Introduction

The notion of a semiring (i.e. a universal algebra with two associative binary operations, where one of them distributes over the other) was introduced by Vandiver [20] in 1934.

Semirings arise naturally in many areas of mathematics and computer science. In fact, semirings provide the most natural common generalization of the theories of rings and bounded distributive lattices. The basic mathematical theory of semirings may be found in [9], [10], [11]. However, you may consult [10] for terminologies.

In this thesis, we study the notions of regularity (in the sense of von Neumann), k-regularity and complete k-regularity for semirings. On the other side, we study the notions of injectivity and P-injectivity for semimodules. In fact, for any cardinal, $2 \leq \alpha \leq \aleph_0$, we introduce the notions of α-regular semirings and α-injective semimodules. In particular, if $\alpha = 2$, these notions coincide, respectively, with Von Neumann regularity for semirings and P-injectivety for semimodules. Characterizations of α-regular semirings are given (***Th***. 4.1.2), and several invariant properties are examined (***Props***. 4.1.4– 4.1.7). Indeed, Some properties of α-injective semimodules are obtained (***Prop***. 4.2.2). The main result of this work is the following theorem : *" **A semiring R is α-regular if and only if every left R-semimodule is α-injective** "* (***Th***. 4.2.4).

Regular semirings with central idempotents are investigated and shown to admit an involution (***Prop***. 2.1.8). Finally, consistent α-systems of equations are studied and applied to obtain a characterization of α-injective cancellative semimodules over a cancellative semiring (***Th***. 4.3.5).

The thesis consists of four chapters :

In ***chapter 1***, we give some basic concepts and results of semirings and semimodules. In section 1.2, some types and special classes of semirings, such as yoked, cancellative, plain and rings of differences of semirings are presented. Particular classes of semimodules such as, free, simple, projective, injective and modules of differences of semimodules are given in section 1.3.

In ***chapter 2***, a review of the work of J. Ahsan, M. Shabir and H. J. Weinert in [2], concerning regular semirings and its characterization in terms of P-injective semimodules is presented. Further more, the class of regular semirings with central idempotents is investigated and shown to admit an involution.

In ***chapter 3***, we give an overview of k-regular and completely k-regular semirings and their characterizations in terms of k-ideals [14] and k-semifields [18], respectively. In addition,

we present S. Ghosh [8] and P. Mukhopadhyay [17] results concerning the k-regularity of $n \times n$ matrices over certain class of semirings.

Chapter 4, covers the notions of α-regular semirings and α-injective semimodules along with their characterizations. It does also include the closure and invariant properties of α-regular semirings. Moreover, the consistent α-systems of equations are studied and applied to obtain a characterization of α-injective cancellative semimodules over a cancellative semiring.

The main results of this thesis are included in chapters (2) and (4).