INTRODUCTION

This thesis is concerned with some problems on the flow of rarefied gases in channels with different cross sections. The dependence of the flow nature on the materials of the walls are investigated in all the problems considered in this thesis. We shall give a survey of the methods of solution used in the treatment of similar problems under consideration in this thesis.

1.1. RAREFIED GAS DYNAMICS:

Rarefied gas dynamics (or super aerodynamics) studies phenomona takeing place for arbitrary ratio of the mean free path of molecules to the characteristic dimension time of the phenomena. The study of the behaviour of rarefied natural gases started as early as the last half of 19th century. At that time it held the interest of such major physicists, as Maxwell and Boltzman. Their work was continued by M. Smoluchwski, Clausing, and M. Knudsen. Most of their work deals with low velocity flowing gas in pipes. At the same time, the development of generalized mathematical methods for solving kinetic equations by Hilbert, Chapman, Enskog and Karlmen was proceeding. In 1964, Tsin suggested the new standard classification of rarefied gas flow based on the Knudsen number $Kn = \frac{\ell}{L}$, where ℓ is the mean free-path, and L is the characteristic length of the studied object.

The theory of rarefied gas dynamics is based on the familiar kinetic gas theory. The main subject deals with the study of the behaviour of highly rarefied gas ($Kn \gg 1$), and slightly rarefied gas ($Kn \ll 1$).

1.2. THE MAXWELL -BOLTZMANN COLLISION EQUATION:

The Maxwell-Boltzmann integro-differential equation for single particle distirbution function in the presence of external force stated in [8]. as

$$\frac{\partial f}{\partial t} + \vec{V}\frac{\partial f}{\partial \vec{r}} + \frac{\vec{F}}{m}\frac{\partial f}{\partial \vec{V}} = \iint 9\,(f'f_1' - ff_1)\,dx\,d\vec{v},$$

where,

f : $f(\vec{r},\vec{v},t)$ is the mean density distribution function.

dx : refers to integration over the so-called collision parameter.

$\vec{V}$: (V_1, V_2, V_3) the velocity of the molecules.

$\vec{r}$: (x_1, x_2, x_3) the phase coordinates.

$\vec{F}$: is the external force, and,

g : is the relative velocity magnitude.

Bhatnager, Gross, and Krook [2] have suggested the use of the so-called relaxation model of the Boltzmann equation or (B.G.K) model which has the form of the non-linear integro differential equation;

$$\frac{\partial f}{\partial t} + \vec{V}\frac{\partial f}{\partial \vec{r}} + \frac{\vec{F}}{m}\frac{\partial f}{\partial \vec{V}} = \frac{1}{\tau}(f_o - f),$$

where,

τ : is the relaxation time (mean time between collisions).

R.h.s: represents the collision term.

f_o : is the local Maxwellian function which is given by;

$$f_o = \frac{n}{(2\pi RT)^{3/2}} \exp\ [-(\mathbf{C}-V)^2/2RT].$$

1.3. BOUNDARY CONDITION FOR THE DISTRIBUTION FUNCTION:

Cases when an impermeable solid surface is present in the flow are frequently of interest in solving problems of rarefied gas dynamics.

The boundary condition for the function (f) at such surface can be obtained by the following consideration; suppose that at time t there is a molecule close to the surface element ds, and that before collision with the surface; it has a velocity in the range between $\vec{c}$ and $\vec{c} + \overrightarrow{dc}$. If the mechanism of interaction of the molecule with surface is known; then, it should be possible to specify the probability $k(\vec{c},\vec{c}\,')\overrightarrow{dc}\,'$, that this molecule will have velocity in the range from $\vec{c}\,'$ to $\vec{c}\,' + \overrightarrow{dc}\,'$ after collision with the surface.

Here by definition $\int k(\vec{c},\vec{c}\,')\overrightarrow{dc}\,' = 1$, then at the solid surface:-

$$f(\vec{c}\,',\vec{r}_o,t) = \int k(\vec{c},\vec{c}\,')\ F(\vec{c},\vec{r}_o,t)\ \overrightarrow{dc} \qquad \vec{c}.\vec{n} < o, \quad \vec{c}\,'.\vec{n}. > o \qquad \ldots\ldots(1)$$

where $\vec{r}_o$ is the radius vector of a point on the solid surface, and $\vec{n}$ is a unit vector of the outside normal to the surface at this point. The function $k(\vec{c},\vec{c}')$ is called the scattering kernel. It may have discontinuities for certain combinations of elements, and also may become infinite i.e. the kernel can be singular.

Condition (1) is the most general form of a boundary condition at a solid surface. However, in particular cases this condition can be written in a much simpler form. Thus, for example, it is theoretically possible to have specular reflection of the molecules, in which only velocity component normal to the surface changes sign after collision, while the tangential velocity component does not change at all.

it is abvious that,in the case of specular reflection;

$\underline{c}' = \underline{c} - 2\underline{n}(\underline{c}.\underline{n})$, and instead of (1) we can get:-

$$f(\underline{c}-2\underline{n}(\underline{c}.\underline{n}), \underline{r}_o, t) = F(\underline{c}, \underline{r}_o, t) \qquad \underline{c}.\underline{n} < o \qquad \ldots.(2)$$

Of more interest from the point of view of practical applications, is the case of diffuse reflection. Here it is assumed that the distribution of molecules reflected from the wall which is entirely independent of the distribution of the incident molecules. If it is Maxwellian at a temperature equal to the wall temperature T_o, then we get;

$$\underline{f}(\underline{c}', \underline{r}_o, t) = n(2\pi RT_o)^{-3/2} e^{-c'^2/2RT_o}, \qquad \underline{c}'.\underline{n} > o \qquad \ldots.(3)$$