SUMMARY

The present thesis is concerned with the maximum principles in differential equations. These principles provide us with one of the most useful and powerful tools in the qualitative and qualitative study of both ordinary and partial differential equations. Since the early nineteenth century, the maximum principles for solutions of elliptic equations and inequalities have been used intensively in the mathematics literature. Throughout the last decades and till now, these principles have been refined and extended to more and more types of equations associated with different conditions on the boundaries of regions, where the equations are considered [see references cited at the end of this thesis]. One of the most important refinements, known as the Hoof maximum principle, asserts that at a maximum on the boundary, the outward normal derivative of the solution of an elliptic or a parabolic equation is positive.

The main objective of this dissertation is to give a complete

survey on the maximum principles and their various versions for ordinary and partial differential equations and some of its applications. We give also some examples illustrating the ideas discussed in this work.

The thesis consists of four chapters. In chapter *I* we discuss and prove different versions of the maximum principle for ordinary linear differential inequalities and equations, and some of its

applications to initial and boundary value problems. At the end of

this chapter the case of nonlinear equations is investigated.

In chapter *II* we introduce and discuss the maximum principles for elliptic partial differential equations and inequalities and some of its generalizations. Applications of these principles to different uniqueness problems are also given.

In chapter *III* we consider solutions of the nonlinear elliptic equation $\Delta u + f(u) = 0$ and obtain bounds for various quantities associated with this problem. We show that it is possible to find functions g and h so that $P = g(u) \, |\text{grand } u|^2 + h(u)$ satisfies an elliptic inequality and by applying a maximum principle we prove that P either attains its maximum on the boundary or at a critical point of the solution u.

In chapter *IV* we give some applications of the material of the previous chapters. We study the torsion problem and the "efficiency ratio" of a nuclear reactor.