Chapter 2

Fermion mixing in the standard model

2.1 Quark mixing in the standard model

In the standard model the mass eigenstates are different from weak eigenstates which have definite gauge transformation properties. For the simple case of two generations of fermions, this produces the Cabibbo mixing of the quarks in the charged current [1,2,3], and the quark weak eigenstates are

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L \qquad u_R, c_R, d_R, s_R, \tag{2.1}$$

and

$$\begin{pmatrix} d' \\ s' \end{pmatrix}_L = V \begin{pmatrix} d \\ s \end{pmatrix}_L = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}_L, \tag{2.2}$$

i.e. the weak eigenstates d',s' are rotations of the mass eigenstates d,s. As we see the rotation appear only for the left handed fermion sector. This is why there is no mixing angle in the right handed fermion sectors because u_R, c_R don't couple to d_R, s_R. The neutral current which is proportional to the operator $(Q\sin^2\theta_w - T_{3L})$ has the important property that it is flavor-diagonal (or flavor-conserving). This follows from the fact that all fermions with the same charge and same helicity have the same transformation properties under the gauge group $SU(2)\times U(1)$, so that the rotation matrices such as the above V commute with the neutral-current operator $(Q\sin^2\theta_w - T_{3L})$. For example, in the (d_L, s_L) sector, the $(1-\gamma_5)$ part of the neutral current is

$$J^0{}_\mu(d,s) = (\bar{d}', \bar{s}')_L\left[-\frac{1}{3}\sin^2\theta_w + \frac{1}{2}\right]\begin{pmatrix} d' \\ s' \end{pmatrix}_L = (\bar{d}, \bar{s})_L\left[-\frac{1}{3}\sin^2\theta_w + \frac{1}{2}\right]\begin{pmatrix} d \\ s \end{pmatrix}_L, \tag{2.3}$$

which is flavor-conserving.

Cabibbo-Kobayashi-Maskawa(CKM) mixing matrix

It is a unitary matrix which contains information on the strength of quark flavor changing (e.g $u \to d$) in the weak decays, so it describe the probability of the transition from one quark (q) to another quark (q'). Also, it specifies the mismatch of the quantum states of the quarks when they propagate freely and when they take part in the weak interactions

In the three-family six quark case, the mixing matrices appear as

$$\mathcal{L} = \frac{g}{\sqrt{2}}(\bar{u}, \bar{c}, \bar{t})_L \gamma^\mu V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L W_\mu^+ + h.c., \tag{2.4}$$

where the unitary matrix

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}, \tag{2.5}$$

can have one complex phase and can be parameterized in a form first introduced by Kobayashi and Maskawa(1973)[4]

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{2.6}$$

where we have used the abbreviations $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$ and δ is the CKM phase responsible for all CP-violating phenomena in flavor changing process in the SM.

All the experimental data are consistent with the qualitative feature that the diagonal elements of the CKM matrix are the largest, and the magnitude of the matrix elements decreases as the element moves away from the diagonal[4]

$$|V_{ud}| = 0.97377 \pm 0.00027, \quad |V_{us}| = 0.2257, \quad |V_{cd}| = 0.23 \pm 0.011,$$
$$|V_{cs}| = 0.957 \pm 0.017 \pm 0.093, \quad |V_{cb}| = (41.6 \pm 0.6) \times 10^{-3}, \quad |V_{ub}| = (4.31 \pm 0.30) \times 10^{-3}.$$
$$|V_{td}| = (7.4 \pm 0.8) \times 10^{-3}, \quad |V_{ts}| = (41.61 \pm 0.12) \times 10^{-3}, \quad |V_{tb}| = 0.9991 \pm 0.000034. \tag{2.7}$$

such that all CKM angles θ_i are small. This implies the dominant decay chain t$\rightarrow$ b$\rightarrow$ c $\rightarrow$ s.

2.2 Neutrino oscillations

The neutrino oscillations means that a beam of neutrinos (produced through weak interaction decays) , corresponding to some definite flavor can spontaneously change, or oscillate, into neutrinos of different flavon, e.g. $\nu_e \leftrightarrow \nu_\mu$ · while traveling in vacuum. This property explain the solar neutrino puzzle. While neutrino masses and mixing were discussed earlier by Sakata and his collaborators [2] , these possibilities, particularly in connection with neutrino oscillation, have been studied in details by pontecorvo(1969). If the neutrinos are not massless, their mass matrix, just as in the case for quarks, will be non diagonal and complex. One needs to transform it into a diagonal form by unitary rotations to get the mass eigenvalues. Thus the mass eigenstates are different from gauge eigenstates[2,3]

$$\nu_\alpha = \sum_i U_{\alpha i}\nu_i, \tag{2.8}$$

where $\nu_\alpha = \nu_e, \nu_\mu, \nu_\tau$ are the weak flavor eigenstates and $\nu_i = \nu_1, \nu_2, \nu_3$ are the mass eigenstates with mass eigenvalues m_1, m_2 and m_3. U is a 3×3 unitary matrix termed as Pontcorvo-Maki- Nakagawa- Sakata matrix $[U_{PMNS}]$ and it can be parameterized like the CKM matrix for quark mixing angles

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}. \tag{2.9}$$

It is clear that flavor neutrinos ν_α $(\alpha = e, \mu, \tau)$ are not the same as neutrinos of definite mass $\nu_i(i = 1, 2, 3)$. The electron , muon and tau neutrinos states have no definite masses but turn out to be coherent combination of the mass states.

Of course there is no reason to expect that these angles are in anyway similar to Cabibbo-Kobayashi-Maskawa angles . The parameter δ correspond to CP violation in neutrino sector.

The Weak charged current

$$J^\mu = \bar{l}\gamma^\mu(1 - \gamma^5)\nu_l \tag{2.10}$$

where $l = e, \mu, \tau$

so the weak charged current mix the neutrinos mass states.

If we assumes that the neutrinos are Majorana fermions, the neutrino mass matrix can be diagonalized by the unitary matrix S_ν

$$S_\nu^T M_\nu S_\nu = M_\nu^d \tag{2.11}$$

Where $M_\nu^d \equiv \text{diag}(m_1, m_2, m_3)$ is the diagonal matrix. This could be achieved by relating the flavor basis as $\nu' = S_\nu \nu$

The charged lepton mass matrix can be diagonalized by two unitary matrices, since it is , in general, neither symmetric nor hermitian;

$$S_l M_l V_l^\dagger = M_l^d \tag{2.12}$$

which can also be made by the following relations

$$l'_L = S_L l_L; \qquad l'_R = \nu_l l_R \tag{2.13}$$

where $M_l^d \equiv diag(m_l, m_\mu, m_\tau)$ is the diagonal matrix.

Combining this relation in the charged current, we obtain;

$$J^\mu = \bar{l}'\gamma^\mu(1 - \gamma^5)\nu' = \bar{l}\gamma^\mu(1 - \gamma^5)S^{\dagger_l}S_\nu \nu \tag{2.14}$$