest method to evaluate the quality level of the encryption method. As a result, the ciphered image's histogram should have a uniform distribution to protect against statistical attacks. The histogram analysis for the plain images and the ciphered images are shown in Figures 7-9

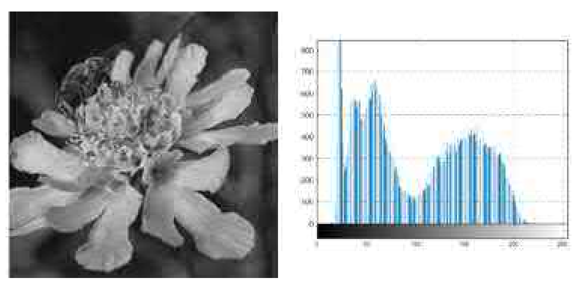


(a)

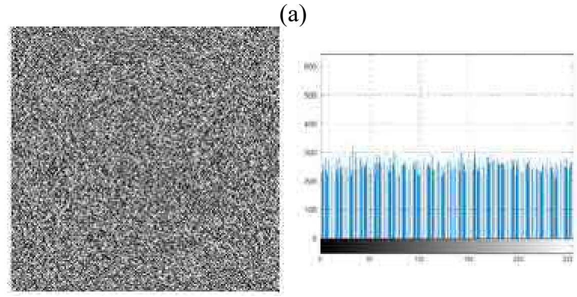


(b)

Figure 7. a) Zelda plain image and its histogram, b) Ciphered image and its histogram

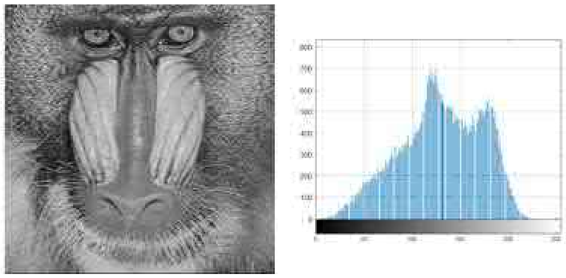


(a)

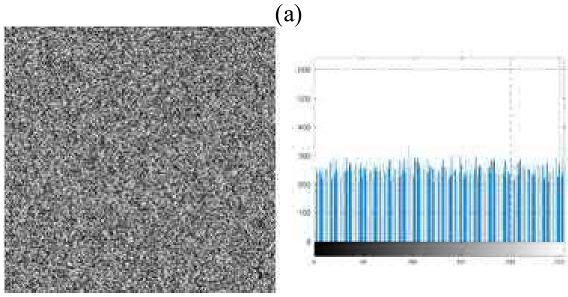


(b)

Figure 8. Flower plain image and its histogram, b) Ciphered image and its histogram



(a)



(b)

Figure 9. a) Baboon plain image and its histogram, b) Ciphered image and its histogram

B. Adjacent pixels Correlations

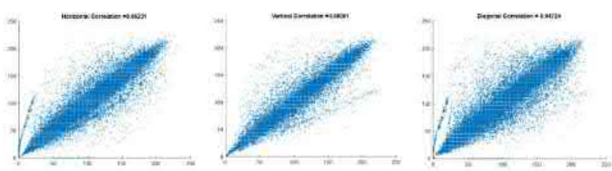
The goodness of the proposed encryption technique is based on the reduction of the correlation coefficients among the adjacent pixels of the enciphered image [11-12]. The correlation coefficients are computed as follows.

$$corr_{xy} = \frac{cov(x,y)}{\sqrt{D(x)D(Y)}} \quad (6)$$

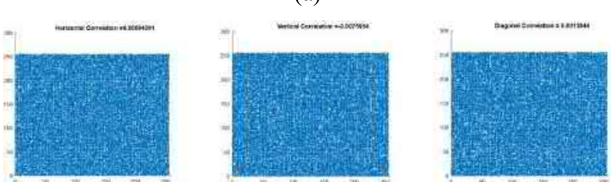
$$E(x) = \frac{1}{n}\sum_{i=1}^{n} x_i \quad (7)$$

$$D(x) = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - E(x)\right)^2 \quad (8)$$

Where D is the variance and E is the mean value. The correlation between various horizontally, vertically, and diagonally adjacent pixels of the plain and encrypted image is displayed in Figure 10.



(a)



(b)

Figure 10. Correlation coefficients for a) Kinda plain image and b) Ciphered image.

The following table summarizes the correlation coefficients for the plain images and ciphered images.

Table1. Correlation coefficients for the plain images and ciphered images

Image	Horizontal Ccorrelation		Vertical Correlation		Diagonal Correlation	
	Plain	Ciphered	Plain	Ciphered	Plain	Ciphered
Kenda	0.96231	0.000944	0.98081	-0.007595	0.94724	0.001584
Flower	0.97361	0.003174	0.97211	0.0046521	0.949365	-0.003383
Baboon	0.71686	-0.00225	0.60911	-0.001816	0.609176	0.001595

C. Information entropy analysis

Entropy is essential estimator factor for the quality of the encryption technique. The greater a ciphered image's entropy, the higher the system security achieved [14-15]. It is computed as follows.

$$\eta = -\sum_i P(x_i)\, log_2 P(x_i) \quad (9)$$

Where x_i is the pixel intensity value ranging from 0:255 and $P(x_i)$ is the probability of x_i occurrence within the entire image. The good encryption algorithm for the 8-level ciphered image should entropy value close to 8.

D. PSNR and SNR

Peak Signal to Noise Ratio (PSNR) and Signal to Noise Ratio (SNR) are used to calculate the image encryption quality. The change in the quality of the plain image in comparison to the ciphered image is defined as follows [16-17].

$$PSNR = 10log\frac{m^2}{MSE} = 10log\frac{m^2}{\sum_{i=0}^{N}\sum_{j=0}^{M}[P(i,j)-C(i,j)]^2} \quad (10)$$

Where m is the maximum value of the plain image and MSE is the mean square error between the original image P and the ciphered image C. The ideal quality of decrypted image is indicated by a high SNR value computed below.

$$NR = 10log\left(\frac{\sum_{i=0}^{N}\sum_{j=0}^{M}[P(i,j)]^2}{\sum_{i=0}^{N}\sum_{j=0}^{M}[P(i,j)-C(i,j)]^2}\right) \quad (11)$$

VI. DIFFERENTIAL ATTACK

Attackers frequently make a minor change to the plain image and use the suggested approach to encrypt it before and after the change, comparing two ciphered images to determine the relationship between the plain image and the ciphered one. It is known as differential attack. To determine the effect of changing one single pixel on the entire ciphered image, two factors are used NPCR (number of pixels change rate) and UACI (unified average changing intensity [18-19]. These are computed as follows.

$$NPCR = \frac{\sum_{i=1}^{M}\sum_{j=}^{N} D(i,j)}{M\ X\ N}\ X\ 100\% \quad (12)$$

$$D(i,j) = C_1(i,j) \oplus C_2(i,j)$$

$$UACI = \frac{1}{M\ X\ N}\left(\sum_{i=1}^{M}\sum_{j=}^{N}\frac{|E_1(i,j)-E_2(i,j)|}{255}\right)X100\% \quad (13)$$

Where the image size is defined by M and N and the two ciphered images are defined by C_1 and C_2
The different analysis results of the plain and ciphered images are shown in the following table.

Table 2. Analysis Results of plain and ciphered images

Image	Entropy		Differential	
	Plain	Ciphered	UACI	NPCR
Kenda	7.5149	7.995	32.7	98.71
Flower	7.382	7.995	32.8	98.45
Baboon	7.331	7.994	32.7	98.42

Conclusion

This paper presents new encryption method based on the chaotic system. The encryption method is mainly dependable on the initial conditions and the control parameter of the chaotic maps used. The proposed algorithm using chaotic maps is faster for image encryption and decryption techniques compared to the non-chaotic image encryption.

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