Introduction

Studying Finsler geometry, one encounters substantial difficulties trying to seek analogues of classical results of Riemannian geometry. These difficulties arise mainly from the fact that in Finsler geometry all geometric objects depend not only on positional coordinate as in Riemannian geometry, but also on directional argument. Moreover, in Riemannian geometry, there is a canonical linear connection on the manifold M, whereas in Finsler geometry there are at least four canonical linear connections: The Cartan connection, the Berwald connection, in addition to the Chern (Rund) and the Hashiguchi connections. However, these are not connections on M but on TM, the tangent bundle of M. In Riemannian geometry, there is one curvature tensor associated to the Riemannian connection and no torsion tensor. In Finsler geometry the situation is much more complicated. Associated to a Finsler connection, there are three curvature tensors, and, in general, five torsion tensors. Consequently, Finsler geometry is much richer in content than Riemannian geometry. In fact, Riemannian geometry emerges as a special case of Finsler geometry, the case in which the metric depends solely on the positional argument.

For a differential one-form $\beta(x, dx) = b_i(x)dx^i$ on M, G. Randers [36], in 1941, introduced a special Finsler space defined by the change $\overline{L} = L+\beta$, where L is Riemannian. His aim was to construct a generalized field theory that would encompass both gravity and electromagnetism. M. Masumoto [24], in 1974, studied Randers space in a more general setting, by assuming that L is Finslerian. V. Kropina [20] introduced the change $\overline{L} = L^2/\beta$, where L is Reimannian. This change has been studied by many others authors such as Shibata [40] and Matsumoto [23]. Both Randers and Kropina changes have many applications in physical theories and as such have been studied by many authors, from both the physical and mathematical aspects ([10], [12], [33], [42], [45], [47]). In 1984, C. Shibata [41] studied the general case of any β-change, that is, $\overline{L} = f(L, \beta)$, which generalizes many changes in Finsler geometry ([20], [24], [53]). In this context, he investigated the change of torsion and curvature tensors corresponding to the above transformation. In addition, he also studied some special Finsler spaces corresponding to specific forms of the function $f(L, \beta)$.

On the other hand, in 1976, M. Hashiguchi [16] studied the conformal change of Finsler metrics, namely, $\overline{L} = e^{\sigma(x)}L$. In particular, he also dealt with the special conformal transformation named C-conformal. This change has been studied by many other authors ([19], [47]). In 2008, S. Abed ([1], [2]) introduced the transformation $\overline{L} = e^{\sigma(x)}L + \beta$, thus generalizing the conformal, Randers and generalized Randers changes. Moreover, he established the relationships between some important tensors associated with (M, L) and the corresponding tensors associated with $(M, \overline{L})$. He also studied some invariant and σ-invariant properties and obtained a relationship between the Cartan connection associated with (M, L) and the transformed Cartan connection associated with $(M, \overline{L})$.