Summary

Mathematical modeling of many frontier physical systems leads to nonlinear ordinary and partial differential equations. An effective method is required to analyze the mathematical model, which provides solutions conforming to physical reality, i.e., the real word of physics. Therefore, we must be able to solve nonlinear ordinary and partial differential equations, in space and time, which may be strongly nonlinear. Common analytic procedures linearize the system or assume that nonlinearities are relatively insignificant. Such procedures change the actual problem to make it tractable by the conventional methods. In short, the physical problem is transformed to a purely mathematical one for which the solution is readily available. These changes, sometimes seriously, the solution. Generally, the numerical methods such as Runge Kutta method are based on the discretization techniques, and they only permit us to calculate the approximate solutions for some values of time and space variables, which cause us to overlook some important phenomena such as chaos and bifurcations, because generally nonlinear dynamic systems exhibit some delicate structures in very small time and space variables. Also, the numerical methods require computer-intensive calculations. The ability to solve nonlinear equations by an analytic method is important because linearization changes the problem being analyzed to a different problem, perturbation is only reasonable when nonlinear effects are very small, and the numerical methods need a substantial amount of computation but only get limited information.

The main goal of this thesis, which consists of four chapters, is to study the approximate solutions of some types of coupled systems of nonlinear partial differential equations (NPDEs). Since, the exact solutions of most model problems are difficult to obtain, so we are interested in this thesis to obtain approximate solutions for some coupled systems models; namely, Cauchy problem of thermo-elasticity in one dimension which is represented by coupled system of parabolic and hyperbolic NPDEs, Manakov system which consists of nonlinear coupled system of Schrödinger equations in one dimension, system of Burgers' equations and coupled Drinfel'd Sokolov-Wilson wave equations, by using some approximate methods such as:

1. Variational iteration method [VIM];
2. Adomian's decomposition method [ADM];
3. Homotopy perturbation method [HPM].

Special attention is given to study the convergence analysis and the error estimate of some of these methods. The main advantages of the methods (VIM, ADM and HPM) are:

- Fast convergence to the solution;
- Do not require discretizations of space and time variables;
- No need to solve non-linear system of algebraic equations as in finite element method and finite difference method;
- No necessity of large computer memory.

Chapter One:

This chapter, is devoted to the presentation of the basic idea of the approximate methods VIM, ADM and HPM, where we used these methods to solve some simple linear and nonlinear differential equations. Also, a brief introduction on FEM is presented, where we used this method to solve a test numerical example of nonlinear differential equations. Moreover, the basic idea of FEM is used to solve numerically the Cauchy problem of thermo-elasticity in one dimension, and used the cubic Hermite basis functions in FEM as shape functions, to obtain accurate solutions. The obtained results from FEM are compared with those obtained results from finite difference method (FDM).

Chapter Two:

In this chapter, VIM which is based on the Lagrange multiplier method is presented to study the approximate solutions of the coupled system of NPDEs. Special attentions are given to study the convergence of the method, where we proved the uniqueness of the solution and proved that the obtained sequence of solutions by using this method is convergent, also we introduced a formula for the maximum absolute error. To verify this theoretical study, we presented some test models solved by the proposed method. These systems are, system of Burgers' equations and Cauchy problem of thermo-elasticity in one dimension.

Chapter Three:

In this chapter, the standard ADM is used to obtain the approximate solutions for some problems: The first problem is: Population dynamics with density-dependent migrations and the Allee effects, where we studied special cases of this problem and represented the obtained results graphically.

The second problem: We used ADM to obtain the approximate solutions of some systems of NPDEs, such as, Cauchy problem of thermo-elasticity in one dimension and coupled Drinfel'd- Sokolov-Wilson wave equations.

Chapter Four:

In this chapter, HPM is presented to solve some coupled systems of NPDEs. This method is used to solve the Manakov system which consists of non-linear coupled system of Schrödinger equations in one dimension and make numerical comparison with the methods (VIM, ADM and HPM), to know the strong and weak points in using these methods. To measure the efficiency of these methods, we computed the conservative quantities and found that these quantities are constants, by using these methods. Also, the proposed method is introduced to solve coupled systems of NPDEs represented by, system of Burger's equations and Cauchy problem of thermo-elasticity in one dimension.

All computations in this thesis are done by using, Matlab Version 7.1, Mathematica Version 5 and Fortran.
Some results of this thesis are published in [55]-[58] in the list of references, at the end of this thesis.