LMI static output feedback design of fuzzy power system stabilisers

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Abstract: The design of a model-free fuzzy power system stabiliser (PSS) lacks systematic stability analysis and performance guarantees. This paper provides a step towards the design of a model-based fuzzy PSS that guarantees not only stability, but also performance specifications of power systems. A new practical and simple design based on static output feedback is proposed. The design guarantees robust pole clustering in an acceptable region in the complex plane for a wide range of operating conditions. A power system design model is approximated by a set of Takagi-Sugeno (T-S) fuzzy models to account for non-linearities, uncertainties and large-scale power systems. The proposed PSS design is based on parallel distributed compensation (PDC). Sufficient design conditions are derived as linear matrix inequalities (LMIs). The design procedure leads to a tractable convex optimisation problem in terms of the stabiliser gain matrix. Simulations results of both single machine and multimachine power systems confirm the effectiveness of the proposed PSS design.

Keywords: power system stability; robustness; linear matrix inequality; LMI; Takagi-Sugeno fuzzy models; parallel distributed compensation; PDC; static output feedback.

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1 Introduction

Power system stabilisers (PSSs) have been used by utilities to damp out the electro-mechanical oscillations that follow disturbances (Kundur, 1994; DeMello and Concordia, 1969). Disturbances occur in power systems due to several reasons, e.g., continuous load variations, set point changes and faults. In such cases, a fixed-parameter conventional PSS may fail to maintain stability or lead to a degraded performance (Larsen and Swann, 1981; Soliman et al., 2008). Different design techniques such as adaptive control (Ghosh et al., 1984; Sastry and Bodson, 1989) and robust control (El-Metwally et al., 2006; Malik and El-Metwally, 1998) have been proposed to enhance the performance of PSSs. The implementation of an adaptive controller needs tough precautions to assure persistent excitation conditions and performance during the learning phase (Sastry and Bodson, 1989).

Recently, fuzzy logic has emerged as a potential technique for PSS design. Besides its ability to accommodate the heuristic knowledge of a human expert, the advantage of a fuzzy PSS is that it represents a non-linear mapping that can cope with the non-linear nature of power systems. Several reported results confirm that a fuzzy PSS outperforms a conventional PSS once the deviation from the nominal design conditions becomes significant (El-Metwally and Malik, 1996). Implementation of a fuzzy PSS for a multimachine power system is reported in El-Metwally and Malik (1993). Tuning the scaling factors of a fuzzy PSS is discussed in Elshafei et al. (2005). An adaptive PSS using online self-learning fuzzy systems is discussed in Abdelazim and Malik (2003) and online tuning of fuzzy PSSs as a direct adaptive one is reported in Feng (2006). Although the performance of a well-designed model-free fuzzy PSS is acceptable, it lacks systematic stability analysis and controller synthesis. The reported work attempts to overcome this drawback by a providing a model-based fuzzy PSS that guarantees stability and performance of power systems. In the past ten years, research efforts on fuzzy logic control have been devoted to model-based fuzzy control systems (Tanaka and Wang, 2001). Stability and performance limits of model-based fuzzy control systems can be achieved via linear matrix inequality (LMI) techniques (Werner et al., 2003).

LMI techniques are proposed as design tools of robust PSS in Rao and Sen (2000), Ramos et al. (2003), Befekadu and Erlich (2006) and Wang et al. (1995). In Rao and Sen (2000), the authors represent the model uncertainty as a linear fractional transformation. An output feedback PSS is designed to guarantee stability for all admissible plants, such that a quadratic performance index, based on the nominal plant, is minimised. In Ramos et al. (2003), pole clustering is used to design a full state feedback for a multimachine power system. In Befekadu and Erlich (2006), a combination of LMI and feedback linearisation techniques is used to design a centralised PSS for a two-area power system. In Wang et al. (1995), a robust decentralised PSS is derived by minimising a linear objective function under LMI and bilinear matrix inequality (BMI) constraints.

In this work, an LMI design of a model-based fuzzy static PSS is proposed. The design guarantees robust pole clustering in a prespecified LMI region. The LMI region is selected such that common specifications of power system stabilisation are achieved. This includes adequate damping and acceptable speed of the time response over wide ranges of active power (P), reactive power (Q) and tie line reactance (X_e). These ranges are selected to include all practical loading conditions and very weak to very strong transmission networks. A power system design model is approximated by a polytopic Takagi-Sugeno (T-S) fuzzy model. Each fuzzy rule (vertex) of the T-S model (polytope) represents an extreme operating point corresponding to the selected ranges. According to the universal approximation theorem (Werner et al., 2003), the resulting fuzzy model can approximate the original non-linear system to an arbitrary degree of accuracy. A stabiliser design is carried out at each vertex of the polytope. The designs are derived under global stability and performance conditions using a common Lyapunov matrix. The design leads to a set of LMIs. The solution of this set of LMIs yields a common positive definite matrix that is used to calculate the stabiliser gains. The total control signal is calculated using a parallel distributed compensation (PDC) control law (Takagi and Sugeno, 1985).

Up to our knowledge, application of a model-based fuzzy control in PSS design, as proposed here, is a novel approach. Model-based fuzzy control system allows us to use an imprecise design model (Sugeno and Kang, 1986; Sugeno, 1999; Qiu et al., 2004). It also enables a decentralised design approach that is independent of the power system size as indicated in the next sections. Furthermore, model-based design relies on LMIs, rather than BMIs to have a tractable solution.

The rest of the paper is organised as follows. Section 2 gives a brief review of T-S fuzzy model and describes how to express power system uncertainties in a polytopic model. Moreover, it describes how to convert this polytopic model into a T-S fuzzy model. In Section 3, LMI conditions that correspond to robust pole clustering in LMI regions are recalled. Sufficient LMI conditions required to calculate the fuzzy stabiliser gains are derived in Section 4. In Section 5, simulation results illustrate the merits of the proposed design. A single machine model is used first to clarify the design steps. Then, a benchmark model of a four-machine two-area test system is utilised to compare the proposed PSS to a well-designed conventional PSS. Section 6 concludes this work.

2 Deriving the T-S fuzzy model for design purpose

2.1 A review of T-S fuzzy model and PDC

A T-S fuzzy model, also called type-III fuzzy model by Sugeno and Kang (1986), is in fact a fuzzy dynamic model (Cao et al., 1997a, 1997b; Johansen et al., 2000). This model is based on using a set of fuzzy rules to describe a global non-linear system by a set of local linear models which are smoothly connected by fuzzy membership functions. T-S fuzzy models include two kinds of knowledge: one is qualitative knowledge represented by fuzzy IF-THEN rules, and the other is a quantitative knowledge represented by local linear models. Identification of T-S fuzzy models has been extensively addressed in the literature, e.g., Sugeno and Kang (1986), Sugeno (1999), Cao et al. (1997b) and Tanaka and Sugeno (1992). There are basically two classes of algorithms to identify T-S fuzzy models. The first is to linearise the original non-linear system in a number of operating points when the model is known. This is adopted in this study. The second is based on the data gathered from the non-linear system when the model is unknown. The *i*th rule of a T-S fuzzy model is written as follows:

Model rule *i*:

IF $z_1(t)$ *is* M_1^i *AND* … *AND* $z_n(t)$ *is* M_n^i

THEN $\dot{x}(t) = A_i x(t) + B_i u(t)$

$y(t) = C_i x(t)$

$M_j^i, j = 1,2,\dots,n,$ is the *j*th fuzzy set of the *i*th rule and $z_1(t),\dots,z_n(t)$ are known premise variables that may be functions of state variables, external disturbances and/or time. Let $\mu_j^i\left(z_j\right)$ be the membership function of the fuzzy set M_j^i and let:

$$h_i = h_i(t) = \prod_{j=1}^{n} \mu_j^i\left(z_j\right)$$

Given a pair $\left(z(t),\, u(t)\right)$, the resulting fuzzy system is inferred as the weighted average of the local models and has the following form:

$$\begin{aligned} \dot{x} &= \sum_{i=1}^{r} h_i \left\{A_i x(t) + B_i u(t)\right\} / \sum_{i=1}^{r} h_i \\ &= \sum_{i=1}^{r} \alpha_i \left\{A_i x(t) + B_i u(t)\right\} \qquad (1) \\ y &= \sum_{i=1}^{r} \alpha_i C_i x(t) \end{aligned}$$

where $\alpha_i = h_i / \sum_{i=1}^{r} h_i,\ 0 \le \alpha_i \le 1,\ \sum_{i=1}^{r} \alpha_i = 1,$ for $i = 1,2,\dots,r.$

The PDC offers a procedure to design a fuzzy controller from a given T-S fuzzy model (Takagi and Sugeno, 1985; Akar and Ozguner, 2000). In the PDC design, each control rule is associated with the corresponding rule of a T-S fuzzy model. The designed fuzzy controller shares the same fuzzy sets with the fuzzy model in the premise parts. For a T-S fuzzy model as described in (1), the following state feedback fuzzy controller is constructed via PDC as follows:

Model rule # *i*:

IF $z_1(t)$ *is* M_1^i *AND* ... *AND* $z_n(t)$ *is* M_n^i

THEN $u(t) = F_i x(t), \quad i = 1,2,\dots,r$

The fuzzy control rules have a linear controller in the consequent parts and the overall fuzzy controller is represented by:

$$u(t) = \sum_{i=1}^{r} h_i F_i x(t) / \sum_{i=1}^{r} h_i = \sum_{i=1}^{r} \alpha_i F_i x(t) \qquad (2)$$

Although the fuzzy controller (2) is constructed using local design structures, the feedback gains must be determined using global design conditions to guarantee global stability and performance. The methods for stability analysis and control design of T-S fuzzy systems are classified into different categories as reported in Tanaka and Wang (2001). The analysis adopted in this paper seeks to find a common Lyapunov matrix for all the local subsystems in a T-S fuzzy model (Akar and Ozguner, 2000; Akhenak et al., 2004; Bergsten et al., 2002; Chadli et al., 2004; Tanaka et al., 1998; Tanaka and Wang, 1997; Kang et al., 1998; Chilali et al., 1999).

Substituting (2) in (1), the augmented system is given by:

$$\dot{x} = \sum_{i=1}^{r} \sum_{j=1}^{r} \alpha_i \alpha_j \left\{A_i + B_i F_j\right\} x(t)$$

Denoting $G_{ij} = A_i + B_i F_j$,

$$\dot{x} = \sum_{i=1}^{r} \alpha_i^2 G_{ii} x(t) + 2 \sum_{i=1}^{r} \sum_{i<j}^{r} \alpha_i \alpha_j \left(\frac{G_{ij} + G_{ji}}{2}\right) x(t) \qquad (3)$$

Theorem 1: The T-S fuzzy model (3) is globally asymptotically stable if there exists a common positive definite matrix *X* such that:

$$G_{ii}{}^T X + X G_{ii} < 0, \quad i = 1,2,\dots,r \qquad (4)$$

$$\left(\frac{G_{ij} + G_{ji}}{2}\right)^T X + X \left(\frac{G_{ij} + G_{ji}}{2}\right) \le 0, \quad i < j,\ \alpha_i \cap \alpha_j \ne \varphi \qquad (5)$$

Proof: See Werner et al. (2003).

Corollary 1: Assume that $B_i = B,\ i = 1,2,\dots,r,$ the equilibrium of the fuzzy control system (3) is globally quadratically stable if a common positive definite matrix *X* exists and satisfies (4) only. This follows directly because definite negativity of (4) implies semi-definite negativity of (5) in case of common *B* (Werner et al., 2003).

2.2 Power system uncertainties

Power systems consist mainly of a set of generating units, a transmission network and loads. These units interact with each other through active and reactive power generation (*P*, *Q*) over the transmission network. Briefly, any power system is composed of a set of inherently interacting subsystems, where each subsystem consists of a generating unit connected to the rest of the system by a tie line whose reactance is the Thevenin's reactance at the terminal bus ($X_e = X_{Th}$). For modelling and design approaches proposed in this work, a subsystem is considerably approximated by a single machine infinite bus (SMIB) system. This assumption is made possible because fuzzy modelling allows imprecision (Sugeno and Kang, 1986; Sugeno, 1999; Qiu et al., 2004). As a result of this approximation, each generator can be decoupled from the entire system. The influence of the rest of the system will be taken care of by the scheduling variables; namely its real and reactive powers (P, Q) and an equivalent tie line reactance (X_e). All possible dynamics at the interface between a generator and the rest of the system are supposed to be reflected by this set of scheduling variables (P, Q, X_e). This decoupling leads to a decentralised design.

The origin of power systems uncertainties are the continuous variations in load patterns and transmission network. Since the system is to be linearised around the equilibrium point, it follows that a different system triple (A, B, C) is obtained for each operating point. It is assumed that the set of variables (P, Q, X_e) of certain subsystem varies independently over the following ranges: $P \in \left[\overset{-}{P} \;\; \overset{+}{P}\right]$, $Q \in \left[\overset{-}{Q} \;\; \overset{+}{Q}\right]$, $X_e \in \left[\overset{-}{X}_e \;\; \overset{+}{X}_e\right]$. These ranges are selected to encompass all practical operating points and very weak to very strong transmission networks. Possible combinations of minimum and maximum values of these variables result in eight operating points corresponding to the vertices of a cuboid in the (P, Q, X_e) space. Consequently, a set of matrices obtained from an operating point can be represented by $(A, B, C) \in \Omega$, where:

$$\Omega = \left\{ \begin{array}{c} (A, B, C) : (A, B, C) = \sum_{i=1}^{8} \alpha_i \left(A_i, B_i, C_i\right), \\ \alpha_i \geq 0, \; \sum_{i=1}^{8} \alpha_i = 1 \end{array} \right\} \tag{6}$$

The set Ω describes a polytope with eight vertices $\left(A_i, B_i, C_i\right)$, $i = 1, 2, \ldots, 8$ calculated at $\left[\overset{-}{P}, \overset{-}{Q}, \overset{-}{X}_e\right]$, $\left[\overset{-}{P}, \overset{-}{Q}, \overset{+}{X}_e\right]$, … ,$\left[\overset{+}{P}, \overset{+}{Q}, \overset{+}{X}_e\right]$ respectively. Changes in load and system topology or most of system parameters lead to uncertainties in the state matrix A. Uncertainties in the input matrix B can only be caused by parametric variations in the excitation system and are not taken into account in this work. Rotor speed deviation is selected as the measured output and then no uncertainties appear in matrix C.

2.3 Dynamic T-S fuzzy model

Each vertex system in the polytope (6) corresponds to a model rule in a T-S fuzzy system which is stated as follows:

Model rule 1:

IF $\left(P \text{ is about } \overset{-}{P}\right)$ AND $\left(Q \text{ is about } \overset{-}{Q}\right)$ AND $\left(X_e \text{ is about } \overset{-}{X}_e\right)$

THEN $\begin{bmatrix} \dot{x} \\ y \end{bmatrix} = \begin{bmatrix} A_1 & B \\ C & 0 \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix}$

Model rule 2:

IF $\left(P \text{ is about } \overset{-}{P}\right)$ AND $\left(Q \text{ is about } \overset{-}{Q}\right)$ AND $\left(X_e \text{ is about } \overset{+}{X}_e\right)$

THEN $\begin{bmatrix} \dot{x} \\ y \end{bmatrix} = \begin{bmatrix} A_2 & B \\ C & 0 \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix}$

,…,

Model rule 8:

IF $\left(P \text{ is about } \overset{+}{P}\right)$ AND $\left(Q \text{ is about } \overset{+}{Q}\right)$ AND $\left(X_e \text{ is about } \overset{+}{X}_e\right)$

THEN $\begin{bmatrix} \dot{x} \\ y \end{bmatrix} = \begin{bmatrix} A_8 & B \\ C & 0 \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix}$

The resulting fuzzy system is inferred as the weighted average of the local models and has the form:

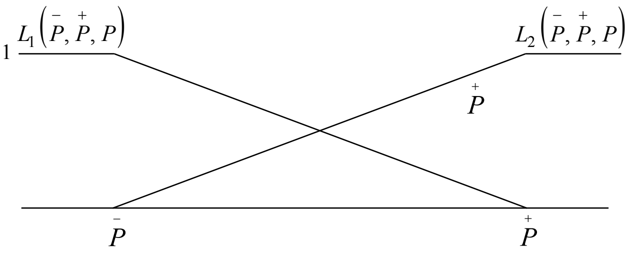
$$\begin{bmatrix} \dot{x} \\ y \end{bmatrix} = \begin{bmatrix} \left(\sum_{i=1}^{8} \alpha_i A_i\right) & B \\ C & 0 \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} \tag{7}$$

Any value $P \in \left[\overset{-}{P} \;\; \overset{+}{P}\right]$ can be expressed as $P = L_1\left(\overset{-}{P}, \overset{+}{P}, P\right) \times \overset{-}{P} + L_2\left(\overset{-}{P}, \overset{+}{P}, P\right) \times \overset{+}{P}$, where $L_1\left(\overset{-}{P}, \overset{+}{P}, P\right)$ and $L_2\left(\overset{-}{P}, \overset{+}{P}, P\right)$ are membership functions of the variable P such that $L_1\left(\overset{-}{P}, \overset{+}{P}, P\right) + L_2\left(\overset{-}{P}, \overset{+}{P}, P\right) = 1$; consequently, these membership functions can be calculated as:

$$L_1\left(\overset{-}{P}, \overset{+}{P}, P\right) = \frac{\overset{+}{P} - P}{\overset{+}{P} - \overset{-}{P}}, \quad L_2\left(\overset{-}{P}, \overset{+}{P}, P\right) = \frac{P - \overset{-}{P}}{\overset{+}{P} - \overset{-}{P}} \tag{8}$$

The membership functions $L_1\left(\overset{-}{P}, \overset{+}{P}, P\right)$ and $L_2\left(\overset{-}{P}, \overset{+}{P}, P\right)$ are labelled '$\overset{-}{P}$' and '$\overset{+}{P}$' respectively. Figure 1 shows the membership functions for the variable P. In a similar manner, membership functions for Q and X_e are defined and labelled M_1, M_2 and N_1, N_2 respectively. The weights are calculated as $h_1 = L_1 M_1 N_1$, $h_2 = L_1 M_1 N_2$, $h_3 = L_1 M_2 N_1$, … , and $h_8 = L_2 M_2 N_2$.

Figure 1 Membership functions of the scheduling variable P

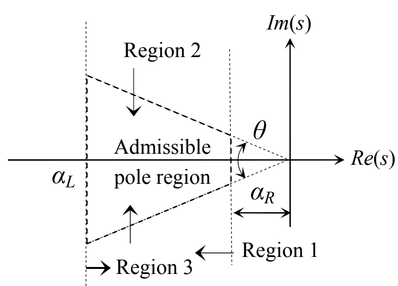


Remark 1: In the proposed modelling approach, it should be noticed that a single machine subsystem is approximated by a separate T-S fuzzy model. As result of this approach, a multimachine power system could be decomposed into a set of T-S fuzzy models that allow for a decentralised design. Interactions between different T-S fuzzy models are guaranteed by a set of scheduling variables (P, Q, X_e) that appear in the premise parts of a model. The sets of different models vary simultaneously and dependently via the network.

3 Representing power system specifications as an LMI region

In power systems, a damping ratio of at least 10% and a real part not greater than –0.5 guarantees better damping characteristics for low frequency oscillations (Ramos et al., 2003). These transient response specifications can be satisfied by clustering the closed-loop poles in the admissible region shown in Figure 2. This ensures a minimum decay rate α_R and a minimum damping $\xi_{\min} = \cos(\theta/2)$. This in turn bounds the maximum overshoot and the settling time of the closed-loop system. To avoid very large feedback gains, the real part of the poles should be placed to the right of the α_L line.

Figure 2 LMI region



Notes: Region 1 guarantees an upper bound on the settling time, Region 2 guarantees sufficient damping of the system and Region 3 prevent controller gains from being excessively large.

The admissible region is expressed as an LMI region defined by three individual LMI regions as shown in Figure 2. The intersection of the LMI regions results in another LMI region. An LMI region is any subset D of the complex plane defined by Chilali and Gahinet (1996) as follows:

$$D = \left\{ s \in C : \Phi + s\Psi + \bar{s}\Psi^T < 0 \right\} \tag{9}$$

where Φ and Ψ are real matrices and $\Phi = \Phi^T$. The region matrices Φ and Ψ are calculated from the values of α_R, α_L and θ as clarified in Chilali and Gahinet (1996) and Cao et al. (1998). An LMI condition for D-stability of a closed-loop system with state matrix A_{cl} is given by the following lemma.

Lemma 1 (Chilali and Gahinet, 1996): The matrix A_{cl} is D-stable if and only if there exists a symmetric, positive definite matrix X such that:

$$\Phi \otimes X + \Psi \otimes (XA_{cl}) + \Psi^T \otimes (XA_{cl})^T < 0 \tag{10}$$

Proof: See Chilali and Gahinet (1996) and Cao et al. (1998).

4 Synthesis of a fuzzy static output feedback PSS

Typically, PSS has the speed deviation as a feedback signal. In such case, attention is oriented towards output feedback design methods. This section studies the design of a static output feedback PSS for power systems described by continuous T-S fuzzy models. Generally, the problem of a static output feedback leads to a BMI which is non-convex. Many papers addressed this problem and present iterative LMI techniques to solve this problem, e.g., He and Wang (2006), Fujimori (2004), Yu (2004), Haung and Nguang (2006) and Crusius and Trofino (1999). Chadli et al. (2002) and Gahinet et al. (1995) present a solution for the static output feedback via an equality constraint. A fuzzy static output feedback PSS shares the same fuzzy sets with the fuzzy model as follows:

$$u(t) = \sum_{i=1}^{r} \alpha_i F_i y(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \alpha_i \alpha_j \left\{ F_i C_j x(t) \right\} \tag{11}$$

where F_i are the local static output feedback gains. By substituting (11) in T-S model (1), we obtain:

$$\dot{x} = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{\ell=1}^{r} \alpha_i \alpha_j \alpha_\ell \left\{ A_i + B_i F_j C_\ell \right\} x(t) \tag{12}$$

For the case of power systems, $B_i = B$, $C_\ell = C$, i, $\ell = 1, 2, \ldots, r$, then (12) can be rewritten as follows:

$$\dot{x} = \sum_{i=1}^{r} \alpha_i \left\{ A_i + BF_i C \right\} x(t) \tag{13}$$

The following theorem gives sufficient conditions in LMI form to ensure D-stability of (13).

Theorem 2: Let $F_i = N_i M^{-1}$, the eigenvalues of (13) lie in the LMI region (9) if the matrices R, M, N_i, $i = 1, 2, \ldots, r$ exist such that the following LMIs hold.

$$R > 0 \tag{14.a}$$

$$\Phi \otimes R + \Psi \otimes \left(A_i R + B N_i C \right) + \Psi^T \otimes \left(A_i R + B N_i C \right)^T < 0, \quad i = 1, 2, \ldots, r \tag{14.b}$$

$$MC = CR \tag{14.c}$$

Proof: Substituting (13) in (10), we get:

$$\Phi \otimes X + \Psi \otimes \left(X \left\{ A_i + B F_i C \right\} \right) + \Psi^T \otimes \left(X \left\{ A_i + B F_i C \right\} \right)^T < 0 \tag{15}$$

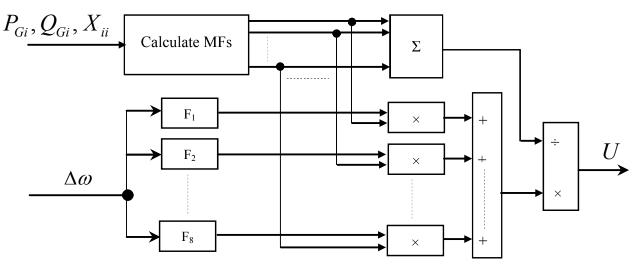
Performing congruerence transformation with ($I \otimes X^{-1}$) on (15) leads to:

$$\Phi \otimes X^{-1} + \Psi \otimes \left(A_i X^{-1} + B F_i C X^{-1} \right) + \Psi^T \otimes \left(A_i X^{-1} + B F_i C X^{-1} \right)^T < 0$$

Putting $X^{-1} = R$, $CR = MC$ and $F_i M = N_i$ leads to LMIs (14).

Remark 2: Since the matrix C is full row rank as the case studied herein, one can deduce from (14.c) that there exists a non-singular matrix $M = CRC^T \left(CC^T \right)^{-1}$.

Figure 3 Schematic diagram for the proposed stabiliser on Gen # *i*



The design steps can be summarised as follows:

1 Determine the ranges $P \in \left[\overset{-}{P} \;\; \overset{+}{P} \right]$, $Q \in \left[\overset{-}{Q} \;\; \overset{+}{Q} \right]$ and $X_e \in \left[\overset{-}{X_e} \;\; \overset{+}{X_e} \right]$ that encompass all practical operating conditions.

2 Define the eight local models of the polytope (6) by calculating $A_1, A_2, \ldots, A_8$, B and C.

3 Define the membership functions as given by (8) according to the ranges of P, Q and X_e in (i).

4 Generate the T-S fuzzy system (7).

5 Define α_R, α_L and θ. Then, compute the LMI region matrices Φ and ψ in (9) as clarified in Chilali and Gahinet (1996).

6 Solve the optimisation problem in (14) to get the static gains of the stabiliser F_i, $i = 1, 2, \ldots, 8$ using an appropriate LMI solver, e.g., Gahinet et al. (1995).

7 Impalement the control law given by (11) as illustrated in Figure 3.

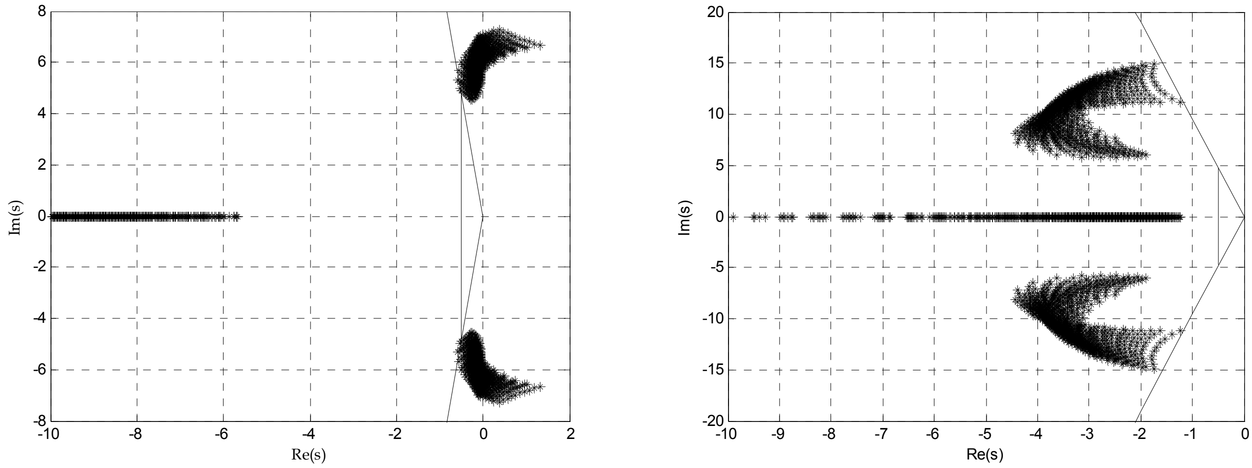
5 Design validation and simulation results

The proposed PSS design is validated in this section based on two different non-linear models. The first is an SMIB model which is used to illustrate the design steps. The second model is a four-machine two-area system which is used as a benchmark problem in the literature. In applying our design algorithm to the multimachine system, each single machine subsystem is approximated by an SMIB model for the design purpose only. The effect of the rest of the system is reflected on the calculation of the line reactance and the power delivered to the system. Consequently, a PSS is designed independently for each machine. The implementation details for the proposed stabiliser are shown in Figure 3.

5.1 *The SMIB test system*

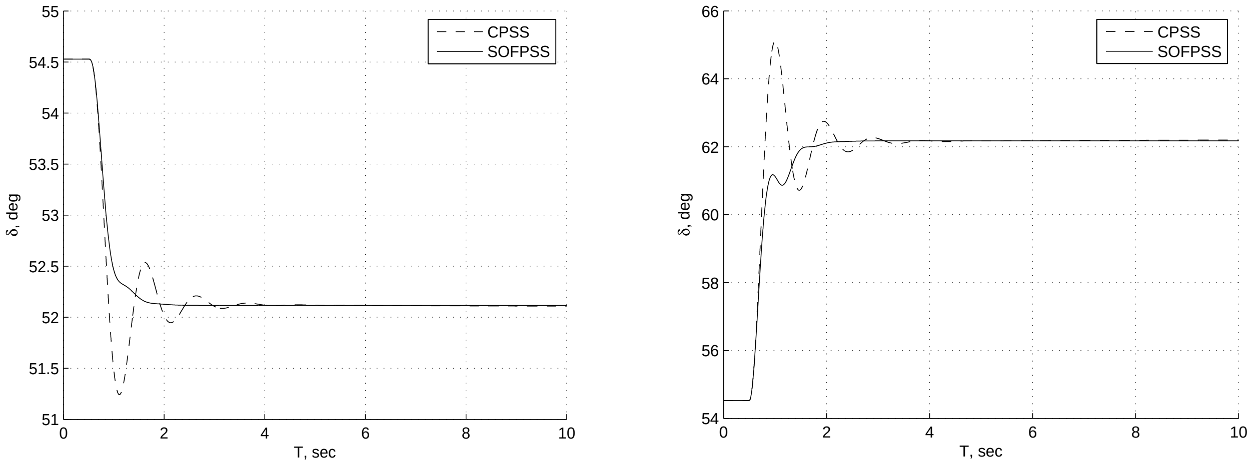
The study in this section will be carried on an SMIB system whose model and data are given in Appendix A.1. P and Q at the generator terminals and X_e are assumed to vary independently over the following ranges; provided that all points included have a steady state load flow solution: $P \in [0.4\ 1.0]$, $Q \in [-0.2\ 0.5]$ and $X_e \in [0.2\ 0.4]$. Figure 4(a) shows the dominant open-loop poles for 1,000 plants as P, Q and X_e vary over their specified ranges. It is noted that most of the plants in this polytope do not have adequate damping and some plants are unstable. The proposed design is carried out for an LMI region bounded by $\alpha_L = -1{,}000$, $\alpha_R = -0.5$ and $\theta = 168°$. The matrices Φ and ψ of the LMI region are computed and listed in Appendix A.2. The optimisation problem (14) is solved to calculate the static feedback gains F_i. The resulting values of the stabiliser gains are listed in Appendix A.3. Figure 4(b) shows the efficacy of the proposed design in clustering the system roots in the predefined LMI region. The time response of three operating conditions is studied and depicted in Figures 5, 6 and 7. The CPSS for the same unit is given in Soliman et al. (2008) and adopted for comparison. It is obvious that the proposed design outperforms the conventional PSS even at the nominal point. The conventional design fails to maintain stability at full load with leading power factor as shown in Figure 6 and fails for the case of overload with unity power factor as shown in Figure 7 as well.

Figure 4 Dominant poles, (a) open-loop (b) closed-loop with the proposed design



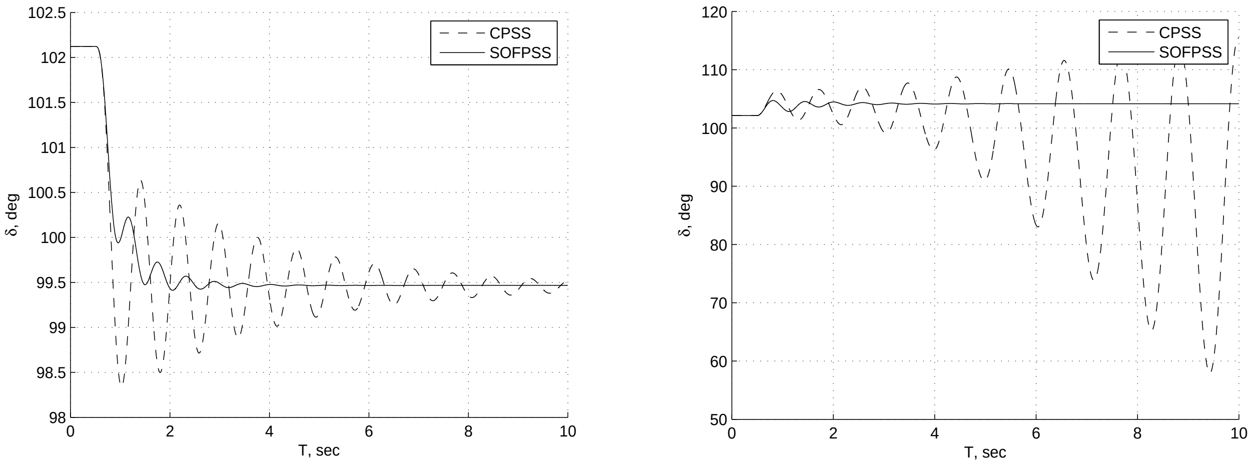
(a) (b)

Figure 5 Rotor angle at $P = 0.9$ pu, $Q = 0.5$ pu and $X_e = 0.4$ pu, (a) due to 2% step change in reference voltage V_{ref} (b) due to 20% step change in mechanical torque T_m



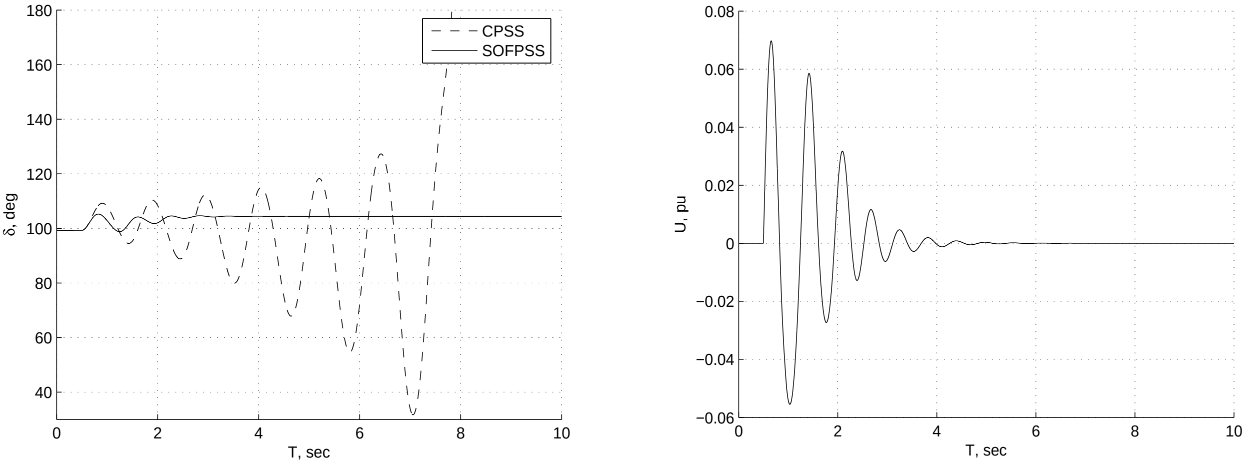
(a) (b)

Figure 6 Rotor angle at $P = 1.0$ pu, $Q = -0.1$ pu and $X_e = 0.4$ pu, (a) due to 2% step change in reference voltage V_{ref} (b) due to 10% step change in mechanical torque T_m



(a) (b)

Figure 7 System response due to 20% step change in mechanical torque T_m at $P = 1.1$ pu, $Q = 0$ pu and $X_e = 0.4$ pu, (a) rotor angle (b) control signal in pu by the proposed design SOFPSS



(a) (b)

5.2 *Decentralised application in a multimachine test power system*

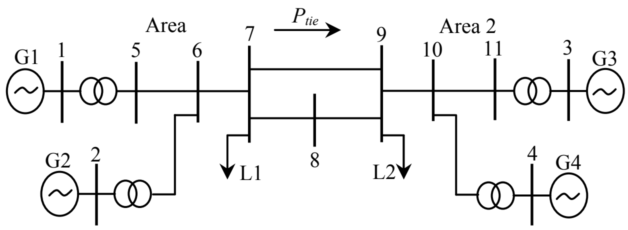
The benchmark two-area model shown in Figure 8 is adopted for simulation studies. The test system consists of two fully symmetrical areas linked together by two 230 kV lines of 220 km length. It is specifically designed in Kundur (1994) to study low frequency electromechanical oscillations in large interconnected power systems. Each area is equipped with two identical round rotor generators rated 20 kV/900 MVA. The synchronous machines have identical parameters except for the inertias which are $H = 6.5$ s in area 1 and $H = 6.175$ s in area 2. Thermal plants having identical speed regulators are further assumed at all locations, in addition to fast static exciter with a gain of 200. Each generator is represented by a seventh order model. The loads are represented as constant impedances and spilt between the areas. Each generator is equipped with a conventional PSS as designed in Kundur (1994) and Soliman et al. (2008) for the same test system. A general procedure to separately design a PSS for each generator includes the following steps:

1 the load flow study is carried out for different loading conditions that may be encountered during the power system operation to obtain the ranges $P_i \in \left[\overset{-}{P_i} \quad \overset{+}{P_i}\right]$ and $Q_i \in \left[\overset{-}{Q_i} \quad \overset{+}{Q_i}\right]$ for different generators, where $i = 1, 2, \dots, n$ and n is the generator index

2 for different network topologies (normal and contingency conditions are assumed), the bus impedance matrix is calculated and different self-impedances are determined at the generator buses to get $X_i \in \left[\overset{-}{X_i} \quad \overset{+}{X_i}\right]$, where $i = 1, 2, \dots, n$ and n is the generator index

3 once all ranges are determined, the steps described in Section 4 are used to find a T-S fuzzy static stabiliser for each generator separately.

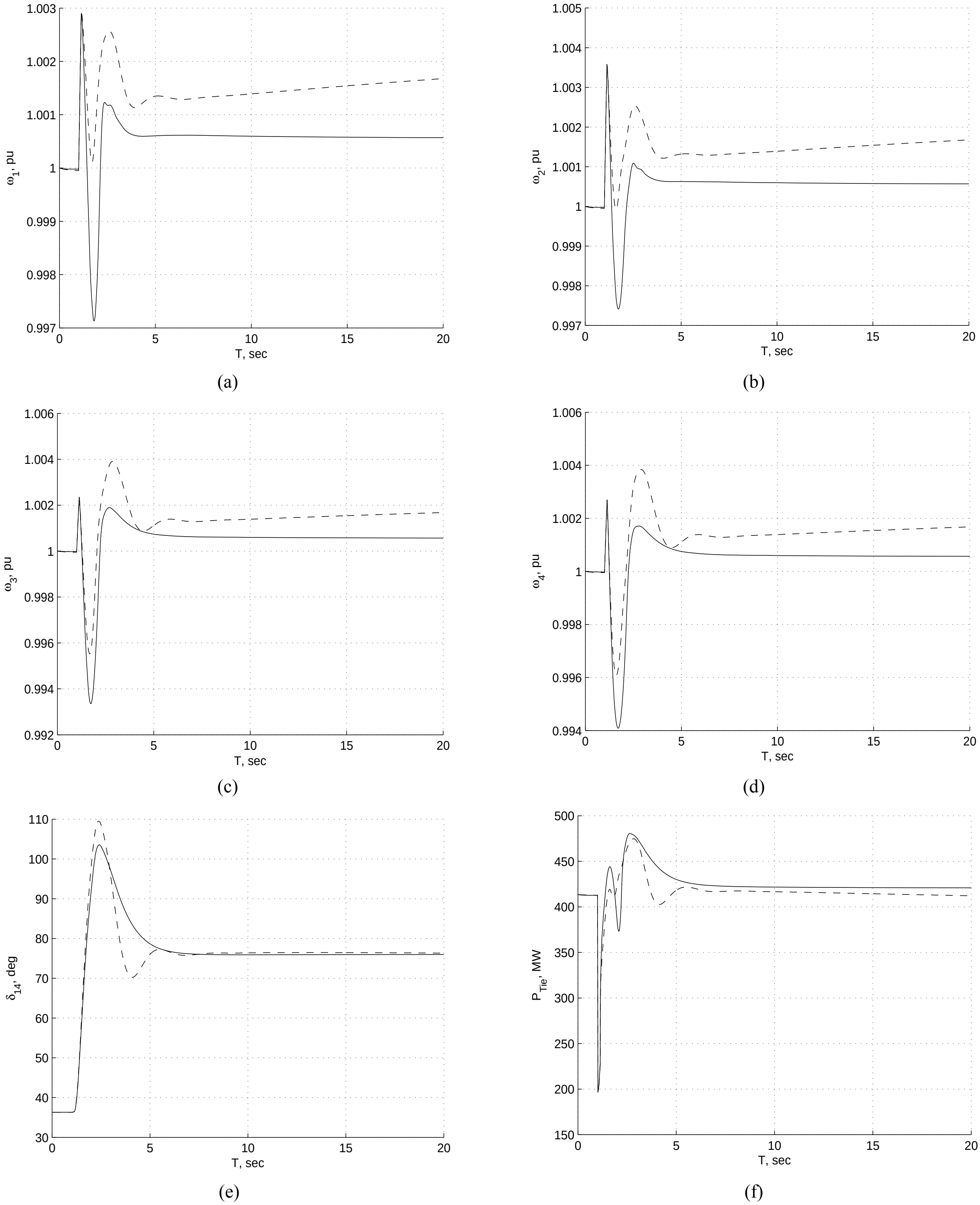
Figure 8 Four-machine two-area test power system



Source: Kundur (1994)

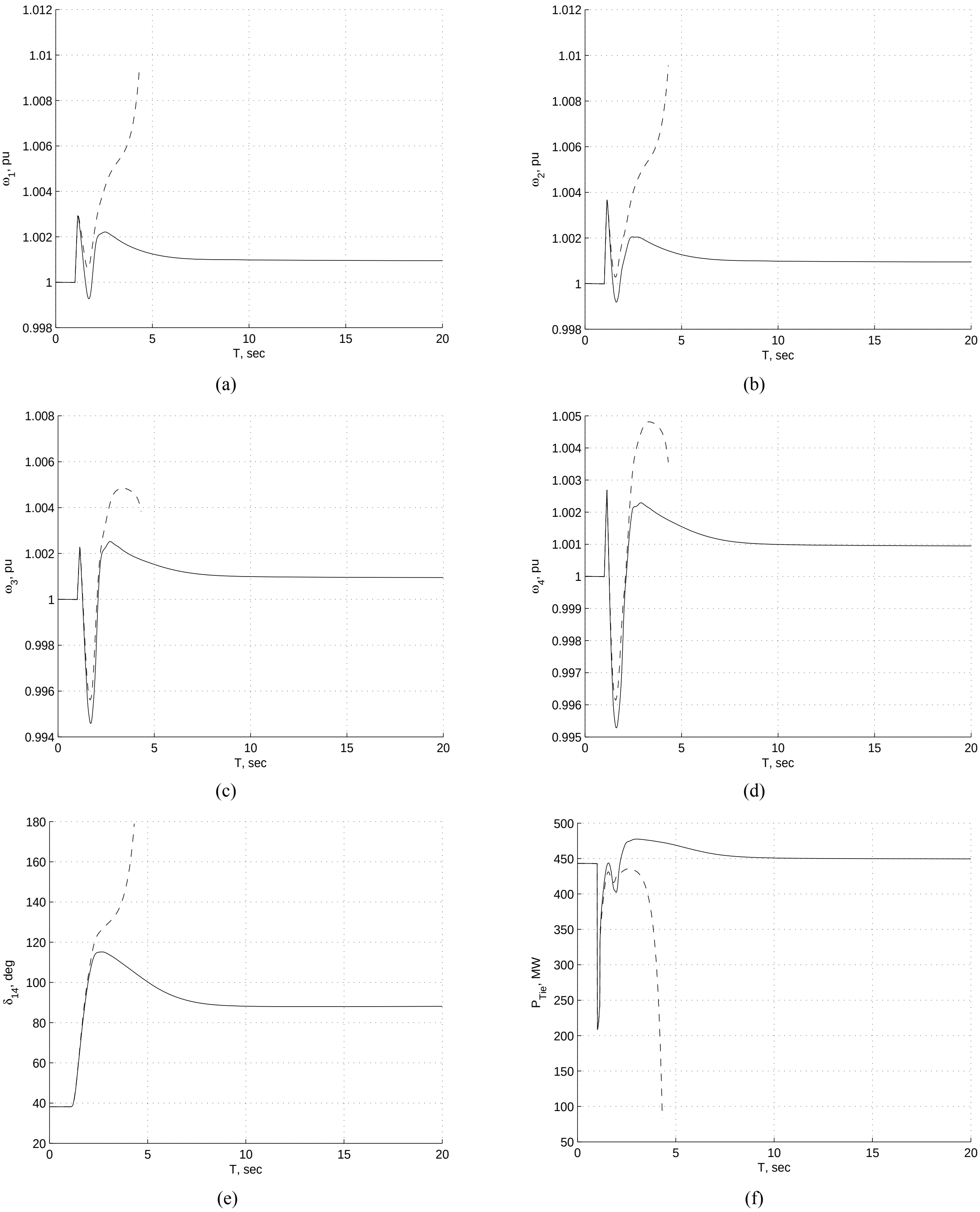
The proposed PSS is compared to the conventional stabiliser at two test points. For fair comparison, all simulation results consider saturation limits of ±0.15 pu on the control signals provided either by CPSS or by the proposed stabiliser. Figure 9 depicts the system response due to a three-phase short circuit at bus 8 when the nominal tie line power is transferred from area 1 to area 2. The fault is cleared after 0.133 sec by opening the two breakers at the ends of the faulty line causing one tie line separation. It is clear that the proposed PSS outperforms CPSS even at the nominal point. If the same fault occurs at larger tie line power, the conventional design fails to maintain system stability; however, the proposed PSS gives the acceptable damping characteristics as shown in Figure 10. The control signals of this case provided by the proposed stabilisers to the four machines are depicted in Figure 11.

Figure 9 System response due to three-phase short circuit at the middle of one tie line cleared after 0.133 sec, (a) rotors speed (pu) for m/cs: 1–4 respectively (b) rotors speed (pu) for m/cs: 1–4 respectively (c) rotors speed (pu) for m/cs: 1–4 respectively (d) rotors speed (pu) for m/cs: 1–4 respectively (e) relative rotor angle (deg.) between m/c-1 and m/c-4 (f) tie line power (MW) from area 1 to area 2



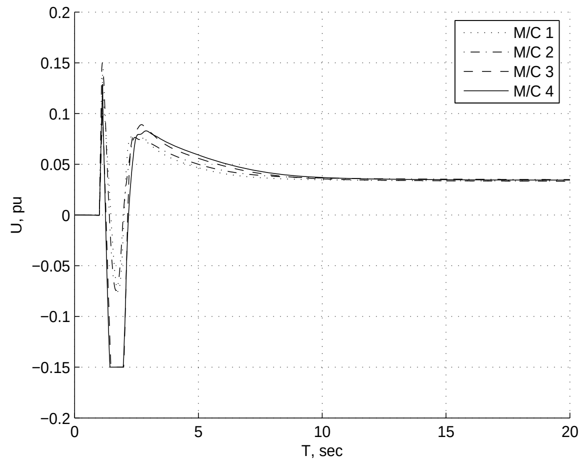
Notes: – – – CPSS and —— proposed stabiliser SOFPSS

Figure 10 System response due to three-phase short circuit at the middle of one tie line cleared after 0.133 sec, (a) rotors speed (pu) for m/cs: 1–4 respectively (b) rotors speed (pu) for m/cs: 1–4 respectively (c) rotors speed (pu) for m/cs: 1–4 respectively (d) rotors speed (pu) for m/cs: 1–4 respectively (e) relative rotor angle (deg.) between m/c-1 and m/c-4 (f) tie line power (MW) from area 1 to area 2



Notes: – – – CPSS and —— proposed stabiliser SOFPSS

Figure 11 The control signal on the four machines provided by the proposed design SOFPSS



6 Conclusions

A design of a PSS that can cope with a wide range of loading conditions and external disturbances has been the objective of the power industry. This paper has provided a step towards this goal. One of the contributions here has been to show that the non-linear model of a power system can be systematically represented in the form of a T-S fuzzy system. This has allowed us to use an approximate design model of the power system to develop a stabiliser that copes with different operating conditions and disturbances. Since the fuzzy model is a polytopic system, the proposed design assures stability and performance for all operating points within the polytope.

A static output feedback fuzzy PSS that guarantees robust pole placement in an LMI region has been designed. The design conditions have been derived via an LMI approach. Simulation results of a four-machine two-area power system have confirmed the superiority of the proposed algorithm in damping the post-fault inter-area oscillations. Compared to a well-tuned conventional PSS, it has been shown that the proposed PSS has a superior capability to cope with larger tie line power.

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Appendix

A.1 Machine data and model adopted for SMIB simulation

$$x_d = 1.8,\ x'_d = 0.3,\ x_q = 1.7,\ T'_{do} = 8,\ M = 13,$$
$$\omega_o = 377,\ V^\infty = 1.0,\ K_A = 200,\ T_A = 0.001,$$
$$x_e \in [0.2\ \ 0.4]$$

$$\dot{\delta} = \omega_o \omega$$
$$\dot{\omega} = \left(T_m - E'_q I_q - \left(x_q - x'_d\right) I_d I_q\right) / M$$
$$\dot{E}'_q = \left(-E'_q - \left(x_d - x'_d\right) I_d + E_{fd}\right) / T'_{do}$$
$$\dot{E}_{fd} = \frac{K_E}{T_E}\left(V_{ref} - V_T + u_{pss}\right) - \frac{1}{T_E} E_{fd}$$

A.2 The matrices of the LMI region

$$\Phi = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -2000 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\psi = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0.99452 & -0.10453 \\ 0 & 0 & 0.10453 & 0.99452 \end{bmatrix}$$

A.3 The static output feedback gains

$$F = \begin{bmatrix} 40.895 \\ 38.738 \\ 35.542 \\ 40.319 \\ 47.211 \\ 38.610 \\ 48.256 \\ 45.479 \end{bmatrix}$$

A.4 Nomenclature

V_t	Terminal voltage
E_q	Induced EMF proportional to field current
E_{fd}	Generator field voltage
V_{ref}	The reference voltage
x_e	Equivalent tie line reactance
x'_d, x_d, x_q	Generator direct-axis transient reactance, direct and quadrature synchronous reactances
δ	Angle between *q*-axis and infinite bus bar
I_d, I_q	Direct and quadrature stator currents
$\Delta\omega$	Speed deviation
ω_o	Synchronous speed (rad/sec)
T_e, T_m	Electrical torque and mechanical torque
T'_{d0}	Open circuit *d*-axis transient time constant
M	Inertia coefficient in seconds
K_E, T_E	Exciter gain and time constant
V^∞	Infinite bus bar voltage
P, Q	Active and reactive power loading, respectively
s, C	Complex operator and complex plane respectively
$\otimes$	Kronecker product
$X > 0$	Positive definite
$X \geq 0$	Positive semi-definite