

Introduction & Summary

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It is known that the famous functional Hilbert space $L_2(\mathbb{R})$, $\mathcal{H}^m(\mathbb{R})$ (Sobolev space) contain elements that are not entire (even not smooth in the space $L_2(\mathbb{R})$). The aim of the present thesis is to introduce and study some Hilbert spaces consisting of entire functions. The second aim of the thesis is to study the Fourier transformation as an operator by which it is possible to define entire functions. For satisfying these aims it was necessary to present some elementary ideas and concepts on analytic functions of a complex variable, generalized function and some fundamental theorems from the theory of real analysis.

The thesis consists of five sections.

The first section, Smooth and Analytic Functions of a complex or Real Variable, deals with analytic functions of a complex variable, analytic functions of a real variable, and the test space and test functions in one dimension.

The second section, Generalized Functions, deals with: The space of generalized functions in one dimension, and derivatives of generalized function.

The third section, Hilbert Spaces and Fourier Transformations, deals with : Abstract Hilbert spaces, Sobolev spaces, and the Fourier transformation in $L_2(\mathbb{R})$.

The fourth section, *Hilbert Spaces of Entire Functions*, deals with : Some theorems of Paley and Wiener, Paley-Wiener spaces, a modified Paley-Wiener theorem, and modified Paley-Wiener spaces.

The fifth section, *Characterization of Fourier Transformations*, deals with : A characterization of Fourier transformation in $L_2(\mathbb{R})$, and a characterization of Fourier transformation in $L_2(\mathbb{R}^n)$.