PREFACE

In his classic paper [69] of 1965, Zadeh introduced the notion of fuzzy sets and their operations. Subsequently, Chang [12], Wong [66], Lowen [35] and others applied some basic concepts from general topology to fuzzy sets and developed a theory of fuzzy topological spaces. We denote by FTS the category of Lowen's fuzzy topological spaces and thier continuous functions.

The thesis consists of four chapters :

The first chapter in this thesis contains basic ideas on fuzzy sets and fuzzy topological spaces, together with their basic properties. The second, third and fourth chapters consist of a new research work done by us. In the second chapter, we introduce for each continuous triangular norm T, a new structure that is a fuzzy T-proximity space, those generalize the fuzzy proximity spaces given in [7] by G. Artico and R. Moresco, which now coincide with our fuzzy T-proximity spaces when T = Min. Some properties of this notion are given. We generate a fuzzy T-proximity given by a classical proximity. Moreover, we introduce a notion of fuzzy T-proximal neighbourhood systems, which are in one-to-one correspondence with the fuzzy T-proximity spaces. Also, we show that every Höhle fuzzy T-uniformity [21] induces a fuzzy T-proximity and that the fuzzy topologies generated by the two structures coincide. Also, we characterize a fuzzy T-proximity, uniquely in terms of its behaviour

on crisp fuzzy subsets.

In the third chapter we study for each lower semicontinuous triangular norm T, a new type of Lowen fuzzy topological spaces which we call the fuzzy T-neighbourhood spaces. Those generalize the fuzzy neighbourhood spaces introduced by R. Lowen in [40] (which now correspond to the case T = Min). Also, we establish chracterizations and aspects of good behaviour for those spaces, including that there is a one-to-one correspondence between fuzzy T-neighbourhood spaces and fuzzy T-neighbourhood systems, we briefly study their level topologies, and we show that all topologically generated spaces are fuzzy T-neighbourhood spaces. We prove that the category T-FNS, of fuzzy T-neighbourhood spaces and continuous functions between them, is a topological category. We show that it is closed in FTS under the formation of both optimal lifts of sources and co-optimal lifts of sinks. Also, we define a functor from the category of fuzzy T-uniform spaces into the category T-FNS of fuzzy T-neighbourhood spaces. We study the continuity of functions in the category T-FNS in terms of fuzzy T-neighbourhood systems.

In the fourth chapter, we introduce the new concept of fuzzy T-neighbourhood systems of fuzzy subsets. This helps us to introduce and study a number of separation properties of fuzzy topological spaces and fuzzy T-neighbourhood spaces, all of which are extensions of the classical R_1 property, regularity and normality in topological spaces. They also generalize existing axioms for the case T = Min, i.e. for fuzzy neighbourhood spaces [9, 29, 44, 51,

52, 55]. Also, we show that for each triangular norm T, the axioms T-R_1 and T-regularity are initial in FTS. We study some relations among these separation axioms, we show that all these separation axioms are good extensions in the sense of R. Lowen [37]. Also, we introduce an axiom of T-complete regularity, on fuzzy topological spaces. We show that it is equivalent to fuzzy T-uniformizability.