ECE447: Robotics Engineering

Lecture 6: Forward Kinematics

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Faculty of Engineering (Shoubra)
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Lecture Outline:

1. Introduction.
2. Basic Assumptions and Terminology.
3. Denavit-Hartenberg Convention.
4. Assignment of Coordinate Frames.

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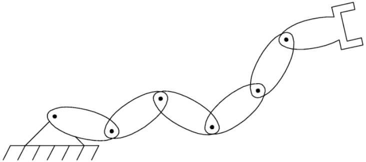
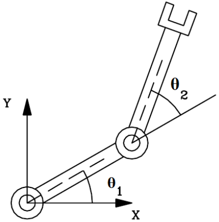
Introduction:

A manipulator is a kinematic chain composed by a series of rigid bodies, the **links**, connected by **joints** that allow a **relative motion**.

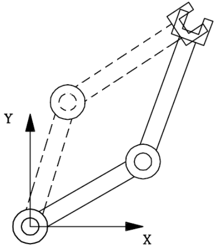


In robotic manipulation we are concerned with two common **kinematic** problems:

Forward Kinematics



Inverse Kinematics



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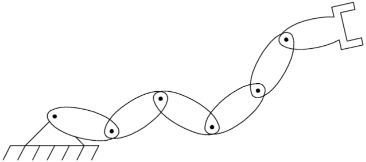

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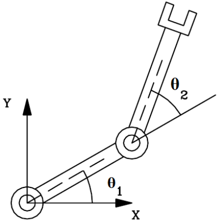

Given: Joint Variables **q** (θ or d)
Required: Position and orientation of end-effector, **p**.

$$\mathbf{p} = f(q_1, q_2, \ldots, q_n) = f(\mathbf{q})$$

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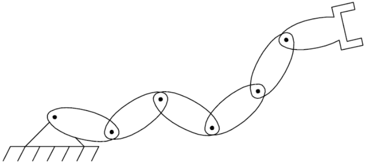

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Inverse Kinematics

Given: Position and orientation of end-effector, **p**.
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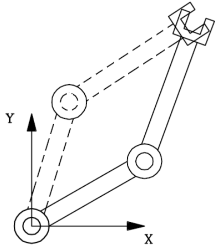
$$\mathbf{q} = f(\mathbf{p})$$



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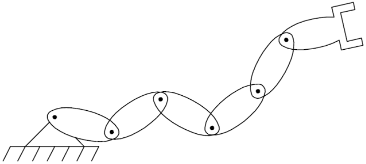

In robotic manipulation we are concerned with two common **kinematic** problems:

In this lecture, we will show how to find the **Forward Kinematics** of a rigid manipulator. Given the joints values and the pose of the end-effector is required.

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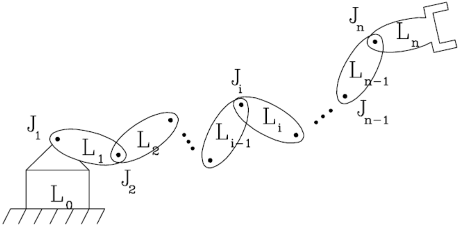
Basic Assumptions and Terminology:



- A robot manipulator is composed of a set of links connected together by joints.
- Joints can be either:
  - revolute joint (a rotation by an angle about fixed axis).
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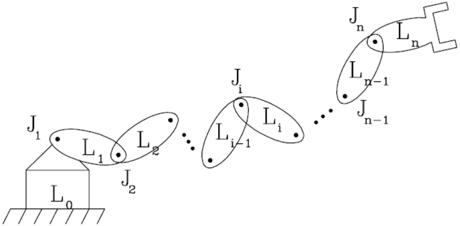

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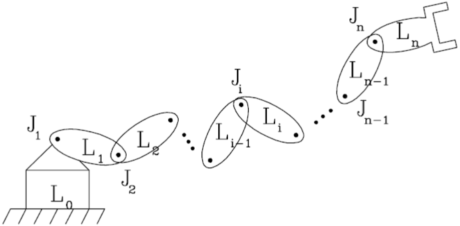

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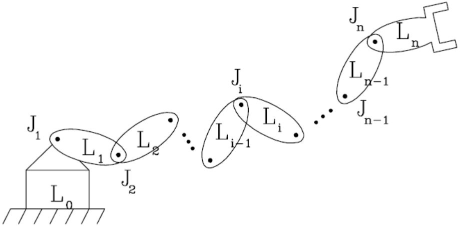

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- We number joints from 1 to n, and links from 0 to n. So that joint i connects links $i-1$ and i.
- The location of joint i is fixed with respect to the link $i-1$.

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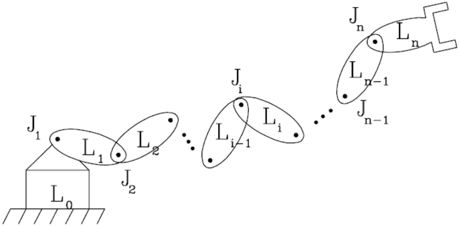

- When joint i is actuated, the link i moves. Hence the link 0 is fixed.

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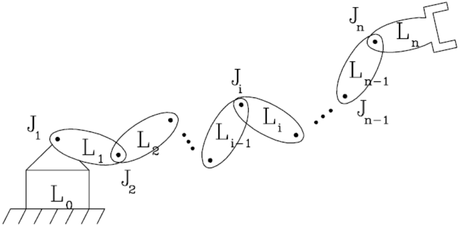

- When joint i is actuated, the link i moves. Hence the link 0 is fixed.
- With the i^{th} joint, we associate joint variable:

$$q_i = \left\{ \begin{array}{ll} \theta_i, & \text{if joint } i \text{ is revolute} \\ d_i, & \text{if joint } i \text{ is prismatic} \end{array} \right\}$$

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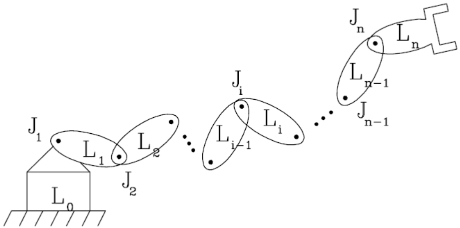

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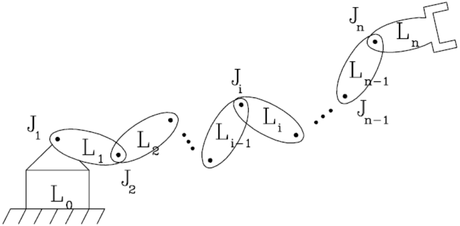

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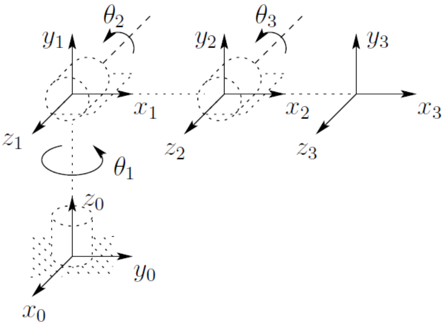
- For each link we attached rigidly the coordinate frame, $o_i x_i y_i z_i$ for the link i.
- The frame $o_0 x_0 y_0 z_0$ attached to the base is referred to as **inertia frame**.

Basic Assumptions and Terminology:

- If A_i is the homogeneous transformation that gives the position and orientation of frame $o_i x_i y_i z_i$ with respect to frame $o_{i-1} x_{i-1} y_{i-1} z_{i-1}$.



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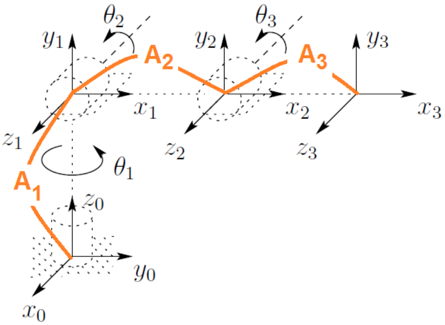

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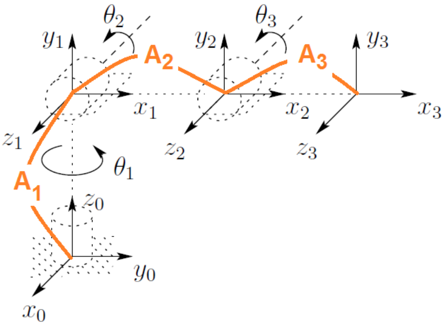
- The matrix A_i is changing as robot configuration changes and it is a function of the joint variables q_i i.e. $A_i(q_i)$.



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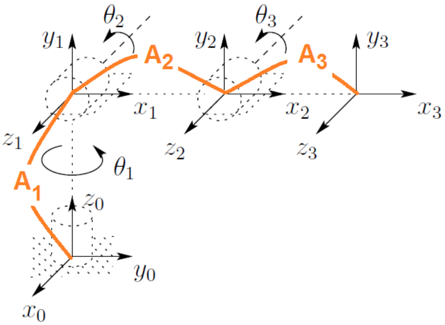
- The matrix A_i is changing as robot configuration changes and it is a function of the joint variables q_i i.e. $A_i(q_i)$.
- The matrix T_j^i is the homogeneous transformation that expresses the position and orientation of frame $\{j\}$ with respect to frame $\{i\}$:

$$T_j^i = \left\{ \begin{array}{ll} A_{i+1} A_{i+2} \ldots A_{j-1} A_j & \text{if } i < j \\ \mathcal{I} & \text{if } i = j \\ (T_i^j)^{-1} & \text{if } i > j \end{array} \right\}$$



Example of elbow manipulator

Basic Assumptions and Terminology:

- Suppose that the position and orientation of the end-effector with respect to the inertia frame are:

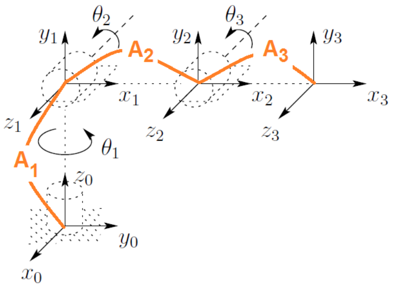
$$o_n^0, \qquad R_n^0$$

- Then the position and orientation of the end-effector in inertia frame are given by homogeneous transformation:

$$T_n^0 = A_1(q1)A_2(q_2)\dots A_{n-1}(q_{n-1})A_n(q_n) = \begin{bmatrix} R_n^0 & o_n^0 \\ 0 & 1 \end{bmatrix}$$

where,

$$A_i(q_i) = \begin{bmatrix} R_i^{i-1} & o_i^{i-1} \\ 0 & 1 \end{bmatrix}$$



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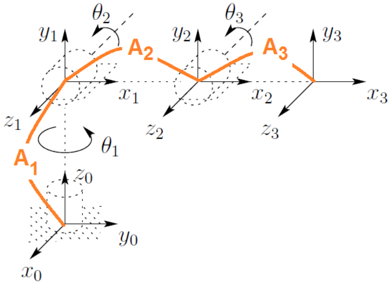
- So, to find the forward kinematics of a manipulator, we need to find all $A_i(q_i)$ and multiply them. (Not simple!)



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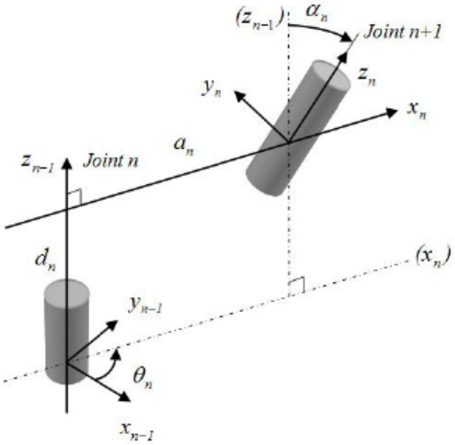
Denavit-Hartenberg Convention:

- The idea is to represent each homogeneous transform A_i as a product of four basic transformations:

$$A_i = \text{Rot}_{z,\theta_i}\ \text{Trans}_{z,d_i}\ \text{Trans}_{x,a_i}\ \text{Rot}_{x,\alpha_i}$$



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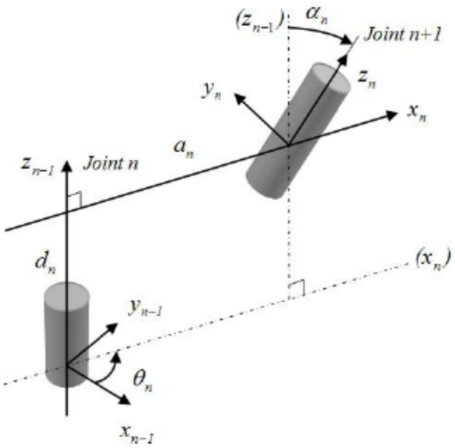

Four DH parameters are required:

1. a_i: link length, distance between z_{i-1} and z_i (along x_i).
2. α_i: link twist, angle between z_{i-1} and z_i (measured around x_i)
3. d_i: link offset, distance between o_{i-1} and intersection of z_{i-1} and x_i (along z_{i-1})
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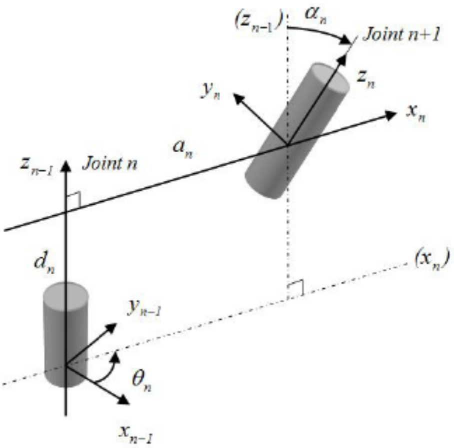

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Three of these DH parameters are constant while the forth is variable θ_i or d_i.

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$$A_i = \text{Rot}_{z,\theta_i}\text{Trans}_{z,d_i}\text{Trans}_{x,a_i}\text{Rot}_{x,\alpha_i}$$

$$= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & 0 \\ s_{\theta_i} & c_{\theta_i} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\alpha_i} & -s_{\alpha_i} & 0 \\ 0 & s_{\alpha_i} & c_{\alpha_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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⇦ If we found the DH parameter, it will be easy to fill this directly

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The Task:

- Given a robot manipulator with n revolute and/or prismatic joints and $(n+1)$ links,
- We need to define coordinate frames for each link so that transformations between frames can be written in DH-convention.

Denavit-Hartenberg Convention:

Example: Suppose the coordinate frames are assigned.

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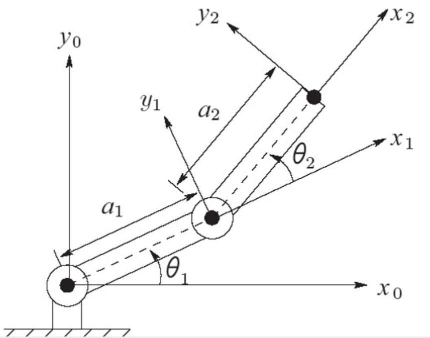
link	a_i	α_i	d_i	θ_i
1				
2				



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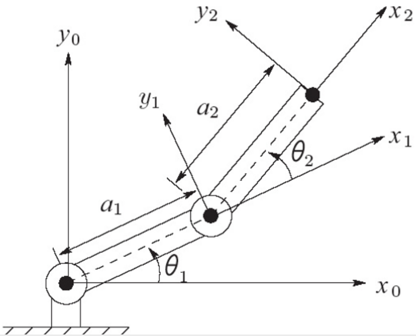
link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1
2	a_2	0	0	θ_2



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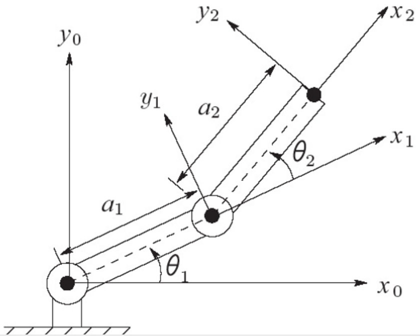
$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



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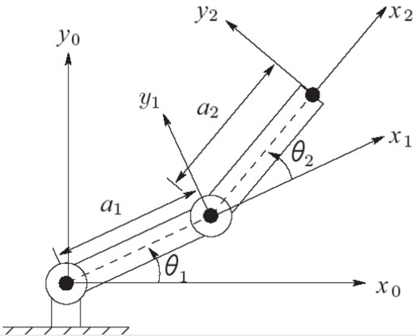
$$T_2^0 = A_1 A_2 = \begin{bmatrix} c_{12} & -s_{12} & 0 & a_1 c_1 + a_2 c_{12} \\ s_{12} & c_{12} & 0 & a_1 s_1 + a_2 s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



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Assignment of Coordinate Frames:

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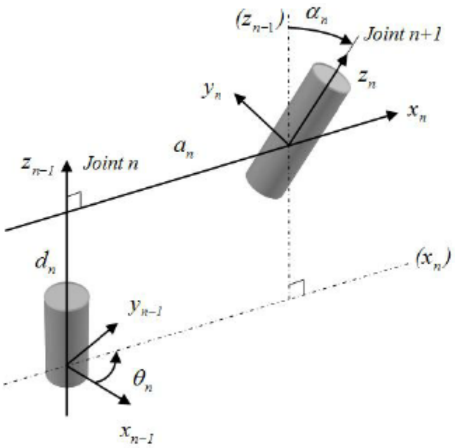

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- Given a robot manipulator with:
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- For a given robot manipulator, we need to assign the $n+1$ frames from 0 to n in such a way to satisfy two conditions:
  1. The axis x_1 is perpendicular to the axis z_0,
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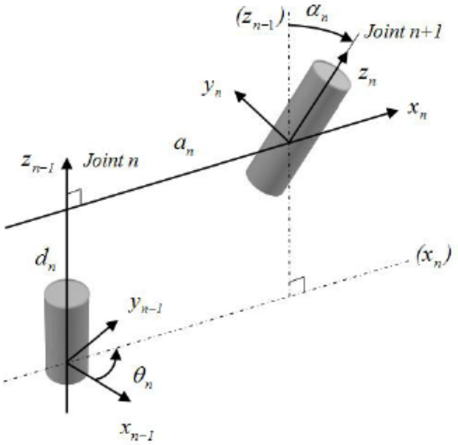

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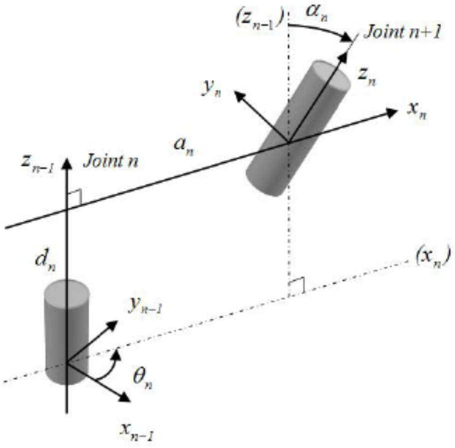
  1. The axis x_1 is perpendicular to the axis z_0,
  2. The axis x_1 intersects the axis z_0.
- This will help to represent each transformation A_i between frame i and frame $i-1$ by the four DH parameters:

$$A_i = \text{Rot}_{z,\theta_i}\ \text{Trans}_{z,d_i}\ \text{Trans}_{x,a_i}\ \text{Rot}_{x,\alpha_i}$$

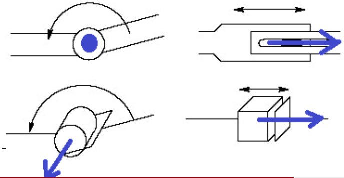


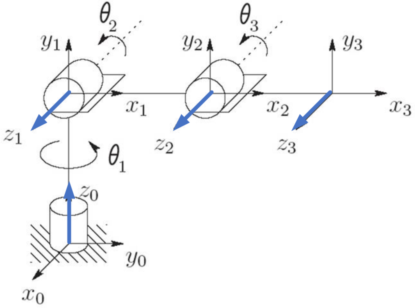
Assignment of Coordinate Frames:

Algorithm for Assigning the Coordinate Frames:

1. **Step 1: Choose z_i-axis along the actuation line of joint $i+1$ for frame 0 to $n-1$:**

- If joint $i+1$ is revolute, z_i is the axis of rotation of joint $i+1$.
- If joint $i+1$ is prismatic, z_i is the axis of translation for joint $i+1$
- z_n is chosen parallel to z_{n-1} and O_n in the center of the end-effector.



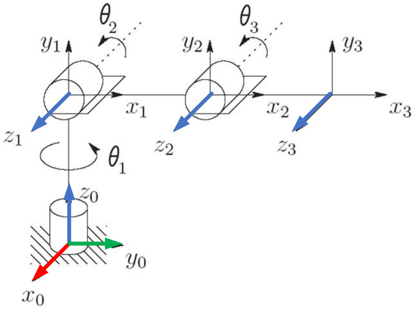


Assignment of Coordinate Frames:

Algorithm for Assigning the Coordinate Frames:

2. **Step 2: Write the inertia coordinate frame 0:**

- The origin O_0 of the base frame can be any point along z_0.
- x_0 and y_0 are chosen arbitrary that follow the right hand coordinate systems.

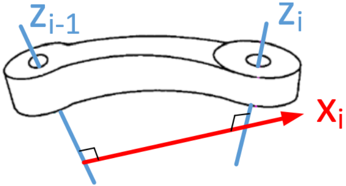


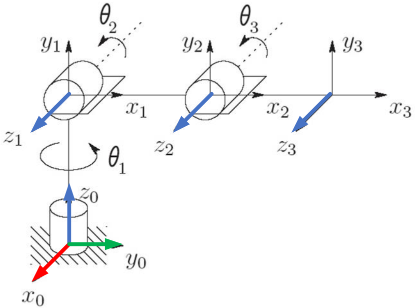
Assignment of Coordinate Frames:

Algorithm for Assigning the Coordinate Frames:

3. **Step 3: Assignment of axes x_i for frame 1 to frame n:**

- To meet the DH conditions, the x_i-axis should intersects z_{i-1} and $x_i \perp z_{i-1}$ and $x_i \perp z_i$.
  - CASE 1: z_i and z_{i-1} are **not coplanar**: then the x_i will be on the common normal to z_i and z_{i-1} and O_i is the intersection of x_i and z_i.



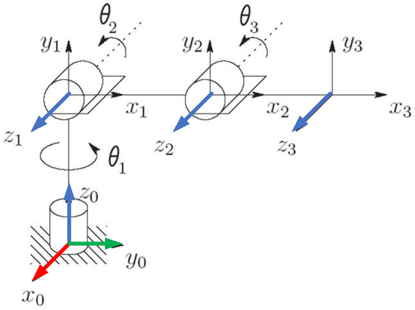


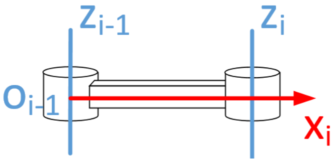
Assignment of Coordinate Frames:

Algorithm for Assigning the Coordinate Frames:

3. **Step 3: Assignment of axes x_i for frame 1 to frame n:**

- To meet the DH conditions, the x_i-axis should intersects z_{i-1} and $x_i \perp z_{i-1}$ and $x_i \perp z_i$.
  - CASE 2: z_i and z_{i-1} are **parallel:**
    x_i is along any of the many normals between z_i and z_{i-1}. However, if x_i is along the normal that intersects at o_{i-1}, d_i will be zero (simple). O_i is the intersection of x_i and z_i.



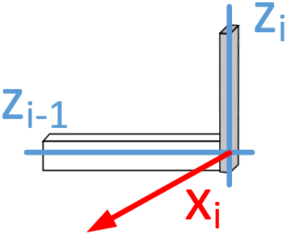


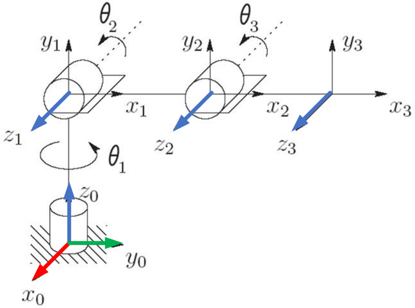
Assignment of Coordinate Frames:

Algorithm for Assigning the Coordinate Frames:

3. **Step 3: Assignment of axes x_i for frame 1 to frame n:**

- To meet the DH conditions, the x_i-axis should intersects z_{i-1} and $x_i \perp z_{i-1}$ and $x_i \perp z_i$.
  - CASE 3: z_i and z_{i-1} **intersect**:
    Choose x_i to be normal to the plane defined by z_i and z_{i-1} O_i is the intersection of z_{i-1} and z_i.





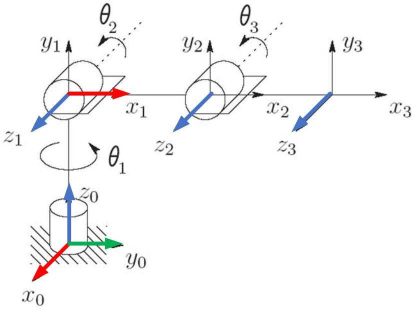
Assignment of Coordinate Frames:

Algorithm for Assigning the Coordinate Frames:

3. **Step 3: Assignment of axes x_i for frame 1 to frame n:**

In this example:

- z_0 and z_1 are perpendicular, x_1 is normal to both of them.



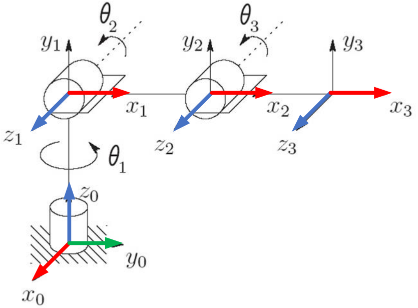
Assignment of Coordinate Frames:

Algorithm for Assigning the Coordinate Frames:

3. **Step 3: Assignment of axes x_i for frame 1 to frame n:**

In this example:

- z_0 and z_1 are perpendicular, x_1 is normal to both of them.
- z_1 and z_2 are parallel, x_2 is normal to both of them along line passing from O_1.



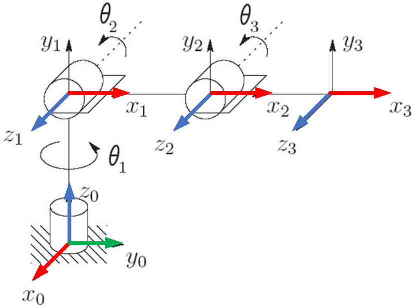
Assignment of Coordinate Frames:

Algorithm for Assigning the Coordinate Frames:

3. **Step 3: Assignment of axes x_i for frame 1 to frame n:**

In this example:

- z_0 and z_1 are perpendicular, x_1 is normal to both of them.
- z_1 and z_2 are parallel, x_2 is normal to both of them along line passing from O_1.
- z_2 and z_3 are parallel, x_3 is normal to both of them along line passing from O_2.

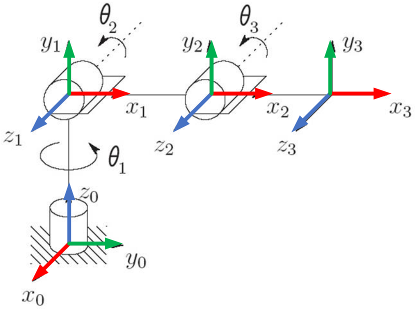


Assignment of Coordinate Frames:

Algorithm for Assigning the Coordinate Frames:

4. **Step 4: Assignment of axes y_i for frame 1 to frame n:**

- y_i are not useful in finding the DH parameters, but we choose them in the direction that follows the RH system.



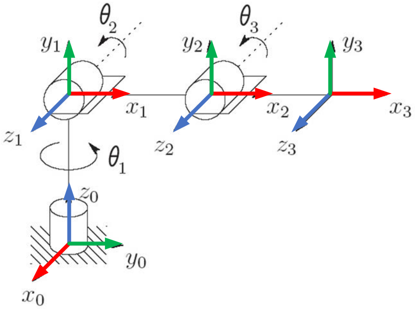
Assignment of Coordinate Frames:

Algorithm for Assigning the Coordinate Frames:

5. **Step 5: Find the DH parameters and write DH table for links from 1 to n:**

Four DH parameters are required:

1. a_i: link length, distance between z_{i-1} and z_i (along x_i).
2. α_i: link twist, angle between z_{i-1} and z_i (measured around x_i)
3. d_i: link offset, distance between o_{i-1} and intersection of z_{i-1} and x_i (along z_{i-1})
4. θ_i: joint angle, between x_{i-1} and x_i (measured around z_{i-1})



Assignment of Coordinate Frames:

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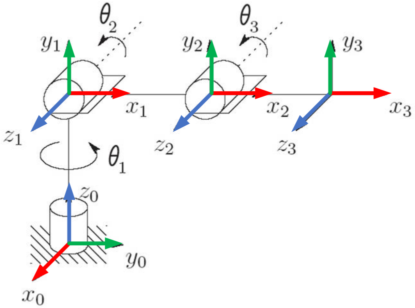
4. θ_i: joint angle, between x_{i-1} and x_i (measured around z_{i-1})

Link	a_i	α_i	d_i	θ_i
1				
2				
3				



Assignment of Coordinate Frames:

Algorithm for Assigning the Coordinate Frames:

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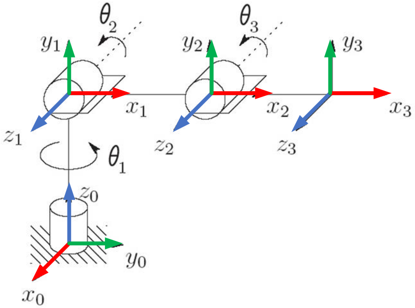
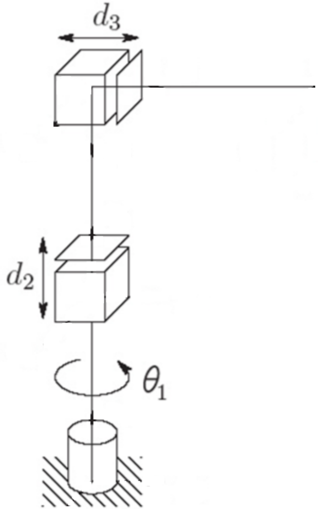
4. θ_i: joint angle, between x_{i-1} and x_i (measured around z_{i-1})



Link	a_i	α_i	d_i	θ_i
1	0	90	a_1	θ_1
2	a_2	0	0	θ_2
3	a_3	0	0	θ_3

Assignment of Coordinate Frames:

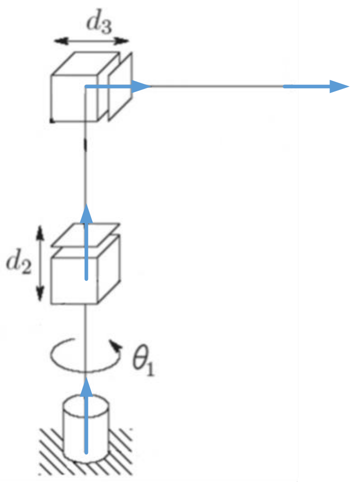
Example:



Assignment of Coordinate Frames:

Example:

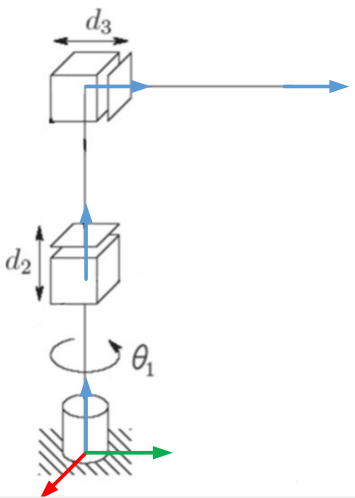
1. Assign z_i along the actuation line of joint i.



Assignment of Coordinate Frames:

Example:

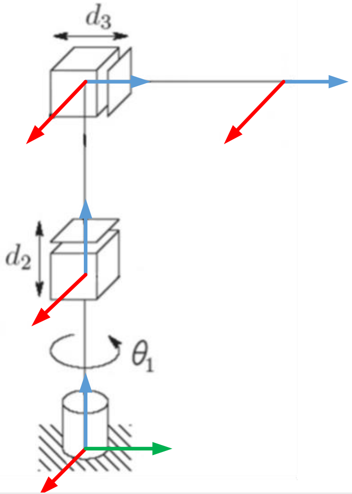
1. Assign z_i along the actuation line of joint i.
2. Choose x_0 and y_0 for frame 0.



Assignment of Coordinate Frames:

Example:

1. Assign z_i along the actuation line of joint i.
2. Choose x_0 and y_0 for frame 0.
3. Find x_i:
    - z_0 intersects with z_1. So, $x_1 \perp z_0$ and z_1.



Assignment of Coordinate Frames:

Example:

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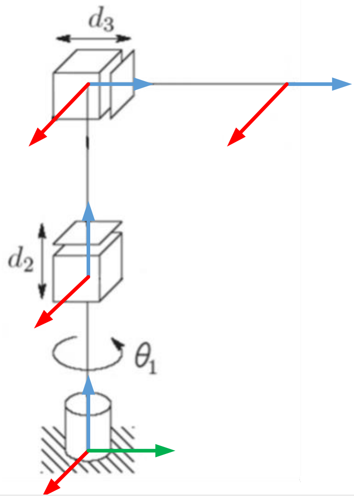
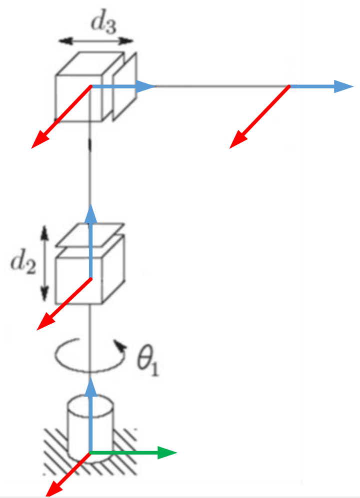
   - $z_1 \perp z_2$. So, $x_2 \perp z_1$ and z_2.



Assignment of Coordinate Frames:

Example:

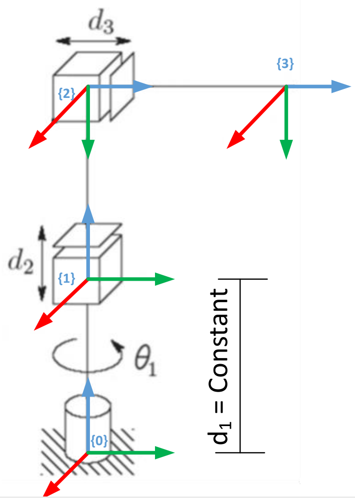
1. Assign z_i along the actuation line of joint i.
2. Choose x_0 and y_0 for frame 0.
3. Find x_i:
   - z_0 intersects with z_1. So, $x_1 \perp z_0$ and z_1.
   - $z_1 \perp z_2$. So, $x_2 \perp z_1$ and z_2.
   - z_2 intersect z_3. So, $x_3 \perp z_2$ and z_3.



Assignment of Coordinate Frames:

Example:

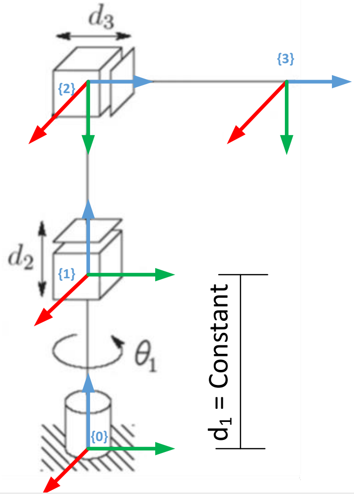
1. Assign z_i along the actuation line of joint i.
2. Choose x_0 and y_0 for frame 0.
3. Find x_i:
    - z_0 intersects with z_1. So, $x_1 \perp z_0$ and z_1.
    - $z_1 \perp z_2$. So, $x_2 \perp z_1$ and z_2.
    - z_2 intersect z_3. So, $x_3 \perp z_2$ and z_3.
4. Complete the coordinate frames with y_i



Assignment of Coordinate Frames:

Example:

1. Assign z_i along the actuation line of joint i.
2. Choose x_0 and y_0 for frame 0.
3. Find x_i:
   - z_0 intersects with z_1. So, $x_1 \perp z_0$ and z_1.
   - $z_1 \perp z_2$. So, $x_2 \perp z_1$ and z_2.
   - z_2 intersect z_3. So, $x_3 \perp z_2$ and z_3.
4. Complete the coordinate frames with y_i
5. Find DH Table for link 1, 2 and 3.



Assignment of Coordinate Frames:

Example:

Four DH parameters are required:

1. a_i: link length, distance between z_{i-1} and z_i (along x_i).
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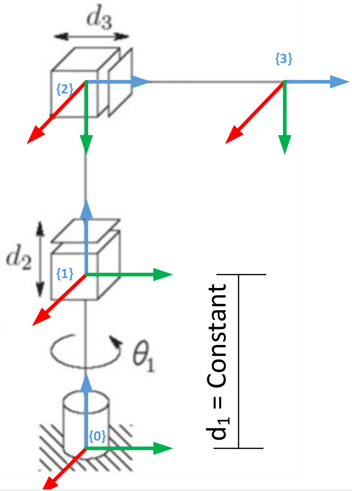
4. θ_i: joint angle, between x_{i-1} and x_i (measured around z_{i-1})

Link	a_i	α_i	d_i	θ_i
1				
2				
3				



Assignment of Coordinate Frames:

Example:

Four DH parameters are required:

1. a_i: link length, distance between z_{i-1} and z_i (along x_i).
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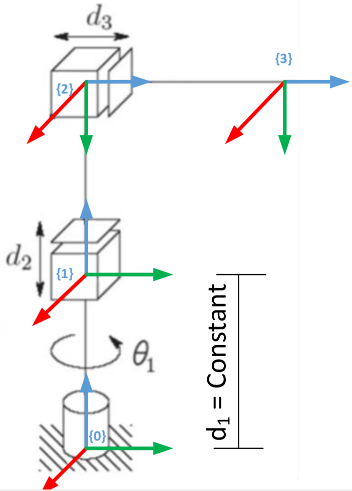
4. θ_i: joint angle, between x_{i-1} and x_i (measured around z_{i-1})

Link	a_i	α_i	d_i	θ_i
1	0	0	d_1	θ_1^*
2	0	-90	d_2^*	0
3	0	0	d_3^*	0



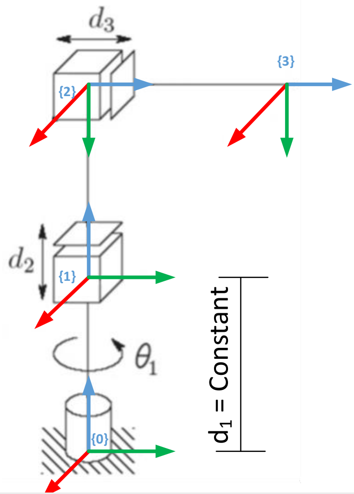
Assignment of Coordinate Frames:

Example:

Four DH parameters are required:

1. a_i: link length, distance between z_{i-1} and z_i (along x_i).
2. α_i: link twist, angle between z_{i-1} and z_i (measured around x_i)
3. d_i: link offset, distance between o_{i-1} and intersection of z_{i-1} and x_i (along z_{i-1})
4. θ_i: joint angle, between x_{i-1} and x_i (measured around z_{i-1})

$$T_3^0 = A_1 A_2 A_3 = \begin{bmatrix} c_1 & 0 & -s_1 & -s_1 d_3 \\ s_1 & 0 & c_1 & c_1 d_3 \\ 0 & -1 & 0 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



End of Lecture

haitham.elhussieny@gmail.com