1-7 Previous work

Many authors [18-55] had been studied the nature of the interaction between quarks inside the heavy meson and the nature of the force acting between its constituent quarks.

Krasemann and Ono [18] assumed that at small distances the interaction between quarks becomes very similar to the electromagnetic interaction between electrons, because the coupling constant tends logarithmically to zero at $r \to 0$ [19],

$$\alpha_S(r) = \frac{12\pi}{25}\frac{1}{2\ln(\mu/\mathrm{r})} \tag{1-8}$$

where $\mu = 0.5\,fm$.

At large distance the string model [20] suggests that $q\bar{q}$ force is completely independent of the inter-quark distance and the quark spin. They proposed a potential model in the following form;

$$V(r) = V_N(r) + V_{\text{int}}(r) + V_C(r) \tag{1-9}$$

where

$$V_N(r) = \frac{-4}{3}\alpha_S(r)/r\,, \tag{1-10}$$

$$V_{\text{int}}(r) = -b\ \exp(\text{-r/c})\ , \tag{1-11}$$

and

$$V_C(r) = a \cdot r \tag{1-12}$$

Where a, b and c are fitting parameters.

Cornell group [21] assumed very simple form for the $q\bar{q}$ interactions given by,

$$V(r) = -\frac{k}{r} + \frac{r}{a^2} \qquad (1\text{-}13)$$

where $k = 0.52$ and $a = 2.34\,\text{GeV}^{-1}$

This form consists of coulomb-type term and linear confinement term, and was used to describe the heavy mesons spectra.

Martin [22] assumed a different shape for the interaction between the quark and the antiquark in case of $c\bar{c}$ and $b\bar{b}$ systems. This potential form is given by,

$$V(r) = A + B\,\text{r}^{\alpha} \qquad (1\text{-}14)$$

where A,B and α are fitting parameters.

Miller and Olsson [23] proposed a potential model consistent with the QCD requirement to describe heavy mesons spectra given by,

$$V(r) = [-\frac{k}{r} + ar]F(t) \qquad (1\text{-}15)$$

where

$$F(t) = 1 + \sum_{n=1}^{3} C_n \sin(\text{n}\pi\text{t}) \qquad (1\text{-}16)$$

$$t = 1/(1 + r/r_0) \quad ; \quad \text{r}_0 = 1\ GeV^{-1} \qquad (1\text{-}17)$$

The function F (t) must satisfy the boundary condition,

F (0)=F (1)=1 (1-18)

The transformation to the variable t maps the radial range into the interval [0,1].

The potential parameters are given by;

$k = 0.216 \pm 0.2$
$a = 0.0649 \pm 0.2$ GeV^2,
$C_1 = 1.827 \pm 1.67$,

$C_2 = 0.541 \pm 0.23$ and $C_3 = 0.336 \pm 0.39$ (1-19)

This potential form is used to fit the different energy states of $c\bar{c}$ and $b\bar{b}$ mesons by using five parameters beside the quark masses.

Clary and Byers [24] proposed a potential form consists of Coulomb-gauge exchange and a scaler confinement terms,

$$V(r) = V_v(r) + V_S(r) \tag{1-20}$$

where

$$V_v(r) = -k \ \mathrm{erf}\left(\sqrt{2}\ \mathrm{mr/r}\right) \tag{1-21}$$

m is the quark mass, $k = -\frac{4}{3}\alpha_s$, and erf is the usual error function,

$$erf(r) = \left(\frac{4}{\pi}\right)^{1/2} \int_0^r e^{-y^2} dy \tag{1-22}$$

and $V_S(r) = \frac{r}{a^2} + C$ (1-23)

Where a is a constant and the parameter C is introduced to contributes the spin-independent corrections for the heavy meson states.

Clavelli, Lichtenberg and wills [25] proposed a potential model to calculate the energy levels spacing in $t\bar{t}$ system.

This potential form is given by,

$$V(r) = -\frac{4}{3}\frac{\alpha_s(r)}{r} + a_1 \ln(\mathrm{a_2 r}) + \mathrm{br} \ , \tag{1-24}$$

where α_S is the strong interaction coupling constant ;

$$\alpha_s(r) = \frac{2\pi}{7}\frac{1}{1+cr}\frac{\lambda r - 1}{\ln \lambda r} \ , \tag{1-25}$$

and

$\lambda = \lambda_0 + \lambda_1 r^{1/2}$ (1-26)

The parameters values of this potential are taken as;

$a_1 = 333 MeV,\quad a_2 = 5.08 fm^{-1}\quad ,b = 605\, MeV.fm^{-1}\quad ,c = 0.05 fm^{-1}$ (1-27)
$\lambda_0 = 11 MeV$ and $\lambda_1 = 137 MeV.fm^{-1/2}$

Frank [26,27] proposed a non-relativistic potential model in the form;

$$V(r) = kr + a - \frac{4\alpha_s}{3r} + V', \qquad (1\text{-}28)$$

Where α_s is the strong coupling constant, V' corresponds to relativistic corrections and include spin-dependent terms, k and a are constants. In this model, the first term represents the confinement part (the effective potential at large distances) and the third one represents the one-gluon exchange potential (the effective potential at small distances).

Lichtenberg [28-30] proposed an intermediated form between the cornell [21] and Song-Lin [31] potentials to calculate the values of the spin -averaged $b\bar{b}$ bottomonium energy levels as;

$$V(r) = -ar^{-B} + br^{B} + C, \qquad (1\text{-}29)$$

where the values of the potential parameters are given by:

$a = 0.62\, GeV^{1/4}\quad ,b = 0.304\, GeV^{7/4}\quad ,C = -0.823\, GeV$ and $B = 3/4$

Fulcher [32,33] calculated the energy levels of the ψ and γ systems by using Eichten and Feinberg [34] potential model, where they found that the spin-dependent potential of a heavy $q\bar{q}$ pair could be written as the sum of three terms, each of which contains a factor of m^{-2} as;

$$V_{SD} = \frac{L.S}{m^2 r}\left[\frac{1}{2}\frac{dV}{dr} + \frac{dV_1}{dr} + \frac{dV_2}{dr}\right] + \frac{S_1.S_2}{3m^2}V_4 + \frac{1}{m^2}\left(\hat{r}.S_1\,\hat{r}.S_2 - \frac{1}{3}S_1.S_2\right)V_3, \qquad (1\text{-}30)$$

Where $S = S_1 + S_2$, V_1 and V_2 are the spin-oribt potentials, V_4 and V_3 carry the radial dependence of the spin- spin and tensor potential and V is a Richardson's potential [35] ;

$$V(r) = Ar - \frac{8\pi}{(33 - 2n_f)r} \qquad (1\text{-}31)$$

where A and n_f are fitting parameters.

The central potential and spin-oribt potential must satisfy Gromes's consistency condition [36];

$$\frac{d}{dr}[V(r) + V_1(r) - V_2(r)] = 0 \qquad (1\text{-}32)$$

They used Gromes's relation to determine V_1 directly in terms of V and V_2 that is

$$V_1(r) = V_2(r) - V(r) \qquad (1\text{-}33)$$

and the spin-oribt can be written as;

$$V_{SO} = \frac{L.S}{m^2 r}\left[2\frac{dV_2}{dr} - \frac{1}{2}\frac{dV}{dr}\right] \qquad (1\text{-}34)$$

They have that

$$V_2 = -\frac{4}{3}\frac{\alpha_S}{r} \quad , V_3 = \frac{4\alpha_S}{r^3} \qquad (1\text{-}35)$$

$$V_4 = 2\nabla^2 V_2 = \frac{32}{3}\alpha_S \delta(r) \qquad (1\text{-}36)$$

where α_S is the coupling constant that controls the size of the fine- and hyperfine-structure effects.

Thus, the spin-dependent potential is given by

$$V_{SD} = \frac{L.S}{m^2 r}\left[\frac{8}{3}\frac{\alpha_S}{r^2} - \frac{1}{2}\frac{dV}{dr}\right] + \frac{32\pi\alpha_S}{9m^2} S_1.S_2\delta(r) + \frac{4\alpha_S}{m^2 r^3}\left(\hat{r}.S_1\,\hat{r}.S_2 - \frac{1}{3}S_1.S_2\right) \qquad (1\text{-}37)$$

Grant, Rosner and Rynes [37] proposed a power law potential form to describe the $c\bar{c}$ and $b\bar{b}$ spectra as;

$$V(r) = \frac{\lambda(r^\alpha - 1)}{\alpha} + C, \qquad (1\text{-}38)$$

Where λ, α, C are fitting parameters.

Their results fit to the data on the spin –averaged levels in case of $c\bar{c}$ and $b\bar{b}$ systems.

Ding, Chao and Qin [38] proposed a color-screened confinement potential with a large string tension; T=0.26~0.32 GeV^2; to study the $c\bar{c}$ and $b\bar{b}$ spectra. By taking into account the color-screening effect, the quark-antiquark potential take the form;

$$V(r) = \frac{-4\alpha_S}{3r} + Tr\left(\frac{1-e^{-\mu r}}{\mu r}\right), \qquad (1\text{-}39)$$

Where the first term is the usual one-gluon exchange Coulomb potential and the second term is the screened confinement potential with screening parameter μ. For $c\bar{c}$ spectra the potential parameters are given by, $\alpha_S = 0.306$, $\mu = 0.156$ GeV and $m_C = 1.6$ GeV, and for $b\bar{b}$ $\alpha_S = 0.275$, $\mu = 0.132$ GeV and $m_b = 4.8$ GeV.