Preface

Lotfi A. Zadeh ([53]) introduced in 1965 the notion of a fuzzy set. This notion has caused great interest among both pure and applied mathematicians, some engineers, biologists, phychologists, economists, and experts in other areas, who use mathematical ideas and methods in their research. By this notion of fuzzy set it is possible to obtain a more distinctive description of some phenomena than the one which is offered by systems based on classical two-valued logic and classical set theory. There are many realized technical applications showing the practical usefulness of fuzzy theory.

One of the first branches of pure mathematics to which fuzzy sets have been applied was the general topology. It was in 1968 that Chang ([10]) introduced the notion of fuzzy topology. Several other authors continued the investigation of such spaces such as Bayuomi ([6, 7, 8, 14]), Eklund and Gähler ([15, 17]), Gähler ([20, 21, 22, 23, 24, 25]), Gähler and Bayoumi ([26, 27, 28, 29]), Geping and Lanfang ([30]), Goguen ([31]), Kandil et al. ([32, 33, 34, 35]), Katsaras and Petalas ([39, 40]), Kerre and Ottoy ([42]), Klein ([43]) and Lowen ([44, 45, 46, 47, 48, 49, 50]). The notion of proximity ([12, 13, 51]) have been generalized to the fuzzy proximity by Artico and Moresco ([1, 2]), Katsaras ([36, 37, 38]) and by Gähler and Bayoumi ([28]). We are interested here on the notion of fuzzy proximity given by Katsaras ([38]). The notion of compactness have been also generalized to the fuzzy case by many authors such as Gähler ([25]) and Lowen ([45, 47]). We study here the fuzzy compactness defined by Gähler ([25]), using the notion of fuzzy filters ([24]), called G-compactness.

The separation axioms for topological spaces have been also generalized to the fuzzy separation axioms by Kandil and El-Etriby ([33]), Kandil and El-Shafee ([34]) and by Wuyts and Lowen ([52]). Whereas the fuzzy separation axioms for fuzzy topological spaces defined in [33, 34] depend on fuzzy points, the fuzzy separation axioms ([52]) depend on usual points.

In this thesis we introduce and study a new kind of fuzzy separation axioms for fuzzy topological spaces related to usual points and ordinary subsets. These axioms are defined, analogously to the separation axioms in the classical case ([11]), using the fuzzy filters ([24]). We denote by GT_i for these axioms and by GT_i-space for the fuzzy topological space which fulfills the axiom GT_i. We study here the cases $i = 0, 1, 2, 3, 4$. The fuzzy filter has been introduced by Eklund and Gähler in [16]. By means of an extension of this notion of fuzzy filter, a point-based approach to fuzzy topology related to usual points has been developed by Gähler in [24, 25]. In this approach several notions are related to usual points, one of these notions is the notion of fuzzy neighbourhood filter at a point which is defined by means of the notion of interior of a fuzzy set. For each fuzzy topological space, the mapping which assigns to each point x the fuzzy neighborhood filter at x can be considered itself as the fuzzy topology. The fuzzy neighborhood filter at a point will be used to define the axioms GT_0, GT_1 and GT_2. Using the notion of fuzzy neighborhood filter at the points of a set we define in this thesis the fuzzy neighborhood filter at this set. By means of the fuzzy neighborhood filter at a set the fuzzy separation axioms GT_3 and GT_4 will be defined. In [27] is defined by means of the fuzzy filters a notion of fuzzy neighborhood structure, called global fuzzy neighborhood structure. There are three types of the global fuzzy neighborhood structure. All fuzzy topologies and stratified fuzzy topologies are global fuzzy neighborhood structures of the first and second type, respectively. They appear in a canonical way as interior operators. Lowen defined in [49] a notion of fuzzy neighborhood structure using the prefilters. For the fuzzy neighborhood structures in sense of Lowen the related fuzzy topological approach differs. Fuzzy neighborhood structures in sense of Lowen are characterized canonically as fuzzy closure operators. A set X equipped with a global (Lowen's) fuzzy neighborhood structure is called global (Lowen's) fuzzy neighborhood space. The fuzzy topological spaces are special global fuzzy neighborhood spaces and then the axioms GT_i will be generalized to the global fuzzy neighborhood spaces and will be denoted by GNT_i. Denote by GNT_i-space for the global fuzzy neighborhood space which is GNT_i. The third type of global fuzzy neighborhood structures is identified, in [27], with the fuzzy neighborhood structure in sense of Lowen. Using this identification we can study, in this thesis, the relation between the axioms GNT_i for the global fuzzy neighborhood spaces and the axioms NT_i ([52]) for the fuzzy neighborhood spaces in sense of Lowen. The axioms GT_i and GNT_i fulfill many properties analogous to the usual axioms and are more general than other kinds of fuzzy separation axioms ([34, 52]). Moreover, the fuzzy proximity spaces ([38]) and the G-compact spaces ([25]) have a good relation with the GT_i-spaces.

The thesis consists of three chapters and is organized as follows:

Each chapter begins with an introduction.

Chapter 1 contains the motivations, ideas, definitions and results about the notions which we shall use throughout the thesis. We recall the notions of fuzzy sets, fuzzy topological spaces, fuzzy filters, fuzzy neighborhood filters, fuzzy proximity spaces, G-compact spaces and fuzzy neighborhood spaces.

Chapter 2 is devoted to introduce and study the main notion of this thesis, that will be the fuzzy separation axioms GT_i for fuzzy topological spaces and the axioms GNT_i for global fuzzy neighborhood spaces, $i = 0, 1, 2, 3, 4$. These new fuzzy separation axioms depend on usual points and ordinary subsets and so the work with these axioms will be more simple and more general. This chapter consists of six