

## Introduction

The concentration of stresses on the edges of holes (punches) is of practical interest, e.g., concentration of stresses around holes (doors, windows, punches, etc) in the body of airplanes ships, bridges, etc. For studying such problems, a solution of the plane problem of elasticity is needed.

The earlier studies of the plane profiles equilibrium were done by G.V. Kolosov and N.I. Muskhelishvili. Indeed Muskhelishvili [1] gave the solution of many problems for a plane weakened by holes (circle, ellipse, etc).

Later, D.I. Sherman and his followers solved several problems related to the deformation of half plane weakened by two holes (circular and elliptical) distributed at certain distances from the straight boundary. They also considered a circular material with holes being blocked by enforcing another materials. The results of these problems are summarized by Sherman [2].

The distribution of stresses about holes in plane, half plane, and domains with circular boundaries are summarized in the monographs of Panasoka, Sabrok and Datchen [3], and Parton and Parline [4]. The works in contact problems are also abstracted in the monographs [5,6]. All previous problems deal with infinite or semi-infinite domains.

The determination of the stress-strain state in a strip weakened by holes, represent a model for many important practical problems, e.g., defects, holes (punches) in the bodies of ships, flying-bodies etc.

The solution of the problems of strip weakened by holes in the form of circles, ellipses etc contains great mathematical difficulties. In the present time only a few of such problems, with circular and elliptical

holes, are solved either for isotropic linear materials [7-17], or for anisotropic materials [18-20] and orthotropic materials [21].

The most effective method for solving boundary value problems of the plane theory of elasticity is by using the tools of the theory of functions of complex variables, by constructing a simple form of analytic functions (polynomials) or rational functions. For the case of deformed bodies occupying simple domains, the theory of conformal mapping is used to map the outside (inside) of a unite circle onto the outside (inside) the boundary of the given domain [1]. While in the case of multiply connected domains, D.I. Sherman [22-25] suggested an effective method for solving finite or infinite doubly connected domain  $S$  bounded by the contour  $L$ . This latter consists of two non-intersecting closed curves,  $L_1, L_2$  where one of these contains completely the other. The problem reduces to the determination of a pair of analytic functions  $\varphi_0(z)$  and  $\psi_0(z)$  satisfying certain given boundary conditions on the contours  $L_j$  ( $j=1,2$ ).

The method suggested above depends on the replacement of the given doubly connected problem by simply connected problem using an auxiliary function  $\omega(t)$ , satisfying Hölder condition, given on the inner contour, together with the analytic continuation. The solution of the obtained fredholm integral equation leads to an infinite system of linear algebraic equations in the coefficients of Fourier series of the function  $\omega(t)$ . The determination of the Fourier coefficients of the function  $\omega(t)$  yields a series form for the potentials  $\varphi_0(z)$  and  $\psi_0(z)$ .

The method of Sherman together with the Fourier integral transformation were used by N.I. Meroninka to solve the problem of strip weakened by one or two circular holes [14,15,16]. The strip is loaded by

uniformly distributed normal forces on the straight contour and at infinity while the contour of the holes are free from external forces. He also considered the bending problem of the same strip.

Then I. Eltaher extended the above mentioned method to the problem of multiply connected strip weakened by holes (elliptic hole, two elliptic holes and  $n$  elliptic holes,  $n > 2$ , distributed in a random way) [7,8,10,11].

In this thesis we follow the methods of D.I. Sherman and his followers to solve the problem of an infinite strip weakened by polygonal hole. The boundary of the strip is loaded by uniformly distributed external forces, while the boundary of the hole is free from external forces.

The first chapter consists of two sections. In the first one, the relations and the equations of two dimensional elasticity to be used in this thesis are summarized. Some mathematical methods (techniques), formulas and relations which are necessary for this work are also introduced (without proof). The second section contains an outline of the method of solution of an infinite simply connected strip problem by means of Fourier transformation.

In the second chapter we formulate the inverse of the conformal mapping  $Z = A(\zeta + \zeta^{-n})$  which maps the exterior of a circle with radius  $\rho > 1$  (in  $\zeta$ -plane) onto the exterior of many axial symmetric curves (in  $z$ -plane). In the second section the parameters  $\rho$  and  $A$  of this mapping, for different  $n$ , are fitted. In the second the inverse of this mapping is represented in terms of Borman-Lagrange series. The coefficients of this series are calculated in  $\zeta$ - plane, using both the original mapping and the Cauchy type integrals. In the last section appropriate formulas for the Cauchy kernel are obtained in  $\zeta$  plane.

The third chapter consists of three sections. In the first we transformed the given problem to a simply connected strip problem which lead to the potentials  $\phi(z)$  and  $\psi(z)$ . In the second these potentials are expressed in series form in terms of the coefficients of the auxiliary function  $\omega(t)$ . In the third section the Fourier coefficients of the auxiliary function  $\omega(t)$  are determined from an infinite system of linear algebraic equations. This system is constructed from the continuation condition of the potentials  $\phi(z)$  and  $\psi(z)$  on the polygonal boundary.

In chapter four we illustrated the treatments and techniques discussed in the previous chapters by studying the problem of a curvilinear crack along the axis of an infinite strip. The strip is loaded by uniform forces. Also we studied different cracks along the strip axes, and examined three cases for different loads on the strip boundary for each crack.