Chapter 1

Introduction and Basic Concepts

In the last few decades, considerable work has been invested in developing new methods for approximate solutions of linear and non-linear differential equations which arise in many scientific and engineering problems. Moreover, there has been a great need for effective algorithms to avoid the onerous work required by traditional methods. So, many approximate and numerical methods, such as, the grid points techniques [17], perturbation techniques [30], spline solutions, and others, have been developed. However, each of these methods suffers from one or more limitations. The grid points techniques define the solution at grid points only. The spline solution requires restrictions on boundary points. The perturbation method suffers from a high computational workload specially when the degree of nonlinearity increases.
In the following four different methods for solving NPDEs are introduced.

1.1 Variational iteration method

In this section, we introduce the analysis of one of the recent methods which has more implementation for solving wide range of linear and non-linear differential equations. This method is called VIM ([1], [4], [31]), which is proposed by He [29] as a modification of a general Lagrange multiplier method. Also, VIM is based on the use of the restricted variations and correction functionals which has found a wide application for the solution of NPDEs [32]. This method does not require the presence of small parameters in the differential equation, and does not require that the nonlinearities be differentiable with respect to the dependent variable and its derivatives. This technique provides a sequence of functions which may converge to the exact solution of the problem. Also, it has been shown that this procedure is a powerful tool for solving various kinds of problems. He [29] has introduced VIM to solve effectively, easily, and accurately a large class of nonlinear problems with approximations converging rapidly to accurate solutions.

In 2006, He and Xu-Hong Wu [31], applied this method to construct the solitary and compact-like solutions. Also, He [32] used VIM for solving the delay differential equations.

He [34] first applied VIM to solve autonomous ordinary differential systems. In this method, the general Lagrange multipliers are introduced to construct functional for the systems. The multipliers in the functional can be identified by the variational theory. The initial approximations can be freely chosen with possible unknown constants, which can be determined by imposing the boundary/initial conditions. Some examples are given to reveal that the method is very effective and convenient.

Wazwaz [64] used VIM for analytic treatment for linear and non-linear ordinary differential equations, he found that this method is capable of reducing the size of calculations and handles both linear and non-linear equations, in a direct manner. However, for concrete problems a huge number of iterations are needed for a reasonable level of accuracy.

Wazwaz [65] introduced a framework for obtaining the analytic solutions to linear and non-linear system of partial differential equations by using VIM. The method reduces the calculations size and overcome the difficulty of handling non-linear terms. Numerical examples are examined to highlight the significant features of VIM. Moreover, the method shows improvements over existing numerical techniques.

Wazwaz [66] introduced the reliable VIM to determine rational solutions for the KdV, $K(2,2)$, Burger's, and cubic Boussinesq equations in a straightforward manner. The study highlights the efficiency of the method and its dependence on the Lagrange multiplier.

Sweilam [54] implemented this method for solving cubic non-linear Schrödinger's equation. Special attention from many authors is given to study a convergence analysis of VIM, one can see for example ([57], [61]). This method is used to solve wide range of problems consisted of ordinary or partial differential equations. Most authors found that the shortcoming arising in the Adomian decompositions method can be completely eliminated by VIM.

The main advantages of VIM are, this technique solves linear or nonlinear problems without discretization of its variables; therefore, it is not affected by computation round-off errors and one is not faced with necessity of large computer memory and time. This method provides the solution of some problems in closed form while the mesh point techniques, such as finite difference method ([17], [35]) provides the approximation at mesh points only.