INTRODUCTION

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The discovery of theories and concepts applicable to decision-making processes has recently increased the complexity of problems eligible for analytical treatment. One of the more pertinent criticisms of current decision-making theory and practice is directed against the traditional approximation of multiple goal behaviour of men and organizations by single, technically-convenient criterion. Reinstatement of the role of human judgement in more realistic, multiple goal settings has been one of the major recent developments in the literature.

Given a multi-objective linear programming problem

$$\{\max(z^1(x), z^2(x), \ldots, z^r(x)) \mid x \in X\},$$

$$X = \{x \in R^n \mid Ax \leq b, x \geq 0\}$$

where A is an (mxn)-matrix and b is an m-column vector. The solution for MOLPP is to maximize all the objective functions simultaneously, except for the trivial case, when all the objective functions have only one optimal solution, it is impossible to achieve this goal.

Instead of searching for an optimal solution for MOLPP, the study of such problems is concerned with the problem of finding all solutions that are efficient, i.e. solutions at which an improve in one of the objectives will contradict with the improving of at least one of the other objectives.

The problem of the formation of a single optimality criterion from a number of essentially noncomparable elementary criteria first introduced by Pareto 1896 [25], the concept of Pareto optimality, or efficient solutions found its way into operations research in the pioneering work of Koopmans [17] in 1951, this was in connection with the activity analysis of production and allocation.

Afterwards, the direct extensions of Koopman's idea appeared in Charnes and Cooper [4], 1961.
Geoffrion, A.M. [11] in 1967 presented an algorithm for maximizing two objective functions via parametric linear programming.

Most authors deal with general optimality conditions. For example, Zadeh, L.A. [33], 1963, Klinger, A. [16], 1964, Dacunha and Polak, E. [5], 1967; and Geoffrion [12], 1968. Yu in his papers [29]-[31] which are published in 1972, 1973, and 1974, proposed a structure of domination over the objective space. Then, he studied the geometry of the set of all efficient solutions and the method for locating it.

Philip J. in [26], 1972, presented algorithms that solve the problems: (i) to decide if a given point in X is efficient, (ii) to find an efficient point in X, (iii) to decide if a given efficient point is the only one that exists, and if not, find other ones, and (iv) the solution of the above problems does not depend on the absolute magnitudes of the coefficients of any objective function.

Belenson and Kapur [2], 1973, introduced an algorithm for finding an efficient solution which is the best in some sense for the MOLPP by applying the linear programming approach for the solution of two person-zero sum games.

In 1974, Isermann [15] presented an algorithm by which all efficient solutions for MOLPP are determined. The procedure comprises these steps. In the first step, an initial efficient basic solution for the considered MOLPP is determined unless the set of efficient solutions is empty. The set of all efficient basic solutions and all efficient extreme rays is established in a second step. Finally, the set of all efficient solutions is constructed as a union of a finite number of sets of efficient solutions.

In 1974, Zeleny [34] introduced a method for locating all efficient extreme points which is based on multi-parametric programming. The k-dimensional parametric space, which can be interpreted as the set providing all possible weighted combinations of k objectives, is decomposed into a finite number of subspaces. These subspaces provide a set of optimal weights imputed to each objective function by corresponding extreme point solution. The decomposition also provides a criterion upon which the decision about nondominance of any particular extreme point can be based.

Zeleny also presented a multicriteria simplex method for locating all efficient extreme points, independent of any parametric considerations. Zeleny developed a technique for generating all efficient solutions from a given set of efficient extreme points.

Yu and Zeleny [32], in 1975 have shown that the set of efficient solutions of the MOLPP is the union of efficient faces of X.

Benveniste [3] in 1977 proposed a simple and economical method for identifying whether the entire set of feasible points is efficient or not.

Ecker and Kouada [7], in 1978 developed a method for finding all efficient extreme points for MOLPP. Simple characterizations of the efficiency of an edge incident to a nondegenerate or a degenerate efficient vertex are given. These characterizations form the basis of an algorithm for enumerating all efficient vertices.

Dauer [6], in 1980 developed a practical technique for characterizing the set of nondominated solutions of a MOLPP as the solution set of a linear algebraic system. Such a system include all nondominated solutions and so the analysis is not limited to a discrete set of solutions as is generated by MOLP algorithms. The algebraic system is constructed using a specific set of nondominated solutions for which appropriate Lagrange multipliers are available.

Sebo [28], in 1981, introduced an algorithm for generating a sequence of efficient extreme points $x^n \epsilon X$ such that $z^n = cx^n$ is an extreme point in the objective space. For each x^n, a multiobjective simplex tableau is obtained representing the current efficient extreme points x^n and the current description of the objective functions.

This thesis, which consists of introduction and four chapters is devoted to the study of theories and methods applicable to MOLPP.

In Chapter I, we present a study on parametric linear programming problems from two sides: (i) parameters in the right hand side of the constraints, and (ii) parameters in the coefficients of the objective function. Each case is studied depending on a scalar parameter and depending on a vector parameter.

In Chapter II, we introduce theory and methods for solving MOLPP. We present three methods for solving MOLPP, the first one is to find a best compromise solution for MOLPP which depends on determining the best weight by using the approach of two person-zero sum games. The second method is an extension to first method to deduce a new approach for obtaining all efficient faces to MOLPP. The proposed algorithm has two important features, first one, it designates the efficient faces without determining the extreme points corresponding to each face. The second feature is that any point generated by it is efficient and there is no need for applying any test for efficiency. The third method is aimed to enumerate the extreme efficient points for MOLPP and decomposing the solvability set of parameters into subdivisions whose interiors are mutually disjoint. This approach is based on the connection between MOLPP and parametric optimization problems. The simplex method is applied to reach this goal.

Chapter III deals with the MOLPP with parameters in the right hand side of the constraints. The method of constraints is used as a natural way of treating such problems. An algorithm for decomposing the solvability set according to the different stability sets of the second kind is presented together with the determination of the efficient points which rest on the corresponding sides.

In Chapter IV, we present Dantzig-Wolfe decomposition principle in linear programming which is the first one to be developed and the most well known.
The Dantzig-Wolfe technique and the second method in Chapter II are combined together to obtain a new technique for solving large-scale MOLPP.