

discrete analogues of corresponding results in differential equations.

In 1981 Hooker, and Patula [11] discussed oscillation of solutions of second order nonlinear difference equations of the form

$$\Delta^2 y_{n-1} + q_n y_n^\gamma = 0, \quad n \geq 1, \quad (1)$$

where $\{q_n\}$ is a real positive sequence and γ is a quotient of positive integers, Δ is forward difference operator defined by $\Delta x_n = x_{n+1} - x_n$. A more general equation of the form

$$\Delta^2 y_n + q_{n+1} f(y_{n+1}) = 0, \quad n \geq 0, \quad (2)$$

has been studied by Thandapani, Györi, and Lalli [31] and Zhang and Chen [37].

In 1985 Szmanda [28] presented several sufficient conditions which guarantee that all solutions of some second order nonlinear difference equations are nonoscillatory.

In 1993 X. He [10] studied the oscillatory and asymptotic behaviour of solutions of the second order nonlinear difference equations of the form

$$\Delta(r_n \Delta x_n) + f(n, x_n) = 0, \quad n \in N(n_o), \quad (3)$$

where $n \in N(n_o) = \{n_o, n_o + 1, \dots\}$ (n_o is a fixed nonnegative integer), Δ is the forward difference operator defined above, $x : N(n_o) \rightarrow \mathbb{R}$ ($\mathbb{R} = (-\infty, \infty)$), $r : N(n_o) \rightarrow (0, \infty)$, and $f : N(n_o) \times \mathbb{R} \rightarrow \mathbb{R}$, is continuous function.

Meanwhile, the stability of solutions of nonlinear difference equations was discussed by Agarwal [1], Elaydi [8] and Lakshmikantham and Trigiante [17]. They offered systematic treatment of the theory of difference equations in terms of Lyapunov functions and comparison principle.

In the present work, we are concerned with the oscillatory and stability behaviour of solutions of nonlinear difference equations.

We improve and extend some of recent results of oscillation and non-oscillation of solutions of second order nonlinear difference equations of the forms

$$\Delta(r_n \psi(x_n) \Delta x_n) + q_{n+1} f(x_{n+1}) = 0, \quad (4)$$

and

$$\Delta(r_n \psi(x_n) \Delta x_n) + q_n f(n, x_n) = 0, \quad (5)$$

where $n \in N(n_o)$ $x : N(n_o) \rightarrow \mathbb{R}$, $r : N(n_o) \rightarrow (0, \infty)$, $\psi : \mathbb{R} \rightarrow (0, \infty)$, f is a real valued continuous function and $\{q_n\}$ is a real sequence. Our results partially generalizes those of [10], [23] and [37].

We are also concerned with the stability of solutions of the nonlinear difference system

$$x_{n+1} = f(n, x_n), \quad \text{for all } n \in N(n_o), \quad (6)$$

where $f \in C[N(n_o) \times S_\rho, \mathbb{R}^k]$, $f(n, 0) = 0$, and $S_\rho = \{x \in \mathbb{R}^k : \|x\| < \rho, \rho > 0\}$, and $\|x\|$ is any norm of the vector $x \in S_\rho$.

We introduce a new notion of l_p -stability for the solutions of the nonlinear difference system (6).

We introduce new notions of Lipschitz stability and total Lipschitz stability of the nonlinear difference system (6) and the perturbed system

$$x_{n+1} = f(n, x_n) + w(n, x_n), \quad (7)$$

where $f, w \in C[N(n_o) \times S_\rho, \mathbb{R}^k]$, $f(n, 0) = w(n, 0) = 0$. Our technique is an extension of the methods employed in the work of Dannan and Elaydi [6, 7] and Soliman [27] for differential equations. Some of our results can be considered as discrete analogues of the corresponding results on differential equations.

The thesis consists of three chapters.

In chapter 1. We state and prove some known published results on oscillation and stability of solutions of nonlinear difference equations.

In chapter 2. We study oscillation criteria for the second order nonlinear difference equations (4) and (5). We give a partial generalization for the works of X. He [10], Li, et al. [23] and Zhang and Chen [37].

In chapter 3. We introduce new results of l_p -stability of solutions of nonlinear difference system (6). Also we apply the notions of uniform Lipschitz stability and uniform total Lipschitz stability introduced on differential equations for the difference systems (6) and (7).

Note :-

Our main results of chapter two were accepted for publication in Journal of Applied Mathematics & Computing [19].

Some results of chapter three were submitted for publication in Electronic Journal of Differential Equations [20].