Introduction

There are many ingenious techniques for obtaining exact solutions of differential equations, but most work only for a very limited class of problems. Symmetry is the key to solving differential equations (even an unfamiliar one).

More than a century ago, the Norwegian mathematician Sophus Lie put forward many of the fundamental ideas behind symmetry methods. Most of these ideas are essentially simple, but are so far reaching that they are still the basis of much research. These method based on the invariance of differential equation under a continuous group of symmetries. These groups, now universally known as Lie groups, have had a profound impact on all areas of mathematics, both pure and applied, as well as physics, engineering and other mathematically-based sciences. It is impossible to overestimate the importance of Lie's contribution to modern science and mathematics [1]. During the past three decades, research on and application of the group properties of differential equations has generated a substantial literature. As evidenced by the literature in book form, for example, Bluman and Cole [2], Ovsiannikov [3], Hill [4], Olver [5-6], Sattinger and Weaver [7], Bluman and Kumei [8], Hans [9], Hansen [10], Ames [11,12], Seshadri and Na [13], Fuchs and Schweigert [14], Dresner [15], Ibragimov [16-20] and Hydon [21]. Also there are a great varity of papers, for example, Bluman [22], Bluman and Cole [23], El-Wakill [24], Saied and El-Wakill [25], Saied [26-28], Clarkson and Hansfield [29], Saied and Magdy [30,31], Saied and Abd El-Rahman [32,33], Abd El-Rahman [34] and Saied and Khalifa [35].

A symmetry group of a system of differential equations is a group of transformation maps any solutions of the system to other solutions.

Similarity transformations which essentially reduce the number of independent variables in partial differential equation (PDE), have been widely used in equations of mathematical physics. Similarity transformations have also been used to convert a boundary value problem to an initial value problem . In other instances Similarity transformations have been used to reduce the order of an ordinary differential equation (ODE) and to convert moving boundary conditions to constant boundary conditions. Mappings have also been discovered which transform nonlinear partial or ordinary differential equations to linear forms [2,5,8,13].

Unfortunately, for systems of partial differential equations the symmetry group is usually of no help in determining the general solution However, one can use symmetry groups to explicitly determine special types of solution. These solutions are found by solving a reduced system of differential equations involving fewer independent variables than the original system. For example, the solutions to PDE in two independent variables which are invariant under a given one-parameter symmetry group are all found by solving a system of ODEs.

This similarity transformation, is just one type of transformation that can be obtained by the use of group procedures, which starts out with a general group of transformations. The reduction of a given PDE (we assume that the given PDE is not of first order since such an equation can be solved by characteristic equations) can be done by two different methods

(i) The "Classical method" for finding symmetry reduction of PDE's is the one-parameter Lie group of infinitesimal transformations, requiring that, the PDE is invariant yields system of linear PDE's for the infinitesimal functions [2-21] this method has been successfully applied to obtain new exact solutions for several physical significant PDE's, see [24-35] for complete survey and references of these subjects.

(ii) The "non-classical method" or "method of conditional symmetries", which requires the original PDE and the surface condition to be invariant under the transformations. The solution set may be larger [8, 3, 36-38].

The present thesis consists of four chapters:

Chapter 1. In this chapter, we introduce the basic ideas of the Lie group of transformations necessary for the study of invariance properties of differential equations.

Chapter 2. In this chapter, we study the exact solutions of (2+1)-dimensional cubic nonlinear Schrodinger (NLS) equation

$$i\psi_t + c(\psi_{xx} + \psi_{yy}) + a|\psi|^2\psi + b\psi = 0$$

This type of nonlinear PDE occurs in a wide variety of physical. For instance in wave propagation in nonlinear and dispersive media, in quantum field theory, propagations of slowly varying electromagnetic wave envelopes in plasma, in the nonlinear optics and other applications .

[The result of this chapter published in [39]].