INTRODUCTION

The thesis is concerned with the L^2-classification of the second-order ordinary linear differential equation

$$- \frac{d}{dx}(p \frac{dy}{dx}) + qy = \lambda y \qquad (1)$$

where the coefficients p and q are real-valued functions defined on the real-interval $[0, \infty)$ and satisfy some basic conditions; $\lambda = \mu + i\nu$ is a complex parameter.
Equation (1) can be written on the form :

$$M(y) = \lambda y \qquad (2)$$

where $M(.)$ is the formally second-order linear differential expression given by :

$$M(y) = -(py')' + qy \quad (\,' \equiv \frac{d}{dx}) \quad . \qquad (3)$$

Thus , with the conditions on p and q , the differential expression $M(.)$ is regular at 0 and is singular at ∞ , this is the standard terminology of [25] chapter V §15.1 and [4] chapter 9 § 2.

According to the original definitions of Weyl [32] sections 2.1 and 2.19; [25] section 17.5; [4] sections 9.2 the differential expression $M(.)$ may be classified as limit-point (LP) or limit-circle (LC) at ∞ according to whether the differential equation (3) has, respectively, exactly one or two, linearly independent solutions in the Hilbert space $L^2(0, \infty)$

when Im $(\lambda) \neq 0$. This classification depends only on the nature of the coefficients p and q. There are no known necessary and sufficient conditions on p and q to distinguish between limit-point and limit-circle cases at ∞, but there is necessary and sufficient condition in terms of certain functions in the Hilbert space $L^2(0, \infty)$.

Closely related with the L^2-classification of M(.) are the strong limit-point (SLP), Dirichlet (D) and conditional Dirichlet (CD) cases at ∞.

There are a number of known connections between the strong limit-point and Dirichlet conditions, for general accounts. (See Everitt [11] and [14] and a number of special results, see Kwong, M.K. [22].)

The purpose of this thesis is to discuss the Lp, LC, SLp, D and CD cases at ∞ for the second-order linear differential expression M(•) given by (3).

Chapter I contains definitions and facts in the theory of linear operators-pertinent to later chapters.

Chapter II is concerned with the linear operators associated with the formally second-order differential expression M(.). Also in this chapter the relation between the concept of deficiency indices and Weyl's L^2-classification is considered.

In chapter III, the concept of Weyl's L^2-classifications is extended to the strong limit-point, Dirichlet and conditional Dirichlet properties at ∞.

An application is given at the end of chapter III, so that M(.) satisfies the strong limit-point and conditional Dirichlet cases at ∞ under some conditions on p and q. Namely, the function q(x) introduced by Everitt in [13] takes the form :

$$q(x) = 2x^2 + xe^x \cos(e^x) + \sin(e^x),$$

while we introduced a function in the form

$$q(x) = ax^2 + b(x \cos(x) + \sin(x)).$$