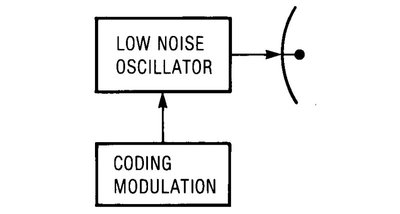
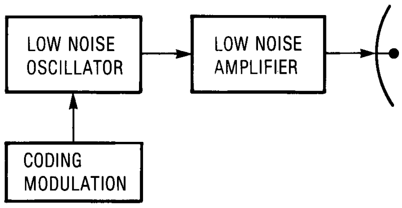
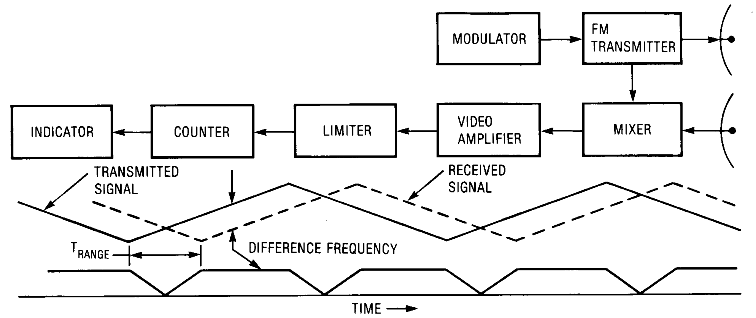
Chapter 13
Continuous Wave Radar



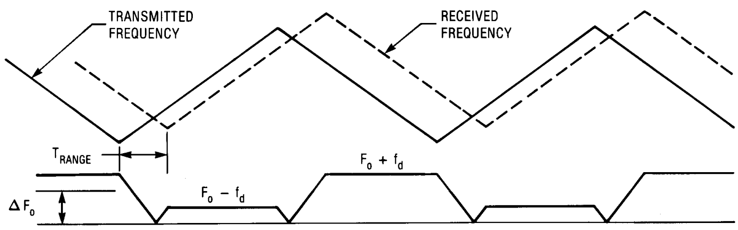
(a) LOW POWER ILLUMINATOR



(b) HIGH POWER ILLUMINATOR



FM-CW range measuring radar block diagram.



Doppler effect on the difference frequency in an FM-CW radar.

Radar System Design

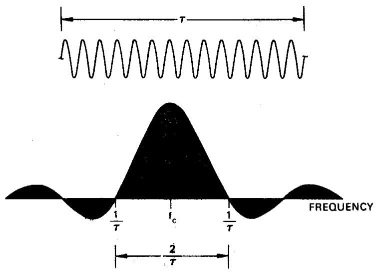
Radar Types

CW systems

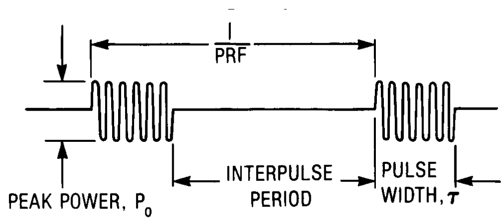
- CW radar
  - No range information
  - single target
  - Unambiguous velocity information
- FM-CW systems
  - measure both range and velocity
  - broaden the transmitted freq. spectrum

Pulsed Systems

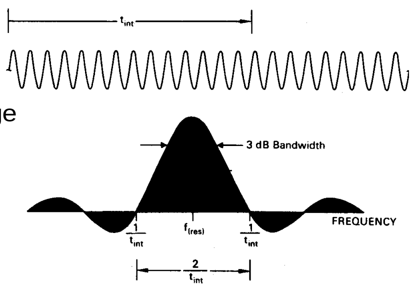
- Pulsed radar
  - measurement of range
- Pulsed Doppler radar
  - measure both range and velocity



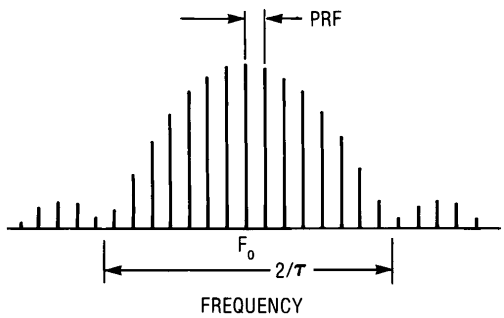
Spectrum of sinusoidal signal of duration, τ.



Pulse modulation terms.



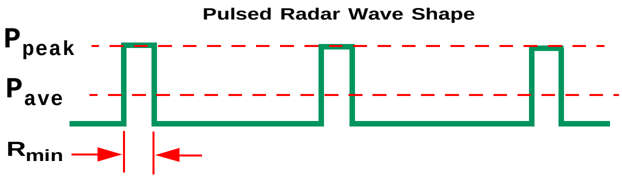
Output characteristic of a narrowband filter to which a signal at least as long as the filter integration time, t_{int} is applied.

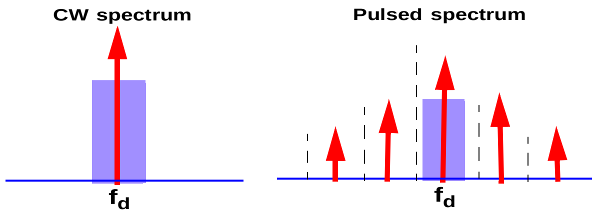


Pulsed radar frequency spectrum.

CW Radar

- Primary useful where no range information is required
- Advantage of CW radar
  - Simplicity; Smaller and lighter
  - Peak transmit is equal to average transmit power → TX is lower than the peak power of a pulsed radar; no high voltage modulators are required for simple CW radar; radar ability to detect targets is determined by the average power
  - Good for short range application; pulsed radars use TR switch or tube to protect receiver → echo returns from short-range targets will not reach the receiver → these targets will not be detected. CW radars do not use TR tubes; TX/ RX isolation is achieved by using other types of duplexer (ferrite circulators) or FM tech.
  - It is generally simpler to extract Doppler information for a CW system than from a pulsed system. Pulsed radar requires additional signal processing (gate filter, delay canceller, FFT)





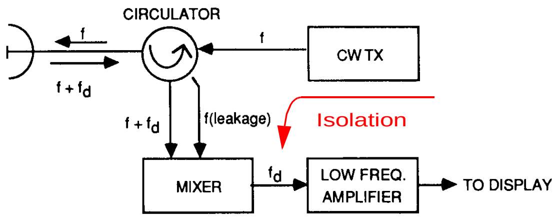
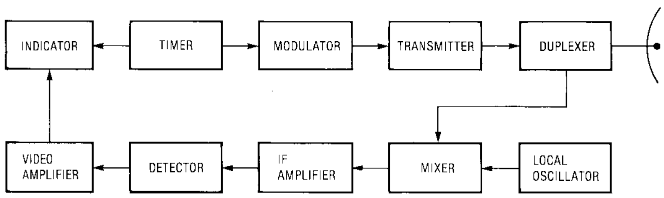


Figure 13-2. Simple CW radar system with a homodyne receiver.

CW Radar

- Disadvantage of CW radar
  - No target range formation for a simple radar ability to determine target range is poor
  - Rather poor TX/ RX isolation
  - can be overcome using proper CW waveform design
- Common applications
  - Simple (encoded), no range information: Weapon fuses, weapon seeker, lightweight portable personnel detector, police radar
  - Radar aircraft altimeter: frequency CW radars capable of aircraft-to-terrain range determination.



Pulsed radar system block diagram.

Table 13-1. Target Parameters and the Waveform Parameters From Which They are Determined.

Target Parameters	*Waveform Parameters*
Presence and bearing	Amplitude
Range	Amplitude, frequency, phase
Size (RCS)	Amplitude
Classification	Amplitude, frequency, polarization
Radial speed	Frequency
Discrimination from clutter	Frequency, amplitude, polarization

Table 13-2. Comparison of CW Radar and Pulsed Radar.

Requirements	*CW Radar Advantage*	*Pulsed Radar Advantage*
Low hardware complexity	X	
High average power	X	
Short-range target detection	X	
Moving target discrimination	X	
Target range determination		X
High transmitter/receiver isolation		X

CW Radar and Doppler Effect

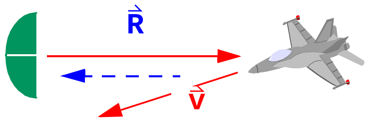
CW radar as a speed monitor device

- Doppler effect (frequency shift): only indicates f_d for targets moving toward or away from radar

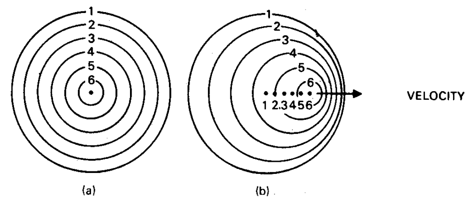
$$f_d = \frac{2v}{\lambda},\ v:\ \text{speed} \qquad c = f\lambda$$

- General form

$$f_d = \frac{2(\vec{v})}{\lambda} \cdot \frac{\vec{R}}{|\vec{R}|}$$



- f = 10G. v = 1mile/hour, f_d = 30Hz.



1. A wave radiated from a point source when stationary (a) and when moving (b). Wave is compressed in direction of motion, spread out in opposite direction, and unaffected in direction normal to motion.

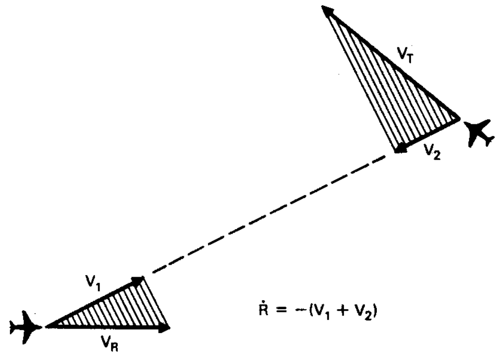


Table 13-3. Doppler Frequency Shifts (Hz) for Various Radar Frequency Bands and Target Speeds.

	Radial Target Speed		
Radar Frequency Band	1 m/s	1 knot	1 mph
L (1 GHz)	6.67	3.43	2.98
S (3 GHz)	20.0	10.3	8.94
C (5 GHz)	33.3	17.1	14.9
X (10 GHz)	66.7	34.3	29.8
K_u (16 GHz)	107	54.9	47.7
K_a (35 GHz)	233	120	104
mm (95 GHz)	633	326	283

Simple CW Radar Systems

Homodyne receiver

- TX/RX Isolation required of circulator: power level of the TX signal and sensitivity
  - HP series 35200 lower power, solid-state, X-band, Doppler radar: 18 dB of isolation is required
  - Circulators with 30 ~ 40 dB of isolation and higher have also been built
  - In high-power radars, more isolation may be required
  - Other isolation tech. such as dual antennas may have to be used.
  - major shortcoming → sensitivity: al low Doppler freq. flicker noise is very strong → amplify received signal at a high freq. → Superheterodyne receiver

Superheterodyne receiver

- IF ~ 60 MHz → flicker noise is negligible
- Baseband Filtering: sweeping LO + a single filter, analog filter bank (IF stage), digital filters or FFT processor (Baseband stage)

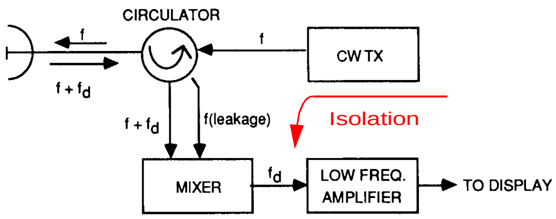


Figure 13-2. Simple CW radar system with a homodyne receiver.

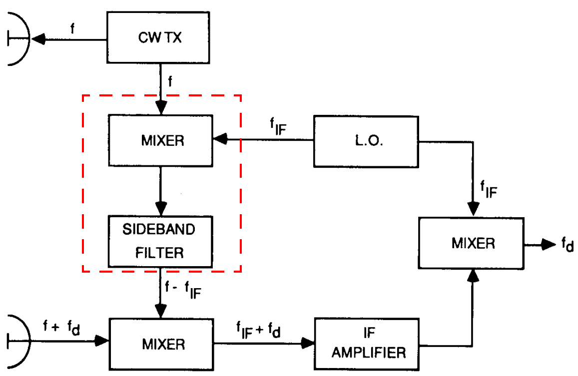


Figure 13-3. Simple CW radar system with a superheterodyne receiver. (Shown in a dual antenna configuration.)

CW Radar Doppler Ambiguity

- For infinite period of time, the received signal (ignoring amp. fixed-phase terms):

$$s(t) = \cos(w_0 \pm w_d)t$$
$$= \underbrace{\cos(w_d t)\cos(w_0 t)}_{I} \mp \underbrace{\sin(w_d t)\sin(w_0 t)}_{Q}$$

- Down to baseband (Video band)

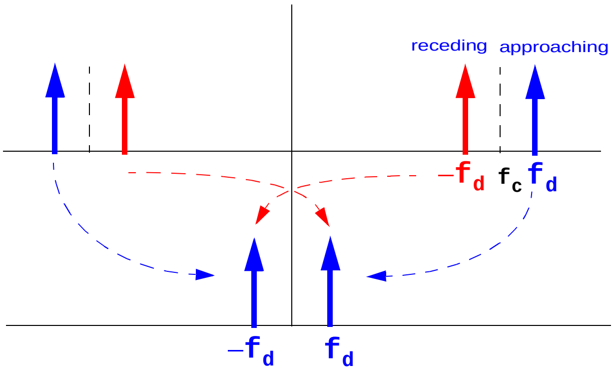
$$s(t) = \cos(\pm w_d)t = \cos(w_d)t,$$
$$S(f) = \frac{1}{2}\delta(f-f_d) + \frac{1}{2}\delta(f-f_d)$$

- Nothing additional is done, then the sign of the Doppler frequency shift will be lost ⇒ Relative target motion (approaching or receding) will be indeterminate.
- The problem of ambiguous relative motion IF signal into two channels, I and Q channels.
- $s(t) = \cos(w_d t) \pm j\sin(w_d t) = e^{\pm j2\pi f_d t}$,

$S(f) = \frac{1}{2}\delta(f \mp f_d)$, sign − for approaching

- Phase detection: DC output → FFT

$\sin(w_d t)[\pm j\sin(w_d t)] = \pm(1/2)$.



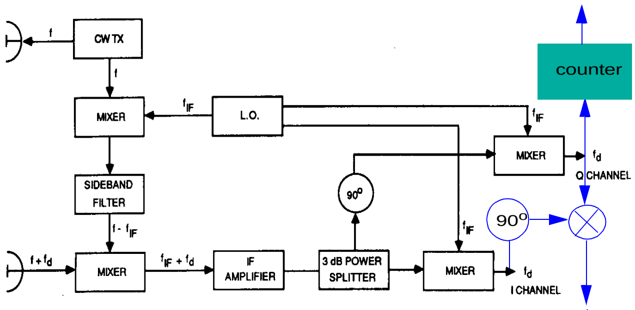


Figure 13-5. CW radar system with an I/Q detector.

CW Radar Spectrum and Resolution

Recall that we talked about spreading of the signal spectrum due to finite length signals from

- Finite time in beam, if θ_B : beamwidth (°) ; θ_s : scan rate (°/sec)

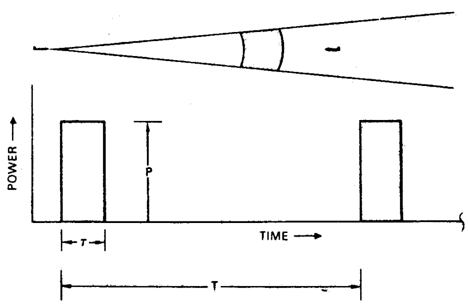
$$t_0 \cong \theta_B / \theta_s$$

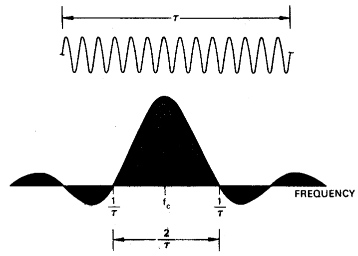
ex: $\theta_B = 2°, \theta_s = 36°/\text{sec}$,
$\Rightarrow t_0 \cong 2/36 = 1/18\,\text{sec}$

Bandwidth of doppler filter

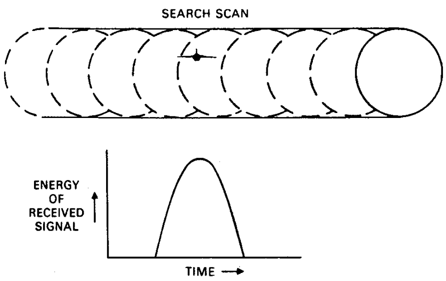
$$BW = 18\text{Hz sec}$$

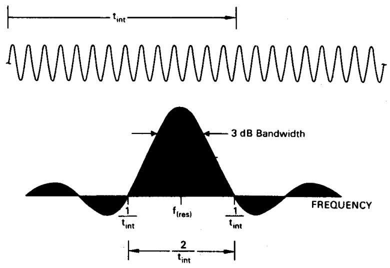
- Cross-section fluctuation during time target is in beam (effectively an amp. modulation)
- Moving Target components. e.g. propeller





Spectrum of sinusoidal signal of duration, τ.





Output characteristic of a narrowband filter to which a signal at least as long as the filter integration time, t_{int} is applied.

Doppler filtering

If we select the IF beamwidth so as to encompass all possible Doppler frequencies, the S/N will be poor. For example: ideally, we would like to use a matched filter

- Analog approaches to optimizing
- Could also use a single tunable BP filter that sweeps over the IF bandwidth.
- Simple digital filters
- Adaptive processing

None of these approaches by themselves will account for sign of f_d .

- Implement at IF or RF stage.

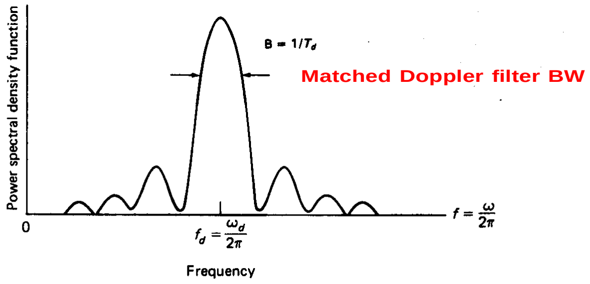
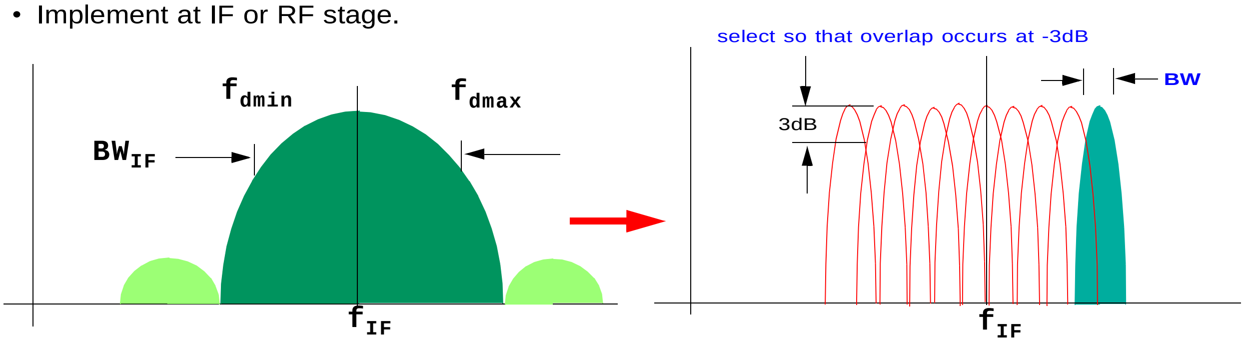
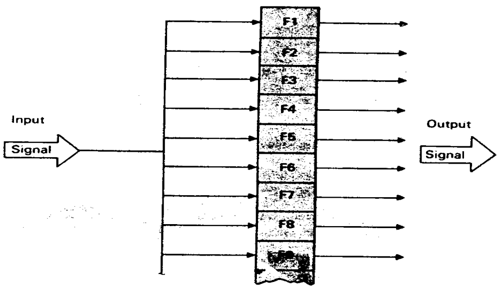


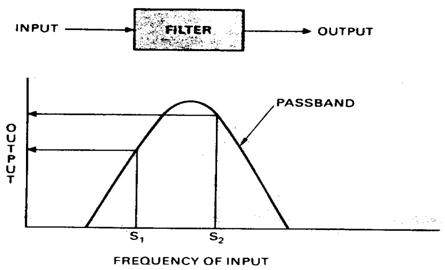
Figure 13-6. Spectral power density function of a signal from a moving target illuminated by a CW radar for a time T_0'.



Doppler filtering



1. The received signals are applied in parallel to a bank of filters.



2. Neglecting sidelobes, each filter passes only a narrow band of frequencies. The closer a signal is to the center frequency, e.g., S_2, the greater the output.

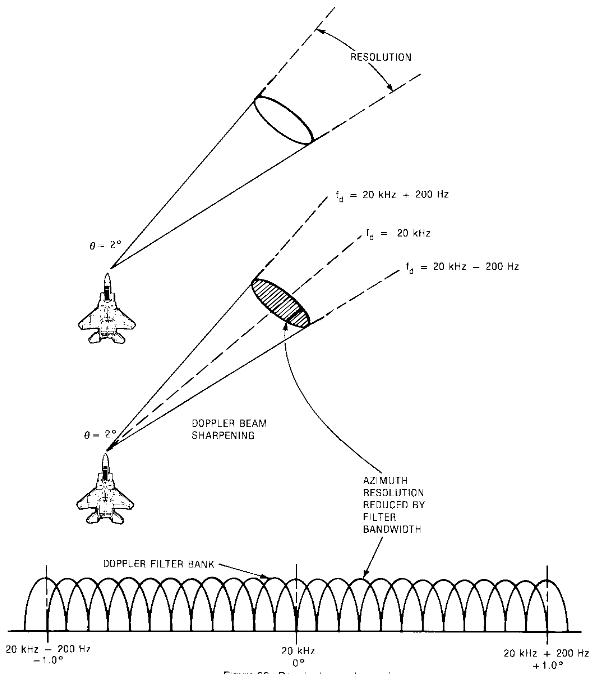
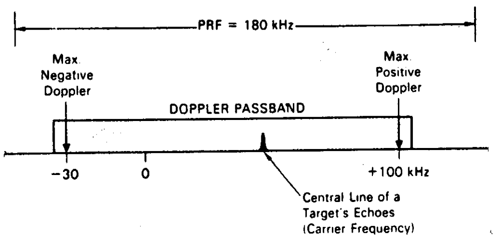


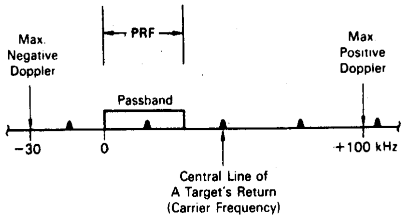
Figure 36. Doppler beam sharpening.

26

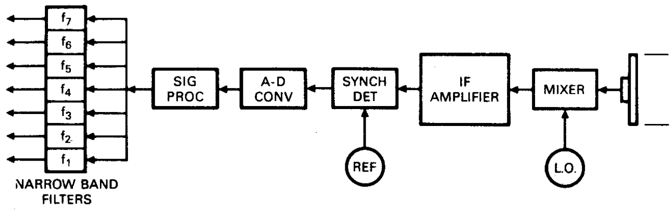
Doppler filtering and PRF

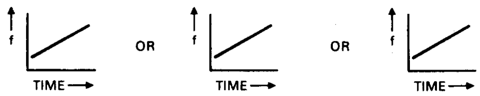


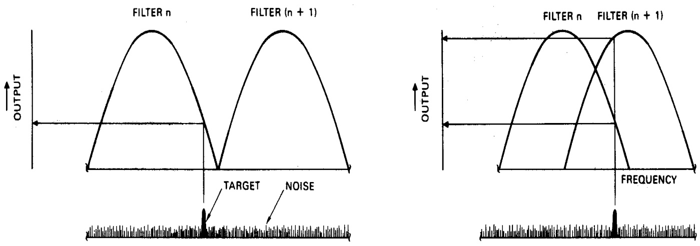
6. If the PRF is greater than the spread between the maximum positive and negative doppler frequencies, the doppler passband should be made wide enough to encompass these frequencies.



7. If the PRF is less than the spread of doppler frequencies, the passband should be made no wider than the PRF so that a target will appear at only one point within the band.







CW Radar Range Equation

A single Pulsed SNR

$$SNR = \frac{PG^2\lambda^2\sigma}{(4\pi)^3R^4kTBFL}$$

- Single pulse
- integrated over a dwell period
  - Coherent or incoherent pulsed radar
  - CW Radar ($\sim 1/\tau$)
- **P**: Peak power P_t. Single pulsed power $= P_t\tau$
- B: IF bandwidth, $B_{IF} \sim 1/\tau$ for a matched filter

Integrated Pulse SNR

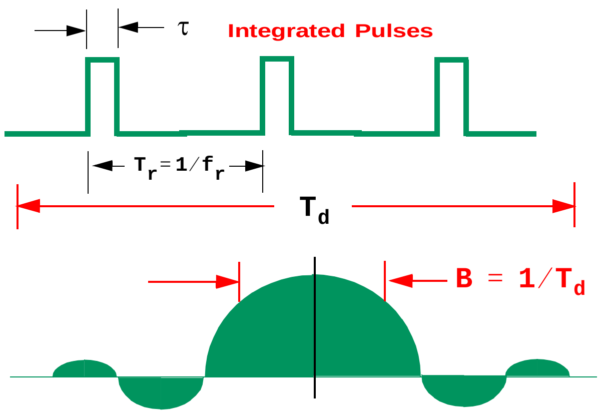
- integrated SNR during a target dwell period, T_d.
- Number of Pulses, n, during T_d. $n = f_rT_d$,
  $f_r = 1/T_r = PRF$ (Pulse repetition freq.)
- $P = P_{avg} = P_tf_r\tau$
- $B = B_{vid} = 1/T_d$

- $S/N = n(S/N)_{single\ pulse}$

$$SNR = \frac{(P_t\tau f_r)G^2\lambda^2\sigma}{(4\pi)^3R^4kT(1/T_d)FL}$$

For CW, $P = P_{avg}$, $B = B_{vid} = 1/T_d$

- For a single (I) channel CW → 3dB loss



CW Radar Maximum Range

Example: Consider a CW police radar with single channel. Dual antenna → a single antenna, a circulator

- $P = 100mW$, $G = 20dB$, $f = 10.525G$, $\lambda = 2.85cm$, $F = 6dB$, $L = 9dB$, $\delta f = 1mph \Rightarrow f_d = 30Hz = B$: Velocity resolution. RCS $\sigma = 30m^2$, Required SNR = 10dB.
- For CW, $P = P_{avg} = 100mW$, $B = 30Hz$
- Max. R = 4.2 km ~ 2.6 miles

$$A_e = \frac{\lambda^2 G}{4\pi} = \frac{(2.85\times10^{-2})^2 100}{4\pi} = 6.467\times10^{-3}$$

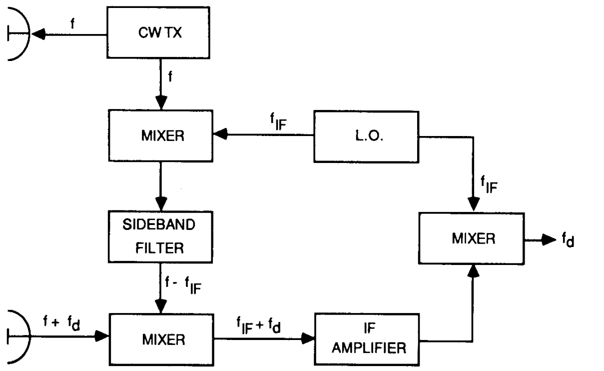


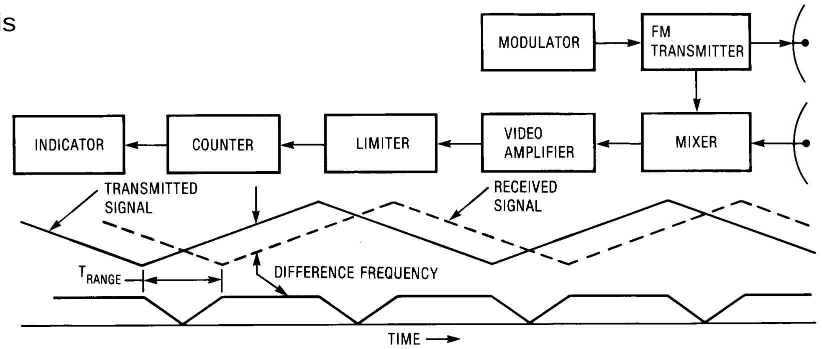
Figure 13-3. Simple CW radar system with a superheterodyne receiver. (Shown in a dual antenna configuration.)

$$S_{i,min} = F_T kTB \times (S_o/N_o)_{min} L$$
$$= [-114_{dBm} + 10\log(B_{MHz})] + F_{T,dB} + (S_o/N_o)_{min,dB} + L$$
$$= -114 + 10\log(30\times10^{-6}) + 6 + 10 + 10$$
$$= -133.229dBm = 4.755\times10^{-14}mW$$

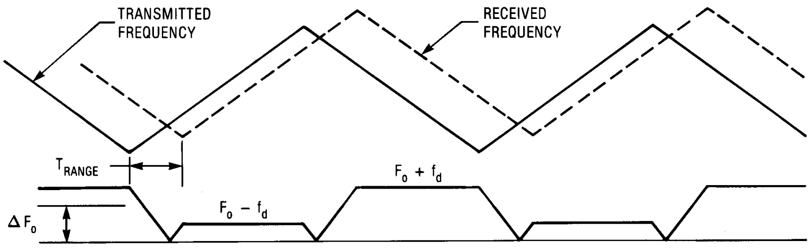
$$R_{max} = \left[\frac{P_t G A_e \sigma}{(4\pi)^2 S_{i,min}}\right]^{1/4}$$
$$= \left[\frac{0.1\times100\times6.467\times10^{-3}\times30}{(4\pi)^2\times4.755\times10^{-17}}\right]^{1/4}$$
$$= 4010.4m = 4.01km$$

CW Ranging

- In order to measure range, it is necessary to place a time marker (modulation) in the transmitted signal
  - amplitude, frequency, phase
  - Pulsed radar → AM.
- CW ranging
  - Frequency-modulated CW (FMCW)
  - Multiple-frequency CW
  - Phase-coded-CW
- FM-CW radar
  - Correlation of frequency of TX and RX signals
  - a measure of target's range and radial speed.
  - Linear frequency modulation



FM-CW range measuring radar block diagram.



Doppler effect on the difference frequency in an FM-CW radar.

FM-CW Ranging (No Doppler)

Frequency Modulation rate

$f_m = 1/\tau$

Target Range

$R = \frac{cT}{2} \Rightarrow T = \frac{2R}{c}$

Δf: Amp. of frequency modulation

We have $\frac{f_b}{T} = \frac{\Delta f}{\tau/4}$

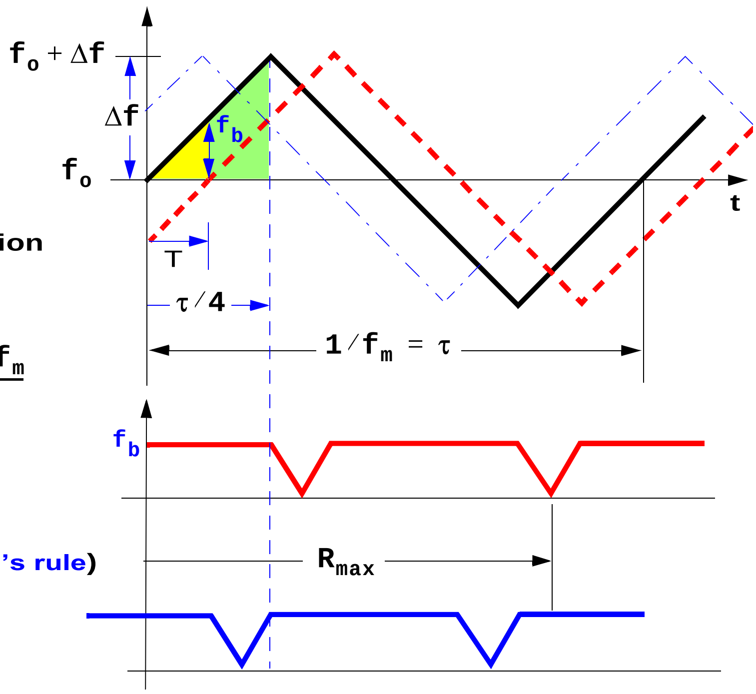
$\Rightarrow f_b = \frac{2R}{c}\left(\frac{\Delta f 4}{\tau}\right) = \frac{8R\Delta f}{c\tau} = \frac{8R\Delta f f_m}{c}$

$$R = \frac{c f_b}{8\Delta f f_m}$$

Transmitted BW = $2\Delta f$ (Carlson's rule)

Maximum unambiguous range

$R_{max} = \frac{c}{2}\tau = \frac{c}{2f_m}$



FM-CW Radar (Low Doppler)

(1) $f_d < f_b$

- For positive slopes

$f_b = f_b^+ = f_b - f_d = \frac{8R\Delta f f_m}{c} - f_d$ (toward)

$f_b = f_b^+ = f_b - f_d$ (negative sign) (away)

- For negative slopes

$f_b = f_b^- = f_b + f_d = \frac{8R\Delta f f_m}{c} + f_d$ (toward)

$f_b = f_b^- = f_b + f_d$ (negative sign) (away)

$f_b = \frac{f_b^+ + f_b^-}{2} = \langle f_b \rangle = \frac{8R\Delta f f_m}{c}$

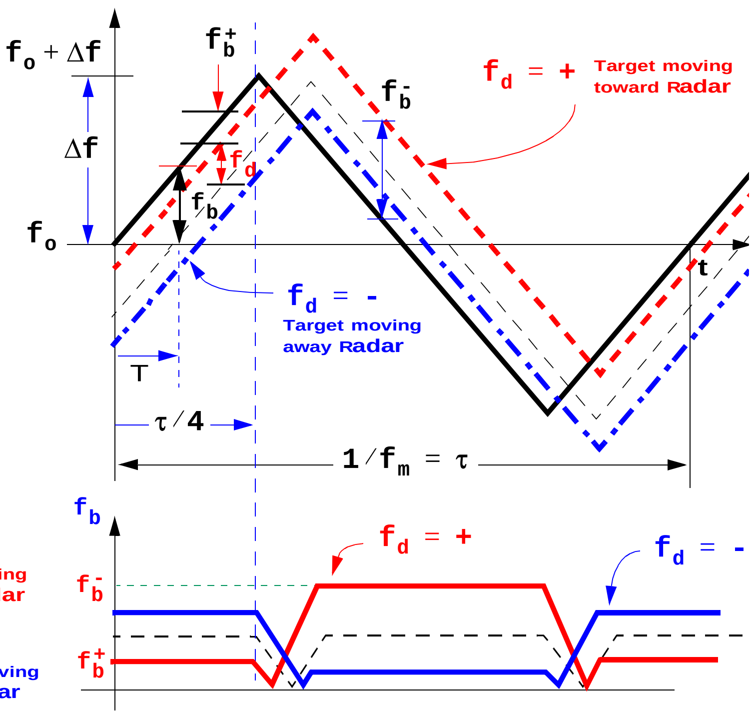
$$\Rightarrow R = \frac{c}{8R\Delta f f_m}\langle f_b \rangle$$

$f_d = \frac{f_b^- - f_b^+}{2} = \frac{2v}{\lambda}$

$$\Rightarrow v = \frac{\lambda}{4}(f_b^- - f_b^+)$$

f_d = + Target moving toward Radar

f_d = - Target moving away Radar



FM-CW Radar (high Doppler)

(2) $f_d > f_b$

- For positive slopes

$f_b = f_b^+ = f_d - f_b = f_d - \frac{8R\Delta f f_m}{c}$ (toward)

$f_b = f_b^+ = f_d - f_b$ (negative sign) (away)

- For negative slopes

$f_b = f_b^- = f_d + f_b = \frac{8R\Delta f f_m}{c} + f_d$ (toward)

$f_b = f_b^- = f_d + f_b$ (negative sign) (away)

$$f_b = \frac{f_b^- - f_b^+}{2} = \langle f_b \rangle = \frac{8R\Delta f f_m}{c}$$

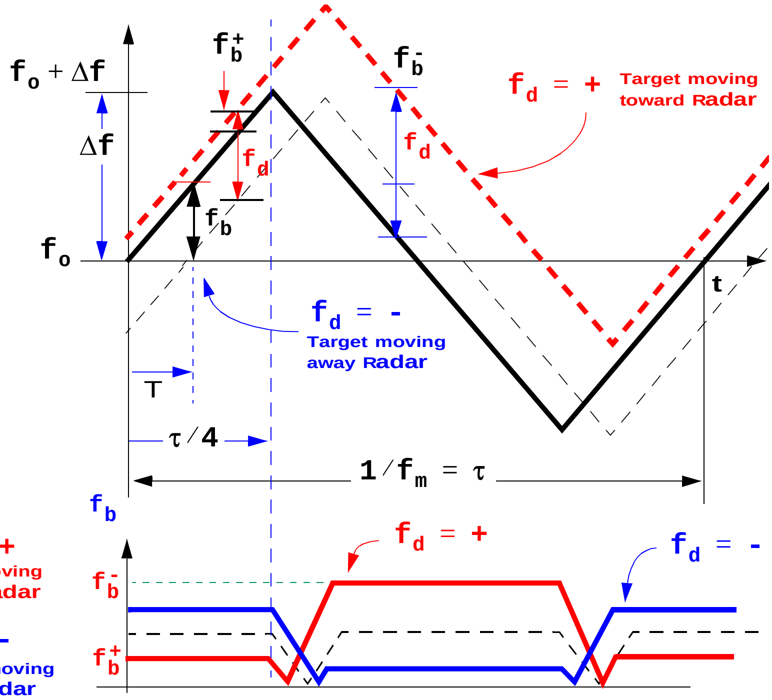
$$\Rightarrow R = \frac{c}{8R\Delta f f_m}\langle f_b \rangle$$

$$f_d = \frac{f_b^- + f_b^+}{2} = \frac{2v}{\lambda}$$

$$\Rightarrow v = \frac{\lambda}{4}(f_b^- + f_b^+)$$

$f_d = +$ Target moving toward Radar

$f_d = -$ Target moving away Radar



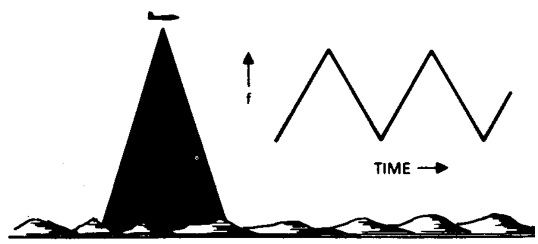
FM-CW Range Resolution

- Range resolution (accuracy) δR : the frequency difference, f_b , can be measured. $\delta f_b \sim B = 2\Delta f$
- For a linear FM, δf_b (thus δR) depends on the BW and linearity of the modulation.
  - Nonlinearity is given by $\delta f / B$. δf : deviation in modulation from linear.
  - Nonlinearity must be much less than f_m / B .
- Maxi. range resolution $\delta R = c/(2B)$, $T \ll 1/f_m$)
- For a given R_{max} and $(\delta R)_{max}$, nonlinearity $\ll f_m/B = \delta R/R_m$.

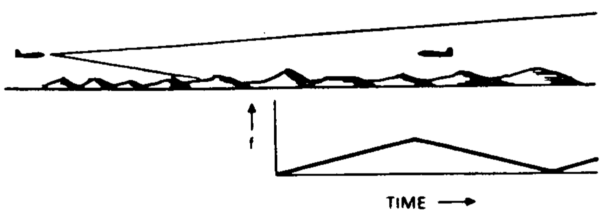
EXample:

- For a range resolution of 1ft and a maxi. unambiguous range of 3000 ft.
- the nonlinearity of the FM waveform must be less than $\delta R/R_m = 1/3000 = 0.03\%$.

ACCURACY: HIGH



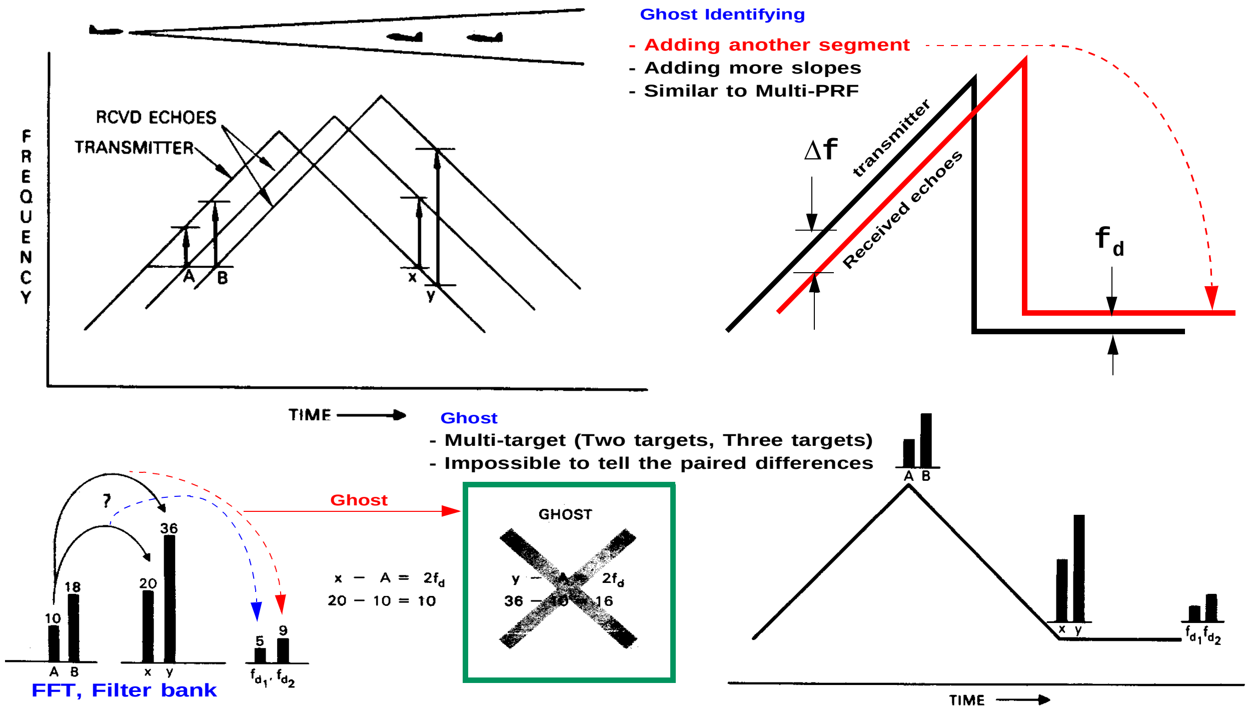
In altimeters, slope of modulation curve can be made steep enough to provide highly accurate range measurement.



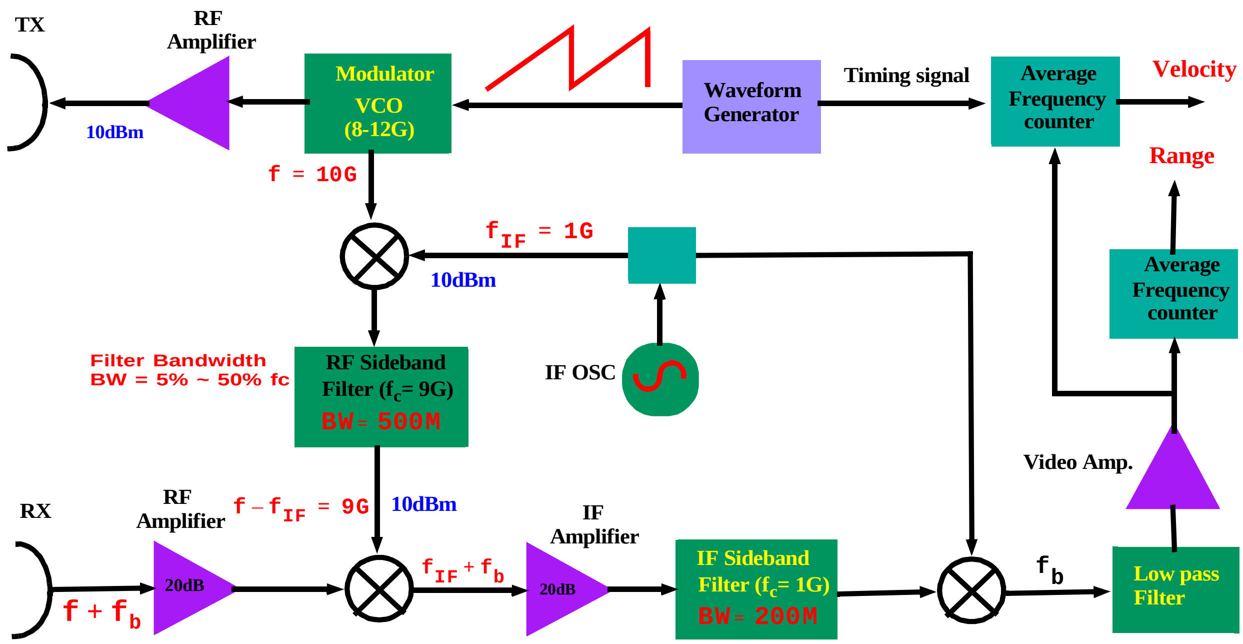
ACCURACY: LOW

For air-to-air applications, slopes must be made shallow to avoid smearing the spectrum of the ground return. The result is low accuracy.

FM-CW for Multi-target



FM-CW Radar Design



Sinusoidal-FM Ranging (1)

- Modulation with an instantaneous frequency of $f(t) = f_0 + \Delta f \cos(2\pi f_m t)$
- The transmitted signal in this case is $s(t) = A_1 \sin[\phi(t)]$, where

$$\phi(t) = 2\pi \int f(t)dt$$

$$= 2\pi f_o t + \frac{\Delta f}{f_m}\sin(2\pi f_m t)$$

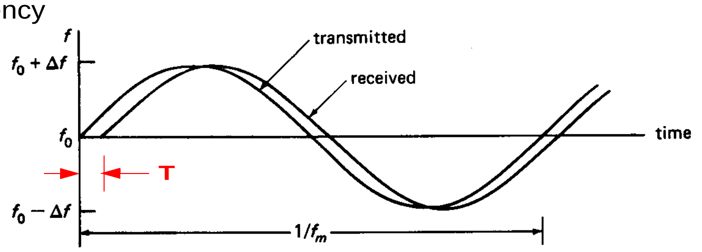


Figure 13-10. Sinusoidal FM.

- The received waveform for a point target is a replica of the transmitter waveform, in other words,

$$r(t) = A_2 \sin[\phi(t-T)] = A_2 \sin\left[2\pi f_o(t-T) + \frac{\Delta f}{f_m}\sin[2\pi f_m(t-T)]\right]$$

- After mixing and filtering of the two signals in the receiver, the output is

$$r(t)s(t) = \frac{A_1A_2}{2}\cos\left[2\pi f_0 T + \frac{\Delta f}{f_m} \times [\sin(2\pi f_m t) - \sin[2\pi f_m(t-T)]]\right]$$

$$\sin A - \sin(A-B) = 2\sin\frac{B}{2}\cos\left(A - \frac{B}{2}\right)$$

$$= \frac{A_1A_2}{2}\cos\left[2\pi f_0 T + \frac{2\Delta f}{f_m}\sin(\pi f_m t) \times \cos\left(2\pi f_m\left(t - \frac{T}{2}\right)\right)\right]$$

- If one assume that $T \ll 1/f_m$,$\langle f_b \rangle = 8R\Delta f f_m / c$

Sinusoidal-FM Ranging (2)

$$r(t)s(t) = \frac{A_1A_2}{2}\left\{\left[\cos(2\pi f_0 T)\cos\left(\frac{2\Delta f}{f_m}\sin(\pi f_m T) \times \cos\left(2\pi f_m\left(t - \frac{T}{2}\right)\right)\right)\right]\right.$$

$$\left.-\left[\sin(2\pi f_0 T)\sin\left(\frac{2\Delta f}{f_m}\sin(\pi f T) \times \cos\left(2\pi f_m\left(t - \frac{T}{2}\right)\right)\right)\right]\right\}$$

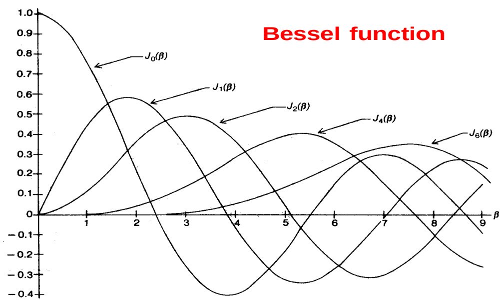
$$= \frac{A_1A_2}{2}\left\{\cos(2\pi f_0 T)\left[J_0\left(\frac{2\Delta f}{f_m}\sin(\pi f_m T)\right) - 2J_2(\ ) \times \cos\left(2\left(2\pi f_m\left(t - \frac{T}{2}\right)\right)\right) + 2(J_4(\ )\cos(\ ))\right]\right.$$

$$\left.-2\sin(2\pi f_0 T)\left[J_1\left(\frac{2\Delta f}{f_m}\sin(\pi f_m T)\right) \times \cos\left(2\pi f_m\left(t - \frac{T}{2}\right)\right) - J_3(\ ) \times \cos\left(2\left(2\pi f_m\left(t - \frac{T}{2}\right)\right)\right) + \ldots\right]\right\}$$

We use expansion of the form

$$\cos(z\cos\theta) = J_o(z) + 2\sum_{k=1}^{\infty}(-1)^k J_{2k}(z)\cos(2k\theta)$$

$$\sin(z\cos\theta) = 2\sum_{k=1}^{\infty}(-1)^k J_{2k+1}(z)\cos((2k+1)\theta)$$



Sinusoidal-FM Ranging (3)

If we add a Doppler component, the net result look like

$$= J_0(D)\cos(2\pi f_d t - \phi_0) + 2J_1(D)\sin(2\pi f_d t - \phi_0)\cos(2\pi f_m t - \phi_m)$$
$$- 2J_2(D)\sin(2\pi f_d t - \phi_0)\cos[2(2\pi f_m t - \phi_m)]$$
$$+ 2J_3(D)\sin(2\pi f_d t - \phi_0)\cos[3(2\pi f_m t - \phi_m)] + \ldots$$

$$D = \frac{2\Delta f}{f_m}\sin(\pi f_m T) \qquad \phi_0 = 2\pi f_o T \qquad \phi_m = 2\pi f_m \frac{T}{2}$$

$$f_d = \frac{2vf_o}{c} \qquad \phi_0 = 2\pi f_o\left(\frac{2R}{c}\right) \qquad \phi_m = 2\pi f_m \frac{R}{c} \qquad D = \frac{2\Delta f}{f_m}\sin\left(2\pi f_m \frac{R}{c}\right) \qquad T = \frac{2R}{c}$$

The harmonics of f_n are modulated by the doppler frequency and weighted in amplitude by a Bessel function

$J_n(D) \cong D^n \propto R^n$ for a small argument

- Advantage: J_o gives a maximum weighting at low D, (i.e. near range) and lower weighting at far range (for example: far clutter)
  - When the received signals are from close-in clutter, higher-order harmonics should be chosen
- Disadvantages:
  - Insufficient use of energy
  - Probably most suitable for a single target

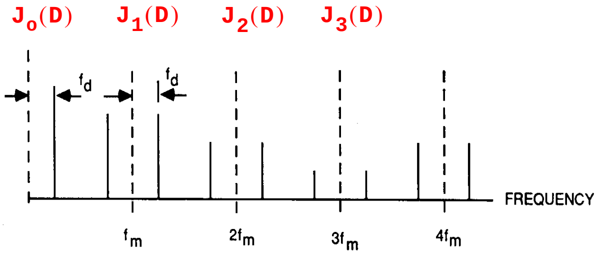


Figure 13-11. Spectrum of the video signal resulting from the detection of a moving target with a sinusoidal FMCW radar.

Multiple-Frequency Ranging (1)

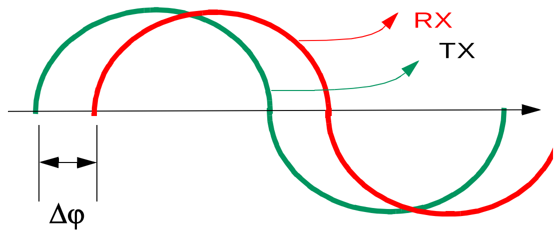
- Target range could be determined by measuring the receive phase difference between the transmitted and received waves
- The relative difference, $\Delta\varphi$, is given by

$$\Delta\varphi = \frac{2\pi d}{\lambda} = \frac{4\pi R}{\lambda} = \frac{4\pi f R}{c} \Rightarrow R = \frac{\lambda\Delta\varphi}{4\pi} = \frac{c\Delta\varphi}{4\pi f}$$

- The maximum unambiguous range

$$\Delta\varphi = 2\pi \Rightarrow R_{max} = \lambda/2$$

  - 15cm at L-band (1GHz) to 1.6mm at 95 GHz
- The approach we intend to look at use two or more CW signals that are very close-in frequency.
- If we have a sufficiently $f_{diff} = f_2 - f_1$, then range measurement may be possible since $\lambda_{diff} = \lambda_2 - \lambda_1$ is large
- We examined the case of two frequency that only differed by kHz



- Advantage: Large unambiguous range
- Disadvantage: poor range resolution

- Two CW signals with f_1 and f_2

Transmitter signal

$$v_{1T} = \sin(2\pi f_1 t + \phi_1)$$
$$v_{2T} = \sin(2\pi f_2 t + \phi_2)$$

Receiver signal

$$v_{1R} = \sin(2\pi(f_1 \pm f_{D1})t - 4\pi f_1 R/c + \phi_1)$$
$$v_{2R} = \sin(2\pi(f_2 \pm f_{D2})t - 4\pi f_2 R/c + \phi_2)$$

Multiple-Frequency Ranging (2)

- Low side of mixing v_{1T} to v_{1R}

$$v_{1d} = \sin(\pm 2\pi f_{D1} t - 4\pi f_1 R/c)$$

$$v_{2d} = \sin(\pm 2\pi f_{D2} t - 4\pi f_2 R/c)$$

- Phase difference between v_{1d} and v_{2d}

$$\Delta\varphi = \pm 2\pi(f_{D1} - f_{D2}) + \frac{4\pi R}{c}(f_1 - f_2)$$

0

$$\cong \frac{4\pi R}{c}(f_1 - f_2)$$

If f_1 and f_2 are nearly the same.

- $\Delta f = f_1 - f_2$. therefore

$$\Delta\varphi = \frac{4\pi R \Delta f}{c} \Rightarrow R = \frac{c\Delta\varphi}{4\pi\Delta f}$$

- Assume $R = 0 \Rightarrow \Delta\varphi = 0$

Determine R necessary to give $\Delta\varphi = 2\pi$

- $R_{unamb} = \dfrac{c2\pi}{4\pi\Delta f} = \dfrac{c}{2\Delta f}$
- If $\Delta f = 1.5kHz$, and then

$$R_{unamb} = \frac{3\times10^8}{2(1.5\times10^3)} = 100km$$

- However, a small Δf also gives a poor range resolution degree
  - Range per (^{o}) degree for $\Delta f = 1.5kHz$

$$\Delta R = \frac{c(\pi/180)}{4\pi\Delta f}$$

$$= \frac{(3\times10^8)(\pi/180)}{4\pi(1.5\times10^3)} = 278(m/°)\ \text{phase}$$

  - If measurement accuracy is 5º, then the range resolution is 5(278) = 1.4 km
  - For small $f_1 - f_2$ as $f_1 - f_2$ becomes smaller, R_{unamb} becomes higher as $f_1 - f_2$ becomes smaller ability to determine R
  - Solution: use more than 2 frequencies.

Three-Frequency Ranging (3)

3-frequency system

- $(f_3 - f_1) \gg (f_2 - f_1)$, let $f_3 - f_1 = 15kHz$ and $f_2 - f_1 = 1.5kHz$
- For $f_3 - f_1 = 15kHz$ (Fine)

$R_{unamb31} = c/2(f_3 - f_1) = 10km$,

$$\Delta R = \frac{c(\pi/180)}{4\pi(f_3 - f_1)} = 28(m/°)$$

- For $f_2 - f_1 = 1.5kHz$ (Rude)

$R_{unamb21} = c/2(f_2 - f_1) = 100km$,

$$\Delta R = \frac{c(\pi/180)}{4\pi(f_3 - f_1)} = 278(m/°)$$

- Example: Assume we are taking measurements with an actual system and measured values as

$$\Delta\varphi_{3-1} = 1.2rad,\ \Delta\varphi_{2-1} = 1.3rad$$

Then by comparing phase

$$R(f_3 - f_1) = \frac{c\Delta\varphi_{3-1}}{4\pi(f_3 - f_1)} = 1.91km$$

$$R(f_2 - f_1) = \frac{c\Delta\varphi_{2-1}}{4\pi(f_2 - f_1)} = 20.7km$$

Infer range of

$$R = 2\times R_{unamb31} + R(f_3 - f_1)$$

$$= 2\times10 + 1.91 = 21.91km$$

- Could use an even greater number of frequency to obtain the best combination of R_{unamb} and ΔR.