INTRODUCTION

There has been a considerable interest with stability and bifurcation of solutions of nonlinear systems of differential equations.

In 1968 La Salle has developed a general theory for examining the asymptotic stability of the trivial solution of the nonautonomous system of differential equation

$$\underline{X}' = \underline{f}(t,\underline{X}) \qquad (I)$$

where $f:[0,\infty)xR^n \to R^n$ is continuous, based on the properties of the derivative of Liapunov function. There are many recent works which are devoted to relax some of the restrictions on Liapunov function.

Recently, Dannan and El-Alyadi introduced asymptotic Lipschitz stability and conjectured that uniformly asymptotic stability does not imply uniformly Lipschitz stability.

The thesis is concerned with improving the theory of asymptotic stability of the trivial solution of (I) by using Liapunov function. A review is given on the preceding progress. New sufficient conditions are given for the case when uniformly asymptotic stability implies Lipschitz uniformly stability of the trivial solutions of (I). The thesis is also concerned with applying Hopf's bifurcation, stability of bifurcating solutions and center manifold theories for the nonlinear

autonomous system of differential equation

$$\underline{X}' = \underline{f}(\underline{X}, \lambda) \qquad \text{(II)}$$

In the case of two and three dimensions, this was done by using the natural parameters given in the system. In fact we consider an immune system recently discussed by Bell-Pimbley and apply the above theories.

This thesis consists of three chapters.

In Chapter One we outline known results and theories of bifurcation and stability of periodic solutions of autonomous system of differential equations

$$X' = f(X) \qquad \text{(III)}$$

Chapter Two is divided into two main parts. First part is devoted for known results about the asymptotic stability and uniform asymptotic stability of nonautonomous system (I). In the second part we improve some results concerning the uniform asymptotic stability for the nonautonomous system (I).

In particular, a sufficient condition was given for asymptotic stability of the trivial solution of (I) using less restricted Liapunov function, and the possibility of obtaining both uniform and Lipschitz asymptotic stability.

In Chapter Three we apply some results of Chapter One and Chapter Two for Bell-Pimbley immune systems and a generalized Liénard equation.