

INTRODUCTION

Differential line geometry consists of three parts. One of them is the theory of ruled surfaces (a ruled surface is defined as a one parametric family of lines). The second is the theory of congruences (a congruence is defined as a two parametric family of lines). The third is the theory of complexes (a complex is defined as a three parametric family of lines).

From these three parts of differential line geometry, only the theory of complexes does not take its final form. The first results in the theory of complexes have been obtained by Tran son. A. [12]. Later, Plücker obtained important results in line geometry. He considers the line as an element of the space of lines. Since any straight line is determined by four parameters, the space of lines is a 4-dimensional space. If these parameters depend on another variable, we obtain a ruled surface. If they depend on two variables we have a congruence of lines. If the parameters of the line depend on three parameters then we have a complex of lines. Before forty years ago, the theory of complexes was studied in some special cases because the method which adopted was very complicated. In 1940, Finikov, C.P. [6] was the first one who used Cartan's differential

calculus in the theory of complexes in metric spaces. The theory of complexes in metric and projective spaces have been recently studied by Kovansoven, N.I. [8] . He gave the principal ideas in the classifications of some classes of complexes based on geometrical properties.

The present thesis is concerned with the theory of complexes in 3-dimensional Euclidean space and particularly those having the following geometrical properties :

1- One of the principal surfaces coincides with the coordinate surface $w^1 = w^2 = w^3 = 0$.

2- Complexes which have double points of orthogonality coincide with the point at infinity and they can be distributed into one parametric family of normal congruences cutting orthogonally an integral surface. The methods adopted in this thesis are based on Cartan's differential calculus. This thesis consists of four chapters.

Chapter I, contains the fundamental concepts of line geometry. We give some theorems and definitions of the line and manifolds of line such as linear complexes and linear congruences.

In Chapter II, we study differential forms and its applications on system of exterior differential equations in geometry. Also in this chapter we mentioned Cartan's common methods for treating the system of exterior differential

equations which we have used in this work.

Chapters III, IV, contain the main results in this thesis. We investigate in chapter III particular kinds of complexes whose one parametric family of principal surfaces coincide with the coordinate surface $w^1 = w^2 = 0$. The differential equations of these complexes are obtained and the existence theorems are proved. The geometrical properties of one of them are investigated by a list of theorems (see theorem (3.4) to theorem (3.10)), which can help us to give its geometrical construction.

In chapter IV, we investigate particular kinds of complexes of lines in E^3 , which can be distributed into special kinds of normal congruences. The differential equations of this complex are obtained and the existence theorem is proved. The geometrical construction, based on its geometrical properties, of such complex is given.
