

# EXISTENCE AND LOCALIZATION OF HARMONIC AND SUB-HARMONIC SOLUTIONS OF SECOND ORDER NON - LINEAR DIFFERENTIAL EQUATION [18]

## § 1.1 Introduction

An important problem in the theory of non-linear differential equations or (non-linear oscillations) is the existence of periodic solutions. Since the earlier work of Poincaré [43] until now, the subject received much attention from many investigators for example, Urabe [57,58,59], Levinson [30], Lefschetz [29], Cartwright and Littlewood [4], Kryloff and Bogoliuboff [28], Coddington and Levinson [8], Choy-Tak-Taam [7], Hale [22,23,24], Cesari [5], Tomas and Tondl [55], Schmitt [49], Elnaggar [11], Elnaggar and Elboughy [14], Elnaggar and Hamad-Allah [16], Peter Girg [42], Mehri and Shadman [34].

There exist many topological or analytical methods, which are available for proving the existence of periodic solutions of certain systems of differential equations, for example, the Isocline method, Delta method ( it is more applicable to equation with oscillatory solutions), Lienard's method (it is most conveniently used to deal with what are called self-excited oscillations, but it is also applicable in other cases), the Index method of Poincaré [43] and the fixed point method of Brouwer [4].