Introduction

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The variety of problems of qualitative properties of a nonlinear system of differential equations of the type

$$x' = f(t, x), \tag{1}$$

and the perturbed system

$$x' = f(t, x) + R(t, x), \tag{2}$$

where $f, R \in C[\Re^+ \times \Re^n, \Re^n]$, and $f(t, 0) = R(t, 0) = 0$, has been successfully studied by different approaches based on Liapunov's direct method, such as, cone-valued Liapunov function and comparison technique. Liapunov's second method is an important ingredient, as it is a powerful tool in stability theory of nonlinear systems of differential equations. Theoretically this method is very appealing and has many applications. Liapunov function and related differential inequalities in Liapunov's second method play essential roles to determine the stability behavior of solutions of linear and nonlinear systems. The major advantage of this method is that the stability in the large can be obtained without any period knowledge of solutions.

Lakshmikantham and Leela [23—25] initiated the development of the theory of differential inequalities through cone and cone-valued Liapunov function. They used the comparison principal to improve and extend different notions of stability such as, eventual stability and L^p-stability for the nonlinear system (1), integral stability and total stability for both nonlinear systems (1) and (2). They used the comparison technique to improve relative stability for the two differential systems

$$\begin{aligned} x' &= f_1(t, x), \quad x(t_0) = x_0, \\ y' &= f_2(t, y), \quad y(t_0) = y_0, \end{aligned} \tag{3}$$

where $f_1, f_2 \in C[\Re^+ \times \Re^n, \Re^n]$, and $f_1(t, 0) = f_2(t, 0) = 0$. Moreover they used the same technique to improve partial stability of a nonlinear system of differential equations of the form

$$\begin{aligned} x' &= F(t, x, y), \quad x(t_0) = x_0, \\ y' &= H(t, x, y), \quad y(t_0) = y_0, \end{aligned} \tag{4}$$

where $F \in C[\Re^+ \times \Re^n \times \Re^m, \Re^n], H \in C[\Re^+ \times \Re^n \times \Re^m, \Re^m]$, and $F(t,0,0) = H(t,0,0) = 0, t \in \Re^+$.

In 1986 Dannan and Elaydi [9] introduced a new notion of stability called Lipschitz stability. Akpan et al. [1-2] discussed ϕ_0-stability, total ϕ_0-stability for the system (1), and exponential ϕ_0-stability of the perturbed differential system (2). Recently, Dannan and Elaydi [10], and Pachpatte [31-32] have obtained results on qualitative behavior of solutions of perturbed nonlinear systems using the nonlinear variation of constants formula of Alekseev [4].

This thesis is concerned with improving and extending some of the recent results of stability for the systems (1)-(2).

We also discuss and introduce the notion of ϕ_0- stability for the impulsive system of differential equations

$$\begin{aligned} x' &= f(t,x) + g(t,y), \quad t \neq \tau_i(x,y), \quad \Delta x|_{t=\tau_i(x,y)} = A_t(x) + B_t(y) \\ y' &= h(t,x,y) \quad t \neq \tau_i(x,y), \quad \Delta y|_{t=\tau_i(x,y)} = C_t(x,y). \end{aligned} \tag{5}$$

where $x \in \Re^n, y \in \Re^m$, $f : \Re^+ \times \Re^n_H \to \Re^n$, $g : \Re^+ \times \Re^m_H \to \Re^n$, $h : \Re^+ \times \Re^n_H \times \Re^m_H \to \Re^m$, $A_t : \Re^n_H \to \Re^n$, $B_t : \Re^m_H \to \Re^n$, $C_t : \Re^n_H \times \Re^m_H \to \Re^m$, $\tau_i : \Re^n_H \times \Re^m_H \to \Re^+$. $\Delta x|_{t=\tau(x,y)} = x(t+0) - x(t-0)$, $\Delta y|_{t=\tau(x,y)} = y(t+0) - y(t-0)$.

On the other hand we are concerned with the oscillation behavior of the Volterra-Stieltjes integro-differential equation (V-S) of the first order in the form

$$p(t)y'(t) + \int_a^t y(s)d\sigma(s) = \beta, \quad t \in I = [a, \infty), \tag{6}$$

where $a \geq 0$ and p, σ are real-valued right-continuous functions which are locally of bounded variation on I and $p(t) > 0$ on I.
We claim that our oscillatory results improve some of those obtained by Jann [19].

The thesis consists of five chapters

In the first chapter our concern is the oscillation criteria for (V-S) equation (6). We give a partial generalization for the work of [19] for (V-S) equation (6).

In the second chapter, we outline some known notations of differential inequalities and different stability notions which will be needed in the next chapters. We extend and improve some recent results of ϕ_0-stability of [1] to a new type of stability namely, ϕ_0-L^p stability.

In chapter three. we discuss and improve the notions of partial, relative and ϕ_0-stability of nonlinear systems of differential equations (3), and (4).

In chapter four we extend the well known notion of ϕ_0-stability for the impulsive system of differential equations (5).

In chapter Five. we extend and improve the notion of (h_0, h)-stability of [26-27] to the called (h_0, h)-integral stability, (h_0, h)-eventual stability and (h_0, h)-total stability of the two systems (1) and (2).

Note.

Some results of Chapter Two and Chapter Five were accepted for publication in J. Applied Mathematics and Computation [13].

The results of Chapter Three were accepted for publication in J. Applied Mathematics and Computation [14].

The results of Chapter Four were submitted for publication in J. Differential equations and dynamical systems. [15].