CHAPTER 1
INTRODUCTION

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One of the greatest remarks in the history of celestial mechanics was the pioneer research of the great French mathematician Louis Joseph Lagrange (1736-1823). His fame came in 1772 from his research [45] for the particular solutions of the Three-Body Problem, in his prize memoir "Essai d'une nouvelle method pour resoudre le probleme des trois corps". Lagrange solved the problem of the one body motion around a fixed center (two-body problem) taking into account the forces which produce the perturbation to the Newtonian law of gravitation. Nevertheless, there are more problems to be considered for the motion of the artificial Earth satellites. In the case of the Three Body-Problem, Lagrange was faced by a quintic (fifth degree) equation. The quintic equation appearing in the stationary solution was called Lagrange's equation. These problems in the artificial Earth satellites and the exact solution of the quintic equation associated with Lagrange's planetary equation are our aim of the present work.

1.1 The Earth Artificial Satellites

The mere anticipation of artificial satellites had already motivated several celestial mechanics theoreticians to turn their attention to the dynamical motion of these artificial objects. The whole succession of analyses was generated for an Earth satellite theoretical orbital motion, employing almost every known perturbation technique. The similarity of an artificial Earth satellite to our natural satellite, the Moon, implies that the wealth of Lunar theory, developed over many years, can be applied with proper interpretation to the motion of an artificial Earth satellite [54].

There are some essential differences between the Lunar motion and that of the artificial Earth satellites. The Moon is sufficiently remote from the Earth to be entirely free of frictional losses produced by motion through the Earth's atmosphere. Moreover, because of the Moon's distance, the orbital perturbations induced by the non-perfect Earth's shape are not as large, in a relative sense, as those induced on the motion of the artificial satellites.

Artificial Earth satellites travel in bound geocentric orbits, which are disturbed by the departure of Earth's shape from a sphere, atmospheric drag, the lunar and solar attractions, the solar radiation pressure, the Earth's magnetic field etc., i.e their orbits are not truly ideal Keplerian orbits. It is impossible to obtain a precise orbit without the knowledge of all the perturbations involved.

(I) Perturbation

Perturbations are classified as either secular or periodic. Secular perturbations are those which increase proportionally with time or to any power of the time. Secular terms in perturbation expressions are important, if they occur, because their presence may imply an unstable dynamical system [54]. Periodic terms are further divided into short-period and long-period perturbations. There are different kinds of perturbation theories depending upon the mode of calculating and expressing the perturbations.

If the perturbation effects are expressed in an analytical form, from which values can be computed at any time by assigning a value to (t), they are called *analytic perturbations*. The absolute and general terms are also used for this class. General perturbation techniques are extremely useful for theoretical investigations, because they lead to theories of orbital motion which determine the position of the body at each instant of time, or the time variation of the body's orbital elements. Various methods are used to obtain the general perturbation, e.g.:

(i) The method of the variation of parameters which exhibits the basic ideas and results of general perturbation theory.

(ii) Nonseparable Hamilton-Jacobi equation such that

$$H_1 = H - H_0 + O(k)$$

where k is a small parameter. Then the solution for H can be considered as a perturbed motion from the known solution of H_0, which is the Hamiltonian for the undisturbed motion.

(iii) The Von ZEIPEL method (Ralph [54]) which depends on obtaining a determining function with the desired properties for transforming canonical variables, so that the new Hamiltonian does not explicitly involve all the new angle