

Multi-path hybrid spectrum sensing in cognitive radio

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Abstract. Inefficient utilization of the authorized spectrum emerges cognitive radio (CR) as a hopeful technology for both present and future telecommunications. It is owing to the potency to leverage the obtainable bandwidth of other wireless communication networks and thereby increase its occupancy. The key feature for the cognitive radio system for distinguishing the blank spectrum is spectrum sensing. This paper is intended to establish a hybrid sensing model for spectrum detection in CR to enhance sensing efficiency of traditional techniques of spectrum sensing, which consists of two parallel paths of hybrid detectors. The first path is formed from two sequential detector stages; in the first phase, energy detector is used to recognize the PU signal existence where the signal has not been identified. Maximum-Minimum Eigenvalue (MME) is used as a second stage to detect the PU signal presence. The second path consists of two parallel stage detectors employing separate ED and MME to detect the PU signal individually, the two results are gathered to make a decision, and then the final detection decision is determined based on the two paths' detection combined results. The proposed hybrid sensing approach adopted for enhancing the sensing performance is validated with conventional methods. Simulation results show that the proposed approach outperforms various traditional and hybrid approaches in terms of maximizing the detection probability on the specified limitations on the false alarm probability, as it can increase the detection probability to 94% instead of 79% for the parallel detector at SNR= -10 dB and Pfa=0.1.

Keywords: Cognitive radio, Spectrum sensing, Hybrid spectrum sensing, Energy Detection, MME Detection.

1 Introduction

Inefficient utilization of bandwidth across various spectrum ranges has been recently recognized as a significant disadvantage in conventional wireless networks with fixed bandwidth deployment regulations. However, cognitive radio technology gives an extraordinary solution to the spectrum usage problem for satisfying the constant rising need for bandwidth [1].

Cognitive radios are intelligent communication technology capable of recognizing their wireless environment and making choices about how and when to use the obtain-

able spectrum (white space), not being utilized by the authorized user. Therefore, cognitive radios must be qualified for scanning vast frequency ranges and locating unused bands to prevent primary users from suffering harmful interference [2].

A cognitive radio executes three essential functions during its employment in radio surroundings: spectrum sensing, spectrum analysis and spectrum decision. In spectrum sensing, the CR frequently monitors the radio spectrum surroundings with the main intention of locating spectrum gaps. However, the spectrum analysis requires an estimate of the obtainable channel capacity. Finally, the CR chooses and changes its operational conditions in spectrum decision such as transmission data rate and bandwidth.

The spectrum sensing is the main function on which the entire existence of cognitive radio lays, and is identified as the challenge of locating unused holes of spectrum by scanning the radio spectrum of the surrounding cognitive receiver in an unsupervised way. Various methods for spectrum sensing are determined, including feature detection (FD), energy detection (ED), and matched filtering (MF) [2]. ED is simpler compared with FD and MF and it also has a fewer computational complexity as well, whereas FD requires more computations but has a preferable performance especially at small signal to noise ratio (SNR). On the other hand, MF offers a good performance but needs complete prior information about the primary user, which might not be available. In addition, there are many other methods for detecting the spectrum such as maximum-minimum eigenvalue detectors, which could also obtain both high detecting probability and low false-alarm probability at the same time without demanding PU prior knowledge [2].

The performance of spectrum sensing is determined by both the sensitivity and selectivity, which are presented by the detection rate and the false alarm probability. Accurate probability of detection improves primary users (PUs) security and decreases the risk of false alarm that results in increasing the secondary users (SUs) channels' utilization [2]. In this paper, a multi-path hybrid sensing model for spectrum detection is proposed to enhance sensing efficiency by maximizing the detection probability on the specified limitations on the false alarm probability.

The rest of the paper is arranged as below. Section 2 shows the related work of this problem. In Section 3, the system model of the sensing scheme, the proposed multi-path hybrid spectrum sensing model, and all the mathematical analysis carried out are introduced. In section 4, simulation results for evaluating the performance of the proposed model, ED, MME detection and two-stage i.e. combination of ED and MME detection technique are presented. Lastly, the final section is a conclusion for the paper.

2 Related work

In early 1990s, Joseph Mitola introduced the concept of software-defined radios (SDRs) [3]. Hardware requirements were minimized using SDR, which introduced a reliable and cheap solution to the users. In this work, an early version of the CR, known later as the SDR new version in 2000, was implemented and the dynamic spectrum sensing concept was also investigated.

S. Haykins in 2005 introduced the CR as a new concept for spectrum efficient usage [4]. Later in 2008, S. Haykins defined the Spectrum sensing as the task upon the entire operation of cognitive radio rests and introduced a lot of sensing techniques as Cyclostationary detector and the multi-taper method (MTM) for spectral estimation [5]. The most used technique; named energy detection was introduced by H. Urkowitz in 1967 for the first time [6], then A. Sahai and D. Cabric in [7] and plenty of researchers used it in CR for the spectrum sensing purpose with numerous number of attempts for enhancing the ED technique performance. Moreover, two classical techniques i.e. the matched filtering (MF) and Eigenvalue -based spectrum sensing were introduced in [7] and [8] respectively.

Considering the spectrum sensing performance enhancement, many methods were developed and introduced. A simple sequential spectrum sensing was introduced in 2009 by Yan Xin and Honghai Zhang [9]. In 2010, Konstantinos Plataniotis introduced two-stage spectrum detection in cognitive radio networks [10]. Then, in 2012, Minny Bhola, Rinkoo Bhatia presented a two-stage spectrum sensing for cognitive radio using cyclostationary detection and energy detection [11]. After that, in 2015, Min Jia, Xue Wang, Fang Ben, Qing Guo and Xuemai Gu developed the concept of energy detection and covariance detection [12]. In 2017, Awani S. Khobragade, R. D. Raut demonstrated a hybrid spectrum sensing method for cognitive radio which has utilized five different techniques for spectrum sensing and combined them to establish a hybrid sensing system based on the principle of Centralized Organization in which the installation of infrastructure has been intended for CR users [2]. In 2019 Mahua Bhowmik, P. Malathi offered and showed a hybrid model for energy efficient spectrum sensing in cognitive radio based in neural network prediction model [1].

In this paper and in the light of the continuous effort to improve the performance of spectrum sensors in terms of seeking to obtain a higher probability of detection and low probability of false alarm, a new multi-path hybrid sensing model for spectrum detection is proposed. This new multi-path hybrid sensing model overcomes the limitations of both hybrid and conventional sensing methods and gives better performance in terms of probability of detection at low SNR, as well as better performance when compared with each method's detection performance.

3 System Model

3.1 Spectrum sensing model

To perform spectrum sensing, the following hypotheses have been tested by the detector: under H_0 , the signal of the primary user is absent and, there is just noise at the receiver input, however, under H_1 , the primary user signal and noise are existed at the input of the receiver. Assuming that bandwidth B and the center frequency f_c of the primary user signal are known, input signal is to be down converted and sampled at the Nyquist rate, $f_s = 2B$. The hypothesis test discrete time model is [13]:

$$H_0: y[m] = w[m], m = 1, \dots, n \quad (1)$$

$$H_1: y[m] = x[m] + w[m], m = 1, \dots, n \quad (2)$$

Where n samples represent period of observation which is equivalent to the sensing time, the secondary user received signal $y[m]$, both of noise $w[m]$ and signal $x[m]$ samples are modeled as independent random variables of Gaussian with zero mean and variance σ_w^2 and σ_s^2 , respectively.

3.2 Energy Detector

Energy measurement is the main method of signal detection when there is noise and is considered the common technique because it could be applied to whatsoever signals. In addition, it demands the least amount of information about the signal, i.e., the signal bandwidth and carrier frequency. In signal processing communications, detection of energy is regarded as a hypothesis testing issue, and evaluating the performance by computing the pair of false alarm and detection probabilities (P_{fa} , P_d). Even though the systems of positive SNR performance are understood properly, it is uncertain whether identical performance and analysis are valid in the extremely negative systems of SNR.

The energy detector decision's statistic measured energy over n samples [13]:

$$\varepsilon(y) = \sum_{m=1}^n y[m]^2 \quad (3)$$

To compute P_d and P_{fa} , we need to determine the probability density function (pdf) of decision statistic under both hypotheses. Because the test statistic is the addition of n Gaussian random variables then its pdf is chi-square χ_n^2 .

The detection is carried out by testing the measured energy threshold. Setting a suitable threshold value can be performed by many methods. In cases of spectrum sensing, threshold γ is adjusted to obey the fixed P_{fa} . Then,

$$P_{fa} = P_r(\varepsilon(y) > \gamma | H_0) = Q_{\chi_n^2} \left(\frac{\gamma}{\sigma_w^2} \right) \quad (4)$$

$$P_d = P_r(\varepsilon(y) > \gamma | H_1) = Q_{\chi_n^2} \left(\frac{\gamma}{\sigma_w^2 + \sigma_s^2} \right) \quad (5)$$

According to the equation, $Q(\cdot)$ is the complementary distribution functions of Gaussian. Note that P_{fa} depends only on the noise variance, thus the threshold can be set regardless of the primary user signal level.

For a certain number of samples n , typically larger than 250, χ_n^2 can be approximated with a Gaussian random variable, i.e. $\chi_n^2 \sim N(n, 2n)$. Then, we can rewrite P_d and P_{fa} as [13]:

$$P_{fa} = Q \left(\frac{\frac{\gamma}{\sigma_w^2} - n}{\sqrt{2n}} \right) \quad (6)$$

$$P_d = Q\left(\frac{\gamma}{\frac{\sigma_w^2 + \sigma_s^2}{\sqrt{2n}}} - n\right) \quad (7)$$

The samples number is a critical value, where if the samples number used in sensing is unlimited, an energy detector is capable of fulfilling any required P_d and P_{fa} at the same time. The least desired samples number is a function of $\text{SNR} = \sigma_s^2 / \sigma_w^2$:

$$n = 2[(Q^{-1}(P_{fa}) - Q^{-1}(P_d)) \text{SNR}^{-1} - Q^{-1}(P_d)]^2 \quad (8)$$

In systems with low $\text{SNR} \ll 1$, the detection needs sufficient samples numbers that fulfill identified P_{fa} and P_d , asymptotically scales as $1/\text{SNR}^2$. This scaling law is a non-coherent detection characteristic, i.e. detectors whose sensing time scale as $1/\text{SNR}^2$ will be named as non-coherent.

Practically to implement a detector of energy, it requires a band-pass filter (BPF), an integrator, a digital-to-analog converter, and a square law device. At first, the bandwidth of input signals is bounded to focus through a BPF. After that squaring, the filtered signal make integration over the observation period T . Lastly, a comparison is made between the integrator output and a threshold to decide whether the primary signal is present or not. Figure 1 shows a typical energy detection block diagram:

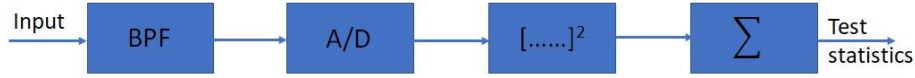


Fig. 1. Energy Detector Block Diagram.

Taking into consideration the previous points, the energy detection drawbacks could be concluded as follows:

- The energy detection performance is very sensitive to the changing noise level.
- It needs longer time than matched filter detection.
- This technique is incapable of distinguishing noise, modulated signals, and interference.
- Spread spectrum signals detection is not possible with energy detection techniques.

On the other hand, energy detection method has several advantages that motivate us to study. For example:

- It is more common as receivers do not require information about the signal of the primary user.
- It has much simpler implementation with respect to other sensing techniques.

Signals can be detected at low SNRs, as long as the power spectral density is known and detection interval is sufficiently long.

3.3 Maximum Minimum Eigen value Detector (MME)

In this technique, more precision and noise robustness are obtained by the development of more complex mathematical derivation of the test statistics [15]. The test statistics has been defined by the ratio of the maximum π_{\max} to the minimum π_{\min} eigenvalue of the covariance matrix \mathfrak{R}_π , which is constructed by performing the following procedures:

First, the n samples obtained vector is divided into k similar sections (k is the smoothing factor), then a matrix D with shape $k \times n_k$ is built where $n_k = n/k$, at last \mathfrak{R}_π is generated:

$$\mathfrak{R}_\pi = \frac{1}{n_k} DD^* \quad (9)$$

We can write it in more details as:

$$\mathfrak{R}_\pi(k) = \begin{bmatrix} g(0) & g(1) & \dots & g(n-1) \\ g^*(0) & g(1) & \dots & g(n-1) \\ \vdots & \vdots & \dots & \vdots \\ \vdots & \vdots & \dots & \vdots \\ g^*(n-1) & g^*(n-2) & \dots & g(0) \end{bmatrix} \quad (10)$$

where

$$g(l) = \frac{1}{k} \sum_{m=0}^{k-1} x(m)x(m-l)^*$$

with $l = 0, 1, \dots, n-1$.

Based on the calculated eigenvalues, the maximum eigenvalue π_{\max} and minimum eigenvalue π_{\min} of the matrix \mathfrak{R}_π is obtained. To employ this statistical test to determine the required threshold for detection, it is necessary to analyze the covariance matrix statistical distribution when the PU signal is absent, which is the noise sample covariance matrix $\mathfrak{R}_W(n)$. According to Random Matrix theory, the MME threshold can be defined as [14]:

$$\gamma_{MME} = \frac{(\sqrt{k} + \sqrt{n})}{2(k+n)} \left(1 + \frac{(\sqrt{k} + \sqrt{n})^{-2/3}}{(kn)^{1/6}} F_1^{-1}(1 - P_{fa}) \right) \quad (10)$$

where F_1^{-1} is the cumulative inverse function of the Tracy-Widom first order distribution as seen in Table 1, γ_{MME} is the MME detection threshold.

Table 1. Tracy-Widom first order distribution numerical values [5]

P_{fa}	0.01	0.03	0.05	0.07	0.09	0.1	0.3	0.6
$F_1^{-1}(1 - P_{fa})$	2.02	1.33	0.97	0.73	0.53	0.45	-0.59	-1.58

The MME detector decision metric is described as:

$$Ym = \begin{cases} H_0 \text{ hypothesis; } MME_T \leq \gamma_{MME} \\ H_1 \text{ hypothesis; } MME_T > \gamma_{MME} \end{cases} \quad (11)$$

$$MME_T = \frac{\pi_{\max}}{\pi_{\min}} \quad (13)$$

The PU signal is assumed to be present if the ratio MME_T exceeds the threshold γ_{MME} , otherwise, it is assumed to be absent.

The false alarm and detection probability can be formulated as [14]:

$$P_{fa} = P_r(MME_T > \gamma_{MME} | H_0) = 1 - F_1 \left(\frac{\gamma_{MME} (\sqrt{k} + \sqrt{n})^2 - (\sqrt{k} + \sqrt{n-1})^2}{(\sqrt{k} + \sqrt{n-1}) \left(\frac{1}{\sqrt{n-1}} + \frac{1}{\sqrt{k}} \right)^{1/3}} \right) \quad (14)$$

$$P_d = P_r(MME_T > \gamma_{MME} | H_1) = 1 - F_1 \left(\frac{\gamma_{MME} n + n(\gamma_{MME} \delta_{\min} - \delta_{\max}) / \sigma_w^2 - (\sqrt{k} + \sqrt{n-1})^2}{(\sqrt{k} + \sqrt{n-1}) \left(\frac{1}{\sqrt{n-1}} + \frac{1}{\sqrt{k}} \right)^{1/3}} \right) \quad (15)$$

where δ_{\max} , δ_{\min} are the maximum and minimum eigenvalues, respectively, of the PU signal covariance matrix, which are all calculated in the same way as the received samples covariance matrix.

Figure 2 illustrates the Maximum-Minimum-Eigenvalue-Detector block diagram, where the covariance matrix is constructed, eigen values and threshold are calculated then the statistical test is performed.

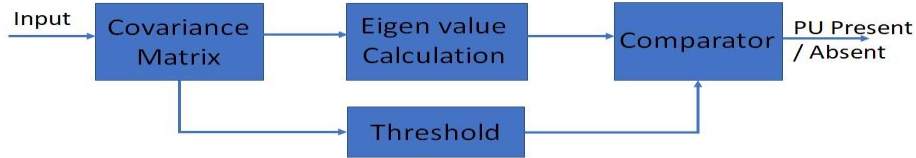


Fig. 2. Maximum Minimum Eigen value Detector Block Diagram.

3.4 Proposed Model

In this paper, we examine the proposed hybrid model in the following system model. A single non-co-operated secondary user is used to detect the primary user signal, while the prime spectrum access links are the authorized band PU connection, and the opportunistic spectrum is the SU link [16,17,18].

The proposed Multi-path hybrid sensing model for spectrum detection that consists of two parallel paths of hybrid detectors is shown in Fig.3. The first path is formed from two sequential detector stages, in the first phase, energy detector is used to recognize

the PU signal existence and if the signal was not identified, Maximum-Minimum Eigenvalue (MME) is used as a second stage to detect the PU signal presence. While the second path consists of two parallel stage detectors employing separate ED and MME to detect the PU signal individually then the two results are gathered to make a decision. Next, the final detection decision is determined based on the two paths combined detection results as illustrated in the process flow chart shown in Fig.4.

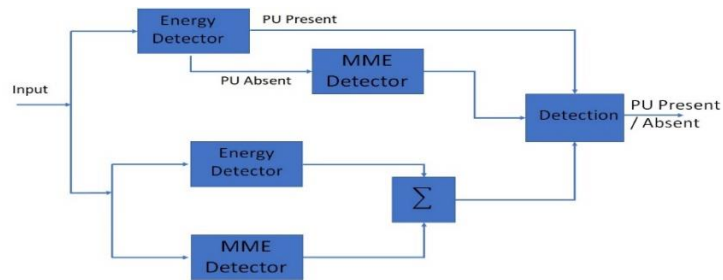


Fig. 3. Block diagram for the proposed multi-path hybrid sensing model.

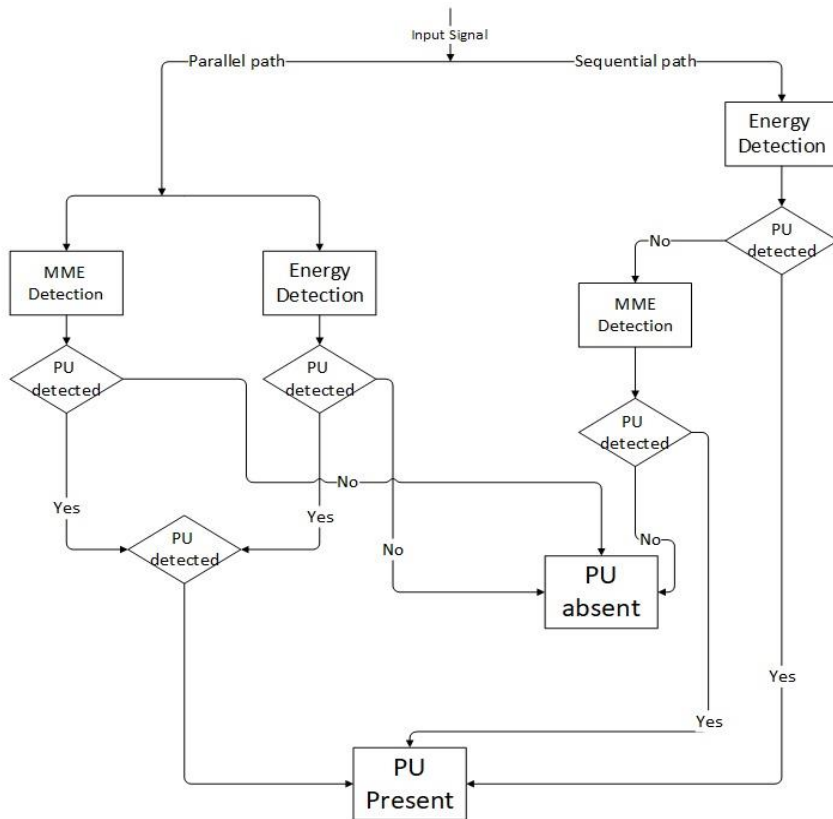


Fig. 4. Process flow chart for the proposed multi-path hybrid sensing model.

4 Simulation Results and Discussion

In this section, MATLAB has been used for evaluating the proposed multi-path hybrid spectrum sensing technique detection performance in comparison with the typical energy detector, MME detector, sequential detector of two stages from ED & MME and the parallel detector of two parallel stages from ED & MME. We carried out different scenarios, for example studying P_d for different techniques, the number of samples effect on P_d , the change in P_{fa} effect on P_d and the mean detection time for different techniques. All of these scenarios used for evaluating the detection performance for the proposed method.

Figure 5 illustrates the detection probability for the proposed multi-path hybrid spectrum sensing technique and compares sensing schemes against signal to noise ratio at simulation parameters $P_{fa}=0.1$ and, $N = 1000$. The obtained results in Fig. 5 indicate that the proposed model detection probability exceeds the other sensing schemes. For example, at $SNR=-15$ dB, the detection probability P_d for ED = 0.07, MMED =0.228, sequential=0.254, parallel=0.283 and proposed model= 0.465. From the previous example, the proposed model achieves a great enhancement in the probability of detection reaches more than 1.5 of parallel detector, which is the most accurate scheme compared with the other schemes.

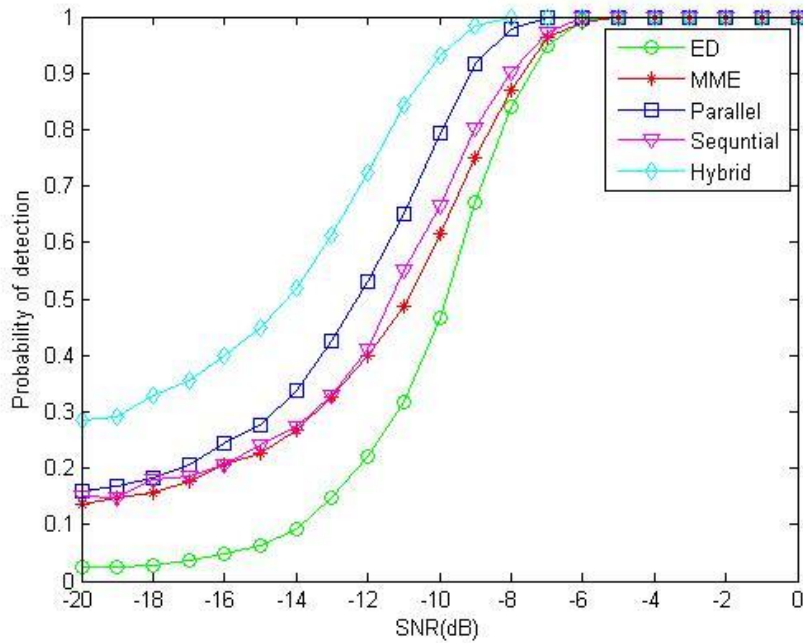


Fig. 5. Performance comparison between the proposed multi-path hybrid sensing model and other sensing techniques, at $P_{fa} = 0.1$ and samples no. $N=1000$ sample.

Figure 6 illustrates the samples number effect on the probability of detection for the proposed multi-path hybrid spectrum sensing technique against SNR curve at $P_{fa}=0.1$ and various samples number. It shows that the increase in the samples number leads to an increase in the detection probability, but also an increase in the complexity of computations that is a critical cost. So as a result, a tradeoff between the samples number and detection probability computational complexity is held to choose an adequate samples number in order to achieve acceptable detection probability based on the system design.

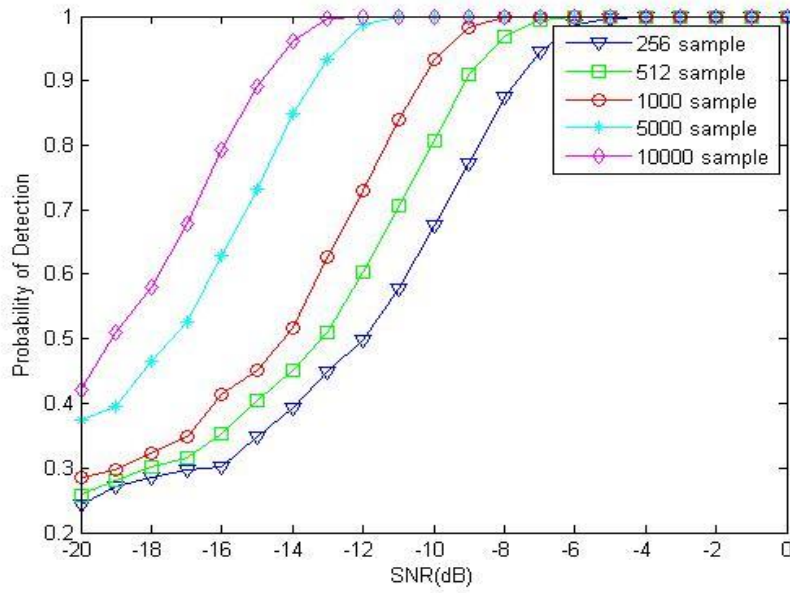


Fig. 6. Performance comparison of different no of samples for the proposed multi-path hybrid sensing model.

Figure 7 illustrates the detection probability for the proposed multi-path hybrid spectrum sensing technique versus signal to noise ratio at $N = 1000$ samples, for different values of false alarm probability, P_{fa} . It also indicates different values for false alarm probability P_{fa} . The detection probability rises when P_{fa} has increased as illustrated in the graph, at $SNR = -15$ dB, the detection probability $P_d = 0.2162$ when $P_{fa} = 0.01$, when $P_{fa} = 0.05$ the detection probability $P_d = 0.3611$ and for $P_{fa} = 0.1$ the detection probability $P_d = 0.4513$, but the maximum acceptable P_{fa} in common wireless medium is 0.1, so it cannot be exceeded.

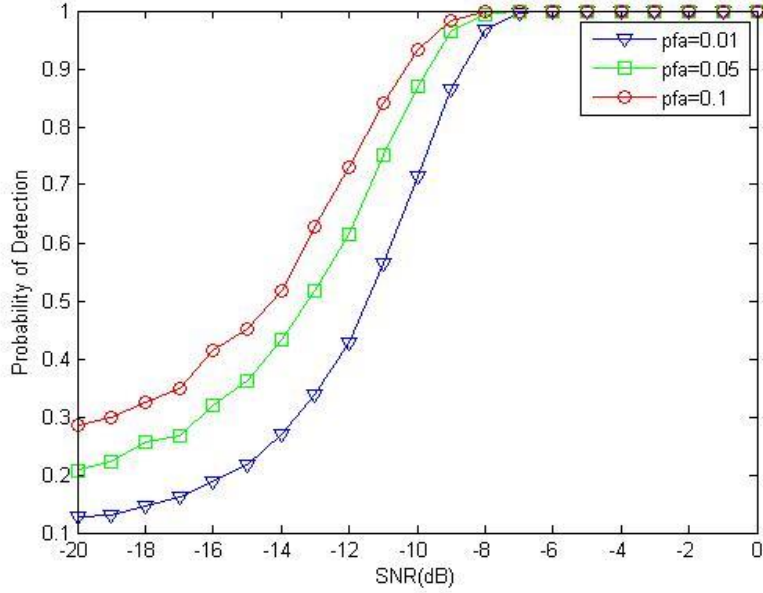


Fig. 7. Performance comparison of different values of false alarm probability for the proposed multi-path hybrid sensing model.

Figure 8 illustrates the mean detection time for the proposed multi-path hybrid spectrum sensing technique and compares sensing schemes against signal to noise ratio at simulation parameters $P_{fa}=0.1$ and $N = 1000$. The obtained results clearly indicate that ED has the lowest mean detection time for all SNR values despite having less detection probability compared with other schemes.

The sequential method has two different time behaviors according to the SNR value, as shown in Fig. 8, due to the working sequence of that approach, which has an ED detector in the first stage and MME detector as a second stage. If the first stage detects the PU correctly, it will end the process; otherwise, it uses the second stage. Noting that the ED has good performance in high SNR environments and typically does not need the second stage, results in taking short detection time. On the other hand, the ED performance degrades in low SNR environments forcing the system to use the MME stage after using the ED results in taking a long detection time. In this case, therefore, this technique has two different time behaviors.

To evaluate the proposed technique versus other sensing methods, we select two points for the comparison sake, one point at small SNR value, i.e. -15dB, and another one at higher SNR, i.e. -4 dB, to include the two extremes of the sequential behavior. Table 2 shows the performance-complexity tradeoff at these two SNR values, for example at SNR=-15dB the proposed model enhances P_d than ED with factor 10, while it takes about quadruple mean detection time than that of ED, which leads to a clear result that the cost of higher detection probability is the longest mean detection time.

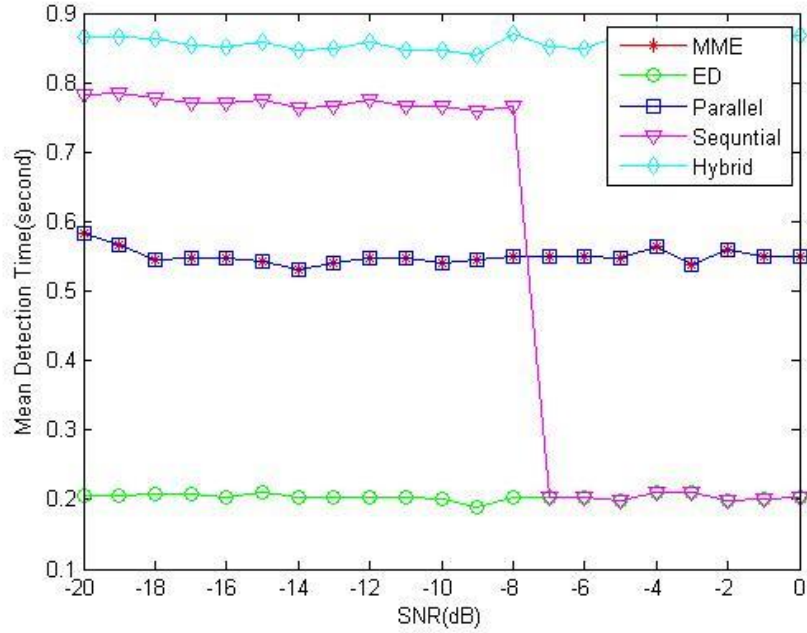


Fig. 8. Mean detection time comparison between the proposed multi-path hybrid sensing model and other sensing techniques.

Table 2. Performance tradeoff between techniques at two SNR values.

	ED		MME		Sequential		Parallel		Hybrid	
	-15	-4	-15	-4	-15	-4	-15	-4	-15	-4
<i>Detection probability</i>	0.038	1	0.181	1	0.181	1	0.211	1	0.355	1
Mean detection time(sec)	0.2	0.2	0.54	0.54	0.78	0.2	0.54	0.54	0.85	0.85

Figure 9 illustrates the ROC curves for the proposed hybrid model under AWGN, Rayleigh channel, and Rician channel for various SNR values. The detection probability is found to decrease when the SNR decreases for both AWGN and fading channels.

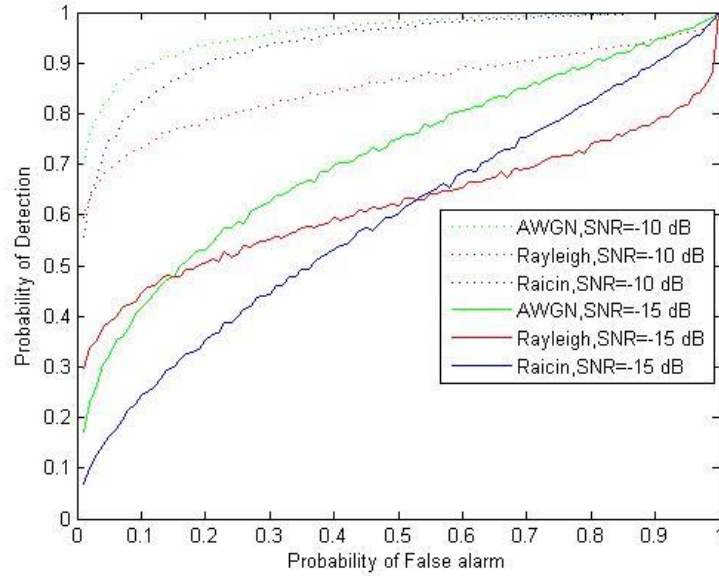


Fig. 9. ROC curves for Proposed hybrid model under AWGN, Rayleigh channel with $m = 10$ and Rician channel with Rice factor $K=10$ dB.

5 Conclusions and Future work

In this paper, we proposed a multi-path hybrid spectrum sensing scheme for cognitive radio to enhance the sensing efficiency. The proposed scheme is a hybrid combination of traditional sensing schemes, namely energy detection and maximum minimum eigen value detector. In addition, performance analysis in terms of its detection performance and mean detection time are carried out which shows the performance trade-offs for the proposed sensing scheme. Maximizing the detecting probability on the given constraints of false alarm probability and does not increase the computational cost is the major aim of this achieved and evaluated work. Moreover, we focus on demonstrating the efficiency of the proposed hybrid model to enhance the sensing performance to solve the problem of spectrum bad utilization. Based on this research, some opportunities for future work could be performed such as examining the proposed hybrid model in a cooperative sensing situation and investigating its impact on the performance of cooperative sensing. Adapting the proposed hybrid model to examine its behavior in the MIMO sensing scenario and evaluate its impact on the sensing performance is another future direction that could be considered.

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