# INTRODUCTION

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The amazingly rapid development of the technology of high-speed computers in recent years has been accompanied by a very significant growth of mathematical science and specially numerical analysis. There are many numerical methods for solving partial differential equations; for the meantime, the finite difference method is considered one of the most important methods for that.

In order to solve numerically partial differential equations, approximations to the partial derivatives are introduced; these approximations convert the differential equations to finite difference expressions, from which we obtain a system of algebraic equations. The resulting system is usually called finite difference equations, and subsequently solved at discrete points within the domain of interest. The difference expressions and their derivations can be found in any textbook on the numerical solution of differential equations for example [6] and [9].

It was twelve years ago when Ismail and Elbarbary established the first technique of approximation restriction in 1995. This earlier restriction type is constructed by supposing a parameter to be determined such that the error is equal to zero at certain points of the domain of the function. If this parameter reduces to zero, we get back the classical approximations. The first enhanced classical approximation was of Padé and the new modified technique was known as restrictive Padé approximation. This new restrictive Padé approximation method was applied over various problems and resulted in satisfying better results than the classified one as solving the convection-diffusion equation in one and two dimensions [18], [17], singularly perturbed initial boundary value problems for parabolic and hyperbolic partial differential equations [19], [22] and solution of the schrodinger equation [23].

This method of restriction has been widely studied to cover more complicated problems and applications in order to reduce the generated error with simplest solution techniques. This led to the restrictive Taylor method that was established in 1998 by Ismail and Elbarbary, which is the new restricted enhancement of the classical Taylor approximation. This gives a new approach for an explicit method to solve first order hyperbolic and parabolic partial differential equation in one dimension [20] and [21]. While the restrictive Padé approximation method usually gives implicit finite difference equation. That was more efficient when compared with restrictive Padé approximation because of its explicit form. These enhanced results by restrictive Taylor method supported me to choose the restrictive Taylor to be my target to discuss and search for new enhancement with it within my study in this thesis.

Another method that is discussed in the thesis is the Adomian decomposition methods [1] and [2] which is highly required to solve many stochastic and deterministic problems in various fields of science like physics and chemical reactions because of its resulted analytical approximation that converges very rapidly.

In the same field of the approximations techniques, other members of my supervisor teamwork of the research students has there own efforts and good results that are established on parallel lines and on other functions and types of equations other than my established types of approximations like to solve "Restrictive Padé Approximation for Variable Coefficients Linear Initial Boundary Value Problem Hyperbolic Partial Differential Equations" [28], "A Restrictive Padé Approximation for the Solution of Generalized Burger's Equation" [30], "A Restrictive Padé Approximation for the Solution of the Generalized Fisher and Burger-Fisher Equations" [42], "A Restrictive Padé Approximation for the Solution of the Generalized Huxley and Burger's-Huxley equations" [32], and "On the General Term of a Cauchy Product of Two Series of the Truncation Error for Some Restrictive Approximations for the Initial Boundary Value Problem for Parabolic and Hyperbolic Equations" [38].

**Chapter (1)**

We study some properties of the polynomial approximation, specially Taylor polynomial approximation and the advanced technique, which is the restrictive Taylor approximation as done in [21]. Also we deal with Adomian decomposition method for the solution of the non-linear differential equations.

**Chapter (2)**

We introduce a new approach to an explicit method for solving the convection diffusion equation ($\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = \gamma\frac{\partial^2 u}{\partial x^2}$). This subject is contained in our paper [24], discussed in 27th International Conference for Statistics & Computer Science and its Applications, Cairo, pp. 493-501 (2002), and publication in Appl. Math. & Computation Vol. 147, pp 335-363, Issue 2, 12 January, (2004).

**Chapter (3)**

We introduce a new approach to an explicit method for solving the two-dimensions initial boundary value problem for parabolic PDE ($\frac{\partial u}{\partial t} = \gamma_1 \ (\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2})$). This subject is contained in our paper [25], discussed in 27th International Conference for Statistics & Computer Science and its Applications, Cairo, pp. 547-555, (2002) and publication in Appl. Math. & Computation Vol. 147, pp 607-615, Issue 3, 13 January, (2004).

Also we solved the two-dimensional convection-diffusion equation ($\frac{\partial u}{\partial t} + C_1\frac{\partial u}{\partial x} + C_2\frac{\partial u}{\partial y} = \gamma_1\frac{\partial^2 u}{\partial x^2} + \gamma_2\frac{\partial^2 u}{\partial y^2}$). This subject is contained in our paper [23], discussed in 1st International Conference on Engineering Mathematics and Physics, MTC, Cairo, pp. 63-71, (2002).

**Chapter (4)**

We introduce a numerical treatment of initial boundary value problem for linear parabolic equations by small parameter with the time derivative term; singularly perturbed parabolic PDE ($\delta\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + f(x,t)$). We derive a new finite different scheme by applying the restrictive Taylor approximation of exponential matrix. This subject is contained in our paper [29], discussed in 28th International Conference for Statistics & Computer Science and its Applications, Cairo, and publication in J. Inst. Math. & Comput. Sc. Vol. 14, No. 1, pp 19-26 (2003).

And also, we solved the singularly perturbed hyperbolic PDE ($\delta \frac{\partial u}{\partial t} + a \frac{\partial u}{\partial x} = f(x,t)$). This subject is contained in our paper [31], discussed in 28th International Conference for Statistics & Computer Science and its Applications, pp 9-16 Cairo, (2003).

**Chapter (5)**

We introduce a new approach to the solution of the general KdV and IKDV equations by Adomian decomposition method then, we compute the conservation laws:

- General KdV ($u_t + \varepsilon_\circ u^p u_x + \mu u_{xxx} = 0$), this subject is contained in our paper [33], discussed in 28th International Conference for Statistics & Computer Science and its Applications, pp 65-77 Cairo, and is published in Applied Math. & Computation Vol. 154, pp 17-29, Issue 1, (2003).
- Generalized IKDV ($u_t + \varepsilon_\circ u^p u_x + \mu u_{xxx} - \delta u_{xxt} = 0$), this paper is sent for possible publication in Applied Math. & Computation, December (2006).

**Chapter (6)**

Due to me and my colleagues in our researches teamwork to be acquainted in a new school for restrictive approximations; we try to compare the two approaches for new algorithms by restrictive Taylor or restrictive Padé approximations we treated this comparison in several applied problems. Also, in chapter 6, we give a comparative study between restrictive Taylor, restrictive Padé approximations and Adomian decomposition method for the solitary wave solution of the general KdV equation, with the work of my colleague Aziza A. A. Rabbou, as we exchange our researches conclusions in order to help each other in clarifying our self-efforts. This subject is contained in our paper [39], discussed in $29^{th}$ International Conference for Statistics & Computer Science and its Applications, pp 45-64, Cairo (2004) and publication in Applied Math. & Computation Vol. 167, pp 849-869, (2005).

**Chapter (7)**

As an analogy to the proof of convergence of restrictive Padé approximation for parabolic and hyperbolic IBVP for PDE done by my professor & the leader of our researches teamwork Hassan N. A. Ismail [34], then I participated with him in sending the next paper [35], which is discussed in details in chapter.7 to the same journal for the proof of, On the convergence of the restrictive Taylor approximation to the exact solutions of IBVP for parabolic, hyperbolic, convection diffusion, and KdV equations. This subject is contained in our paper [35], discussed in $29^{th}$ International Conference for Statistics & Computer Science and its Applications, pp 11-22, Cairo, (2004).

Some other techniques driven of my approximations were considered by the others, there were further studies that applied our algorithms as new numerical approximations to support them in their studies, as the work of Davod Khojasteh Salkuyeh in 2006 [54], he compare our results in solving the convection-diffusion equation [24], of the title “On the Finite Difference Approximation to the Convection-Diffusion Equation”. Also the work of Mustafa Gulsu and Turgut Ozis in 2005 [11] of the title “Numerical Solution of Burger’s Equation with Restrictive Taylor Approximation” and the work of Mustafa Gulsu in 2006 [12] of the title “A Finite Difference Approach for Solution of Burger’s Equation”.

This chapter contains a sufficient condition for producing zero local truncation error LTE for RTA of some chosen IBVPs of types of PDEs. Which is that the exact solution of the given problem belongs to a semi-group of linear operators. For which has the LTE series expansion is of identically zero coefficients. A paper for this idea is sent for possible publications.