Chapter (1)

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The motion of fluids in the annular region between two eccentric cylinders has received considerable attention in the last few decades due to the wide range of applications of this technical problem. Some important examples of these applications are the flow in rotation turbo machinery, journal bearing lubrication, petroleum and food industries. Besides of its importance in these technological applications, such types of flow are very useful in the field of viscometry and rheometry. From a scientific point of view, this flow gives ample information about the mechanical properties of material solutions and their melts. This information is very important in the processing of these materials in almost all branches of industry.

When a solid surface slides over another one, the frictional resistance can be greatly reduced by separating the surfaces by means of a thin film of lubricating oil. For example, in machinery engines, the sliding motion of the piston in the cylinder as well as the rotational motion of the crankshaft are lubricated with engine oil to reduce friction, ***Sergio et al.*** [1] and ***Abdallah et al.*** [2]. The motion of lubricating thin films fluid undergoing shearing flow and their induced forces are classified as lubrication theory. This theory served the designer well for decades, but the recent increase in size and speed of rotating machinery strained the theory. Hence, there is a continuous need to develop it ***Capone et al.*** [3] and ***Harnoy et al.*** [4].

On the basis of the lubricant theory, the standard technique for treating hydrodynamic lubrication problems involves a truncation of the Navier-Stokes equations. This truncation depends on the value of the Reynolds number "Re". The smallness of Re justifies the neglect of all inertial terms. But, with increasing Re the standard approximation cannot be a straight forward, ***Zhang et al.*** [5]. The significant changes in journal bearing characteristics for "$Re \gg 1$" may be attributed to three changes; namely, laminar, secondary flow and turbulent flow, ***Dai et al.*** [6].

With the trend toward higher operating speeds and larger sizes of rotating machinery or using low kinematic viscosity lubricants such as water or liquid metals, fluid film journal bearings have been operated at high Re, ***Hashimoto et al.*** [7]. As the speeds of the machinery using journal bearings increased, the interest in the development of journal bearing theory increased considerably. The users of such machinery were reporting large ***vibrational amplitude*** under certain conditions of loading and speed, which in turn caused large forces to be transmitted to the system foundation and to the system component parts, ***Kirk et al.*** [8]. ***Newkirk et al.*** [9] reported the first recorded instance of bearing instability. He demonstrated that, under certain conditions of speed and loading, the journal center did not remain fixed but processed or orbited about the equilibrium position. This phenomenon was termed oil whip or whirl motion. ***Kulinski et al.*** [10] considered the effect of inertia on the flow within the journal bearing by using a perturbation procedure for small eccentricity without calculating the pressure distribution, load and torque. In a series of papers, ***DiPrima*** alone [11] and in collaboration with ***Stuart*** [12, 13, 14] studied the non-local effects in the hydrodynamic stability of viscous flow between two long rotating eccentric cylinders. Using a perturbation series in terms of clearance ratio, they obtained the linearized inertial contribution to the Stokes approximation at small clearance ratios and for small Re. Their solution of the zero-order, i.e. Stokes flow approximation, is identical to that obtained in lubrication theory. In these papers, the emphasis is placed on the resultant force exerted by the fluid on the cylinders and on the distribution of these forces over the cylinders.

According to the analysis reported by ***Truesdell et al.*** [15], a secondary flow takes place when the fluid moves in the annular regions of cross-sections different from the concentric circular case. So, when one cylinder is displaced to an eccentric position, new effects appear. First of all, a pressure gradient is created which tends to restore concentricity. The pressure is of a special interest in hydrodynamic lubrication theory and has been studied for very small clearance ratios while neglecting inertial effects. A second effect of the eccentricity is the creation of an eddy adjacent to the outer cylinder in the region of the largest clearance. A third effect observed in the course of the annulus flow experiments is the separation of fluid from the outer cylinder. This phenomenon does not occur for small eccentricities and, therefore, cannot be studied by perturbing the solution of the Navier-Stokes equations for the flow between concentric cylinders.

The pioneering studies of flow between eccentric cylinders considering viscous fluid at effectively zero-Re begin with the work of ***Muller*** [16]. Flow at large Re was studied by ***Wood*** [17]. ***Jeffrey*** [18], ***Duffing*** [19] and ***Reissner*** [20] discussed the problem using bipolar coordinates as a reference system. From a mathematical standpoint, all of these authors determine a stream function which satisfies a biharmonic equation with constant values at the boundaries. ***Wannier*** [21] also discussed the problem, using a complex variable technique for the solution of the biharmonic equation satisfied by the stream function. The essential difference between the work of these various authors is being in the completeness with which they discuss the implications of their solutions to the boundary value problem.

Attempts have also been made to analyze the modifications in the results, which follow from taking into account the effects of inertia. ***Kamal*** [22], considered the case when the inner cylinder is rotating and the outer cylinder is kept stationary. He derived the effects of inertia as a perturbation of the Stokes solution, linearizing all the equations in the perturbing velocities. However, this solution in terms of the perturbed stream function is incorrect, ***Sood, et al.***[23]. The perturbation analysis of ***Ballal et al.*** [24] is one of the most acceptable theoretical analyses for stokes flow only, but that inertial solution is incorrect, ***Szeri et al.*** [25].

Zidan et al. [26] investigated the Couette flow for a second-order fluids in the annular region between two eccentric cylinders superimposed on simple shearing as well as on axial constant pressure gradient in terms of a bipolar system of coordinates. The authors made two important conclusions for rheometry. One conclusion was that the superposition of an axial component on the rotational Couette flow creates a second-order modification of the axial velocity component. The other conclusion was a proposal for the construction of a measuring instrument, rheometer, based on the flow of a second-order fluid in the annular region between two eccentric cylinders in order to measure the material parameters of these fluids.

On the basis of the last reference work, ***Abdel Wahab et al.*** [27] designed the so-called "new eccentric cylinder rheometer" which concerns only the Couette flow of a second-order fluid in the annular region between two eccentric cylinders. From a mathematical standpoint, the authors determine the modified first-order stream function which allows the pressure periodicity condition. No attention is paid for the effect of the inertial terms on the fluid flow in that work. So, the measurements of the material parameters have been investigated at very slow motion.

Most of these earlier literatures treated the problem by considering some restrictions on the geometry of the flow. ***San Andres et al.*** [28] analyzed the problem numerically using Galerkin's procedure with B-spline test functions. They worked out an accurate numerical solution of the exact equations and applied it to the wide-gap for different values of the rotational velocities of the two cylinders. This analysis was recently extended to account for heat transfer in a fluid with temperature-dependent viscosity by ***Dai et al.*** [29].

Sood et al. [23] used a finite difference method to solve the full Navier-Stokes equations for the problem numerically, without making the linearized inertial approximation. In this work, they considered only the case at which the inner cylinder is rotating and the outer cylinder is stationary. They showed, for a particular geometry of the system, a shift in the separation point on the outer cylinder, resulting from the inertial contribution. They also show schematically the distribution of the normal thrust on the inner cylinder.

A three-dimensional spectral algorithm for the solution of Stokes flow between eccentric rotating cylinders have been considered by

Roberts et al. [30]. They calculated the load and couple on the inner cylinder and found their results to be in good agreement with those available from lubrication theory. The numerical treatment of eccentric cylinder flows has been hindered in the past because the partial differential equations that model the basic flow contain the Re, clearance ratio and eccentricity as parameters. Numerical solutions are difficult to obtain at some values of these parameters, especially at large values of Re and eccentricity.

From this survey, it is clear that the problem of fluid flow between two eccentric cylinders is more significant for applications in technology, specially lubrication. Therefore, numerous papers concerning this flow problem have been published. Up to date, to the best of our knowledge, no one has solved this problem exactly, therefore the solution is carried out by either numerical or asymptotic methods.

In the present work, we consider the steady nonlinear flow of a Navier-Stokes fluid, in the annular region between two infinitely long eccentric cylinders with parallel axes. The flow in the annular region is created by the rotation of the inner cylinder about its own axis with a constant angular velocity while the outer one is kept at rest. The solution of the problem is performed in terms of the cylindrical bipolar coordinates. The partial differential equations that emerge from the applied method of successive approximations are solved using the Green's function technique.

Hence, the primary object at present is to analyze the effect of the inertial contribution added to the Stokes flow and the technological applications in a journal bearing. In order to achieve a full treatment of these effects, a valuable discussion of the Stokes approximation is presented. Moreover, in order to develop the equations determining the nonlinear flow, the solution of the full Navier-Stokes equations must be considered. The effect of this inertial flow on the forces, torque, and separation of the flow and static characteristics of the journal bearing are discussed.

The present thesis includes five chapters, where the Introduction is being the first one, besides the discussion and an Appendix at the end of this thesis.

The definition of the boundary value problem as well as the formulation of the problem, the perturbation method, the solution of the Stokes flow are discussed in chapter two. The second-order approximation and the mathematical form of the inhomogeneous partial differential equation which govern the emerged stream function are included in the same chapter.

Chapter three is devoted to the solution of the nonlinear flow, the velocity field and the pressure function are determined at the end of this chapter. Chapter four includes an analysis of the flow of an inertial and of a non-inertial viscous fluid in the annular region between two eccentric cylinders.

Chapter five is directly concerned with the calculation of practical parameters for journal bearing configurations. The pressure distribution in the gap of a journal bearing is used in order to obtain an analytical description of the fluid film force. Therefore, we provide an additional insight into the effects of the eccentricity on the fluid film force and the static characteristics of the journal bearing.