CHAPTER I

INTRODUCTION

(1.1) Electrohydrodynamics (EHD)

Electrohydrodynamics includes that part of fluid mechanics concerned with electrical force effects. It can also alternatively be considered as that of electrodynamics which is concerned with the influence of moving media on electric fields. As a matter of fact, electrohydrodynamics is a combination of these two areas. Many of the interesting problems in electrohydrodynamics involve both an effect of the fluid motion on the field, and conversely, an influence of the fields upon the motion. Although electrohydrodynamics is a relatively new subject, yet it treats an area which is not new. Its applications form the basis for major industries.

The subject deals with interaction between electric fields and free or bound charges in fluids. If an electric field interacts with multiproperty systems, surface interactions become of great importance. Waves which propagate along the interface between two fluids are called surface waves. In case a deformation takes place in the interface between two fluids in presence of an electric field, there are additional energy storages. When the electromagnetic energy is stored predominately in the electric field, the interaction is called an electrohydrodynamic interaction.

Electrohydrodynamic instability at the interface between two fluids has received attention in the context of many and varied physical configurations. Most often considered are situations where the fluid is much more conducting and has an electrical relaxation time short compared with the electromechanical dynamics of interest, as for example, in the case of an Ohmic liquid interfaced with an insulating gas. Examples include the Rayleigh limit for charge on a conducting drop, instability of a planer layer of liquid stressed by normal electric field and instability and breakup of conducting fluid jets and drops in electric fields [71] , [107] .

Perhaps the simplest configuration exhibiting the destabilizing effect of an electrical field on an otherwise thermally stable layer of dielectric liquid is the parallel plate system considered experimentally by Gross and Porter [40] and Turnbull[109,II].Attempts to explain this phenomenon have been made by several authors including Turnbull [109,I], Roberts [87], Takashima [106],Bradley[14] and the most recently. Hoburg [44], Richardson [86], Castellanos and Velarde [17], Terasawa et al [108] and Worraker and Richardson [114]. Worraker and Richardson [114] studied its motivation in the use of strong d.c. electric fields in the augmentation of single phase heat transfer in non polar dielectric liquids.

Michael [74] considered the effects of electrostatic forces on the stability of wave motion at the surface of highly conducting fluids, his results for the linear problem showed that the electrostatic forces can have destabilizing effects on the wave motion at the surface. However, Michael's nonlinear analysis remains valid only for wavenumbers away from the linear cutoff value and breaks down for the wavenumbers near the latter.

The disperesion equation for small amplitude motions of a plane abrupt interface between two Ohmic fluids subject to an initially normal electric field has been derived by Melcher and Smith [72]. In that work, the fluids have differing mechanical properties (mass density and viscosity), electrical properties (conductivity and permittivity), and experienced interfacial surface tension. Kath and Hoburg [56] studied an interfacial instability between fluid components driven by an equilibrium normal electric field in an appropriate limiting case of the surface coupled theory.

Considerable attention has been given to an instability process which can result from injection of a unipolar ion flux into an insulating region with imposed potential difference between electrodes (Schneider and Watson [89] and Atten and Lacroix [5]).

The stability of the interface in the presence of a periodic electric field was studied by Yih [116]. It was shown that the stability is governed by a Mathieu equation, that the interface can be unstable even if the electric field is at all times weaker than that needed for instability in the case of a steady field.

The electrohydrodynamic stability of two cylindrical interfaces influenced by a periodic tangential field is studied by El-Dabe et al [33]. Also electrohydrodynamic stability of a fluid layer bounded from above and below by two different fluids is studied by El-Shehawey et al [35].

The nonlinear analysis of electrohydroynamic fluctuations did not have sufficient curiosity. Very few works on the finite amplitude disturbance have been published by Mohamed and El-Shehawey [75], Shivamoggi [94], Michael [74] and Kant and Malik [53].

The electrohydrodynamic equations governing the motion of a fluid stressed with electric forces may be summerised as follows:

The continuity (or conservation of mass) equation reflects the fact that matter can neither be created nor destroyed in the fluid. This equation is

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho\vec{V}) = 0 \qquad \text{or} \qquad \frac{d\rho}{dt} + \rho\nabla.\vec{V} = 0 \quad . \tag{1.1}$$

In its first form, it means that a change in fluid density at a fixed point in space ($\frac{\partial \rho}{\partial t} \neq 0$) is due to changing momentum flow $\rho\vec{V}$ as mass is carried in or out of a small volume about the fixed point with the fluid velocity $\vec{V}$.

The force equations describe the volume forces acting in the fluid due to fields (forces at a distance) and the surface forces acting at the interface, which are exerted through the stresses of contact and are normal to the interface for the nonviscous fluids considered here since these fluids cannot support a shearing stress. The volume equation of motion for a dielectric fluid is

$$\rho\left(\frac{\partial V_i}{\partial t} + V_k \frac{\partial V_i}{\partial X_k}\right) = \rho F_i + \frac{\partial}{\partial X_k} \Pi_{ik} \ , \tag{1.2}$$

where $\vec{F}$ is the external force per unit volume and the subscripts i and k refer to the ith and kth component of the vector.

The stress tensor Π_{ik} is defined as

$$\Pi_{ik} = \tau_{ik} + T_{ik} \quad , \tag{1.3}$$

where τ_{ik} is the ordinary hydrodynamic stress tensor given by

$$\tau_{ik} = -P\delta_{ik} + \mu\left(\frac{\partial V_i}{\partial X_k} + \frac{\partial V_k}{\partial X_i} - \tfrac{2}{3}\delta_{ik}\frac{\partial V_e}{\partial X_e}\right) + \eta\delta_{ik}\frac{\partial V_e}{\partial X_e} \quad , \tag{1.4}$$

and P, μ and η are the pressure and the coefficients of viscosity respectively, δ_{ik} is the Kronecker delta.

T_{ik} is the electric component of the Maxwell stress tensor (Melcher[71]), namely

$$T_{ik} = \varepsilon E_i E_k - \tfrac{1}{2}\left(\varepsilon - \frac{\partial\varepsilon}{\partial\rho}\Big|_{\text{Temp.}}\rho\right)E^2\delta_{ik} \quad , \tag{1.5}$$

where $\vec{E}$ is the electric field intensity and ε is the dielectric constant of the medium.

The set of equations(1.1) and (1.2) form, in general, the basis for the study of electrohydrodynamic problems.

The electric body force acting on a unit volume of the fluid is simply the divergence of the electric stress tensor and is accordingly given by

$$\nabla . T_{ik} = q\vec{E} - \tfrac{1}{2}E^2\nabla\varepsilon + \nabla\left(\tfrac{1}{2}\rho\frac{\partial\varepsilon}{\partial\rho}\Big|_{\text{Temp.}}E^2\right) \quad . \tag{1.6}$$

The second term on the right hand side is due to the inhomogeneity of the fluid and the third term arises from electrostriction effect.

Finally the set of equations of motion can be written in the following form

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho\vec{V}) = 0 \quad , \tag{1.7}$$

$$\rho \frac{d\vec{V}}{dt} = - \nabla P + \rho\vec{g} + \mu\nabla^2\vec{V} + (\eta + \tfrac{1}{3}\mu)\nabla(\nabla.\vec{V}) + (\nabla.\tilde{\varepsilon}\,\vec{E})\vec{E} - \tfrac{1}{2}(\vec{E}.\vec{E})\nabla\tilde{\varepsilon} + \tfrac{1}{2}\nabla(C\tilde{\varepsilon}\vec{E}.\vec{E}) \ , \tag{1.8}$$

where $\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{V}.\nabla$,

$\vec{g}$ is the body force acceleration (due to gravity, centrifugal forces, etc.) and the electrostriction constant $C \equiv \frac{\rho}{\tilde{\varepsilon}} \frac{d\tilde{\varepsilon}}{d\rho}$.

The last three terms in eq. (1.8) are forces of electrical origin. The first of these terms may be interpreted as the force on free charges in the fluid, the second is due to a non-uniform dielectric and the last is caused by the dependence of $\tilde{\varepsilon}$ on the fluid density.

The two of Maxwell's equations that are applicable for a non-conducting dielectric fluid in an applied electric field are

$$\nabla \wedge \vec{E} = 0 \ , \tag{1.9}$$

and

$$\nabla.\, \tilde{\varepsilon}\,\vec{E} = 0 \ , \tag{1.10}$$

assuming that no free charge or magnetic field are present.

The solutions of the above equations are subjected to the boundary conditions at the free surfaces and at rigid boundaries.

The interface between two fluids can be defined as a surface in three dimensions that is always composed of the same particles [71].

Since the unit normal vector $\vec{n}$ of the interface depends only on the orientation of the surface then

$$\vec{n} = \frac{\nabla F}{(\nabla F \cdot \nabla F)^{1/2}} \tag{1.11}$$

where $F(x,y,z,t) = 0$ is the equation of the interface. All points (x,y,z) that satisfy this condition at a certain time are on the interface separating the two fluids.

The boundary condition at the interface separating region (1) from region (2)results from the integration of equations of motion across the boundary thus:

(i) The normal velocity should be continuous across the interface

i.e. $$V_{n1} = V_{n2} \tag{1.12}$$

(ii) The tangential component of the electric field at the interface must be continuous.

(iii)If there are no surface charges at the interface, then the normal electric displacement is continuous namely

$$\vec{n}.(\tilde{\varepsilon}_1\vec{E}^{(1)} - \tilde{\varepsilon}_2\vec{E}^{(2)}) = 0 \tag{1.13}$$

(iv) The tangential component of the stress tensor Π_{ik} is continuous at the interface between two fluids.

(v) The normal component of the stress tensor is discontinuous across the surface of separation by surface tension force "σ" ,i.e.

$$n_i(\Pi_{ik}^{(2)} - \Pi_{ik}^{(1)}) = - \delta_{ik}\, \sigma(\frac{1}{R_1} + \frac{1}{R_2}) , \qquad (1.14)$$

where R_1 and R_2 are the principal radii of curvature to be considered positive if the corresponding centres of curvature lie on the assumed positive side of the surface element ds .

(1.2) Surface Tension and Adsorption

In all physico-chemical systems it is possible to distinguish clearly between the conditions governing mechanical equilibrium,and those relating to equilibrium in physico-chemical changes such as chemical reactions and phase changes. In the particular case of capillary systems [64] mechanical equilibrium is determined not only by those factors such as hydrostatic pressure and gravitational attraction common to all systems, but also forces associated with surface tension ; while among the physico-chemical changes which must be considered are modifications of the chemical composition at the boundaries between bulk phases, that is, adsorption [27].

(i) Surface Tension

When two fluids, for example a liquid and a gas, are in contact with one another , they are in general separated by a thin layer, called an interface, interphase or surface phase, whose properties are different from those of the two bulk phases.

To define the tension in this surface at a point P, we imagine a curve AB in the surface, passing through this point and dividing the surface into two regions (fig. 1.1). If across an element $\delta\ell$ of AB, region 2 exerts a force $\sigma\delta\ell$ tangential to the surface, then σ is called the surface (or interfacial) tension at this point and for this particular method of dividing the surface. The surface is said to be in a state of uniform tension if

(1) At each point, σ is perpendicular to the dividing line and has the same value whatever the direction of this line and

(2) σ has the same value at all points on the surface.

In this case σ can be called the surface (or interfacial) tension on the surface. It has the dimensions of force per unit length and is usually expressed in dyne cm^{-1}.

To understand the idea of surface tension more clearly, we shall consider first a portion of a spherical surface separating two phases (the effect of gravity will be neglected). A spherical cap, whose section is shown in fig.(1.2), is subjected to a surface tension σ round the base of the cap, and to normal pressures P_1 and P_2 at each point on the surface.

We now consider the condition for equilibrium of all the forces resolved along the OZ direction (fig. 1.3). Let δA be an

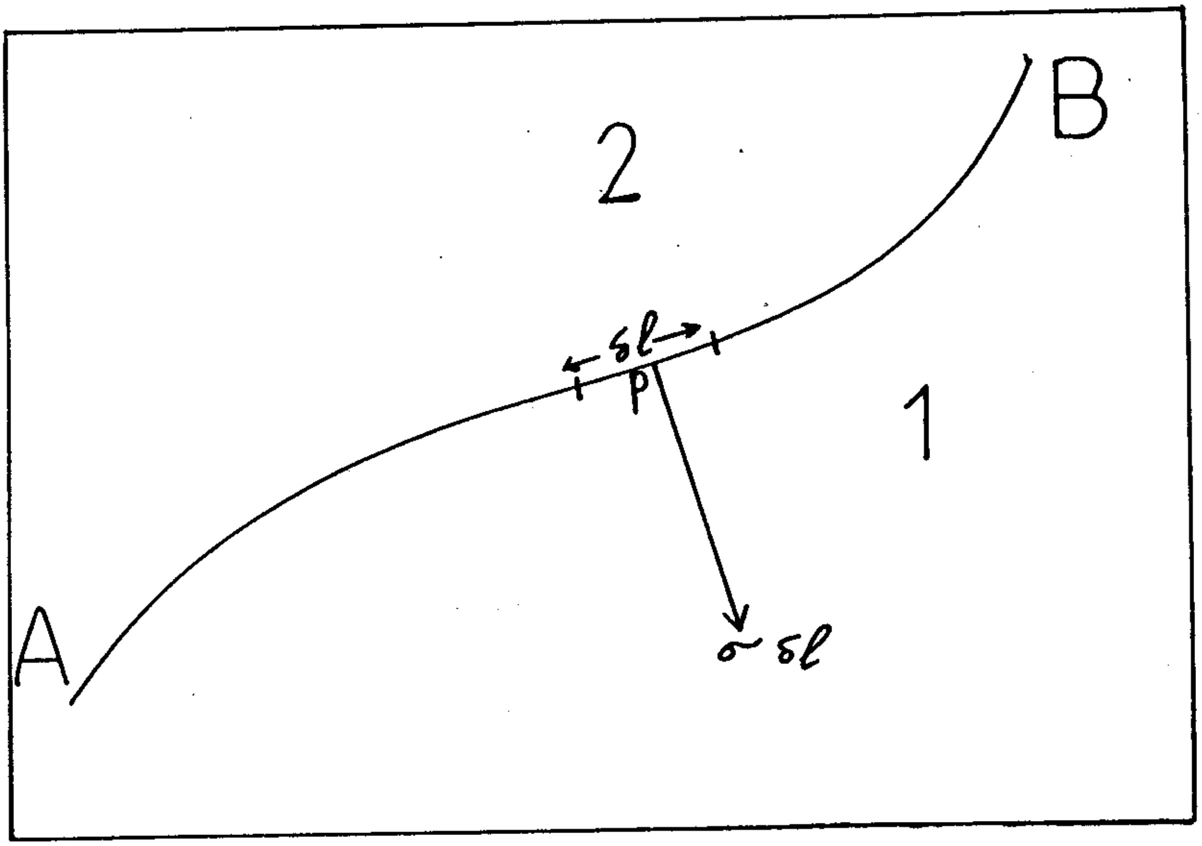

Fig. (1.1): Definition of surface tension at a point P on a line A B in the surface.

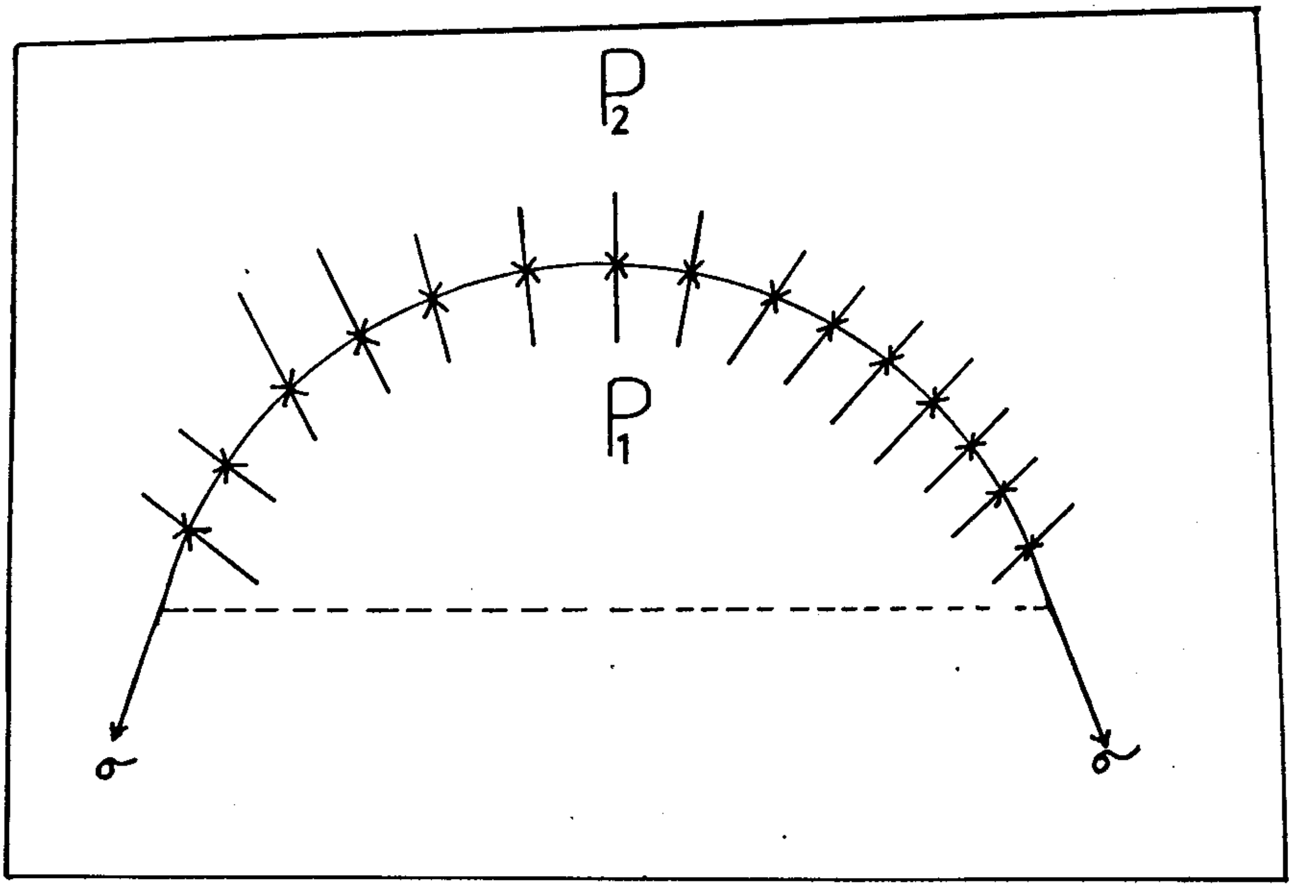

Fig. (1.2): Equilibrium of a spherical cap.

element of the surface and $\delta A'$ its projection on the plane α. The force exerted normal to the element of surface δA is $(P_2 - P_1)\delta A$, and the resolved part of this force along OZ is $(P_2 - P_1) \cos\theta\, \delta A = (P_2 - P_1)\delta A'$. The sum of all such resolved forces is $(P_2 - P_1)A' = (P_2 - P_1)\Pi r^2$, where A' is the area of the circular base and r is its radius. At the same time the surface tension exerts a force $\sigma\delta\ell$ on each element of length $\delta\ell$ of the circumference of the base. The resolved part of this force along OZ is $(-\sigma \cos\theta'\delta\ell)$ and the sum of all such contributions is

$$- \sigma \cos\theta' \int_{0}^{2\Pi r} \delta\ell = - \sigma . \frac{r}{R} . 2\Pi r = - \frac{2\sigma}{R} . \Pi r^2 \quad . \qquad (1.15)$$

Since the system is symmeterical about the axis OZ, all forces normal to OZ must cancel. The condition of mechanical equilibrium is therefore that the forces along OZ also cancel:

$$(P_2 - P_1)\Pi r^2 - \frac{2\sigma}{R}\Pi r^2 = 0 \quad ,$$

or

$$P_2 - P_1 = \frac{2\sigma}{R} \quad . \qquad (1.16)$$

This fundamental equation shows that, because of its surface tension, a spherical surface of radius of curvature R maintains mechanical equilibrium between two fluids at different pressures P_2 and P_1. The phase on the concave side of the surface experiences a pressure P_2 which is greater than that on the convex side.

In particular, if $R = \infty$, $P_1 = P_2$. Hence a plane surface can exist only if the pressures of the fluids on the two sides are equal.

If, instead of a spherical surface, one considers any surface whatever, the condition of mechanical equilibrium at each point in the surface is given by Laplace's formula,

$$P_2 - P_1 = \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \quad , \qquad (1.17)$$

where R_1 and R_2 are the two principal radii of curvature of the surface at the point considered.

(ii) Adsorption

The phenomenon of adsorption was discovered more than a century and a half ago, as when a gas is allowed to come to equilibrium with a solid or liquid surface, the concentration of gas molecules is always found to be greater in the immediate vicinity of the surface than in the free gas phase, regardless of the nature of the gas or surface. The process by which this surface excess is formed is termed adsorption. In any solid or liquid, the atoms at the surface are subject to unbalanced forces of attraction normal to the surface plane. The balance of forces is partially restored by the adsorption of gas molecules.

Adsorption processes may be classified as physical or chemical, depending on the nature of the forces involved. Physical adsorption is caused by molecular interaction forces. The formation of a physically adsorbed layer may be likened to the condensation of a vapour to form a liquid. Not only is the heat of physical adsorption of the same order of magnitude as that of liquifaction, but physically adsorbed layers, particularly those many thick molecular diameters, in many respects like two-dimensional liquids. On the other hand , chemical adsorption, involves transfer of electrons between the solid (or adsorbent) and the gas (or adsorbate). The proces essentially involves the formation of a chemical compound between the adsorbate and the outermost layer of adsorbent atoms [16].

Here, we consider the physical adsorption which depends on the properties of surfaces, which include, for example, electron emission or electrical resistance at contacts.

(iii) General equations of change for adsorption and interfacial tension.

Consider fig.(1.4) which illustrates a moving surface element with area dS(t), dependent on time. The densities of the liquids 1 and 2 are represented as Heaviside step—functions in the distance perpendicular to the surface, so that there will be no difficulties in defining the position of the surface plane. The two dashed lines parallel to the surface line on fig. (1.4) represent surfaces linked to specific solvent molecules near the surface. Therefore, the distances $d_1(t)$ and $d_2(t)$ generally vary with time. The unit normal to the surface is called $\vec{n}$ and is directed from liquid 1 to liquid 2. We remember that diffusion is always defined relative to a solvent reference frame, experimentally. Thus, the fluxes through the boundaries $-d_1(t)$ and $+d_2(t)$ which are called $\int \vec{J}_i . \vec{n}\, ds$ $(i=1,2)$ are only due to diffusion and not to convection, as those boundaries

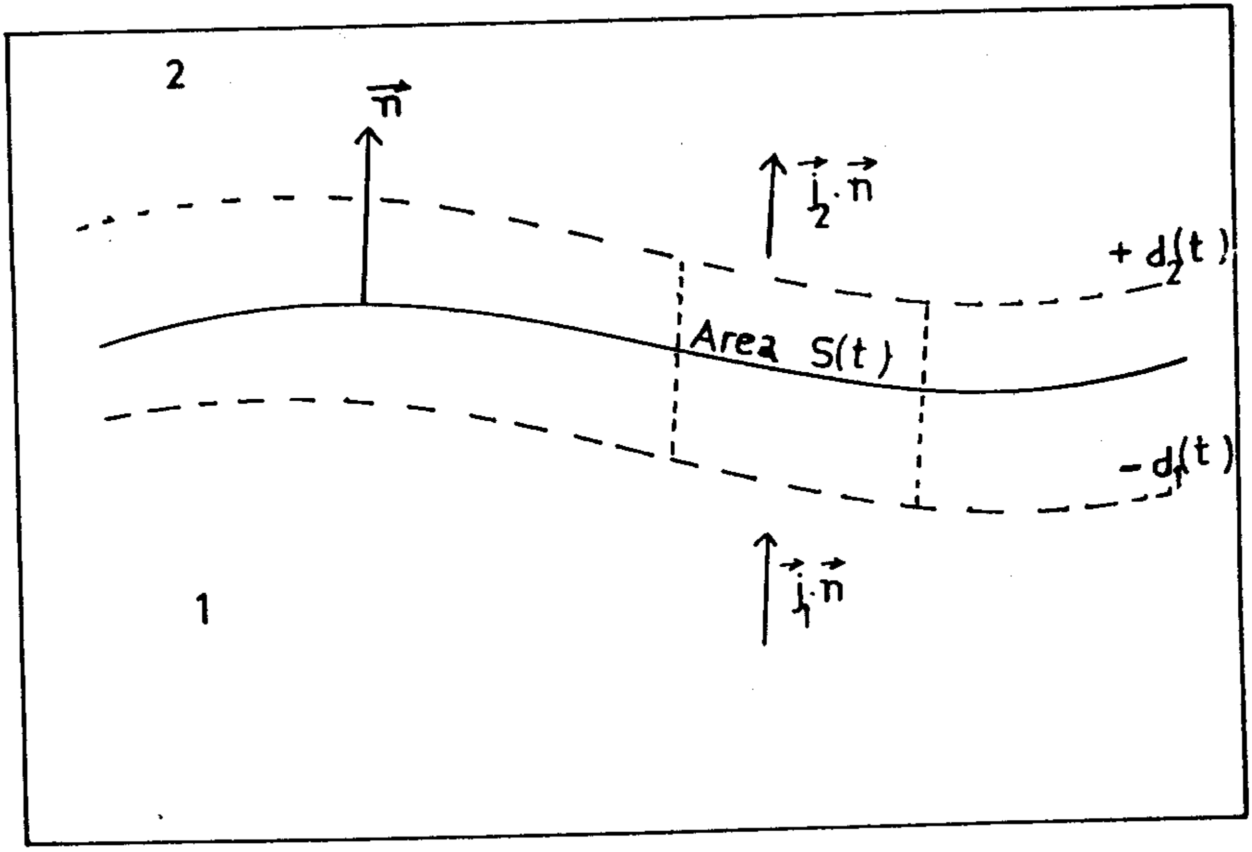

Fig.(1.4): A moving element of interface with diffusion fluxes from an adjacent liquid.

follow the solvent molecules. In addition to the two components of the bulk liquids we consider only a single third component having an uneven distribution. The three components undergo no chemical reactions. For the total accumulation of the third component in the volume element encircled by dashed lines in fig. (1.4), we obtain

$$\frac{dQ}{dt} = \int_{S(t)} (\vec{J}_1 - \vec{J}_2).\vec{n}\, dS. + \oint \vec{J}_s . \vec{n}_s\, ds \ , \qquad (1.18)$$

where Q is the total fluxex through the boundaries.

Because of the smallness of d_1 and d_2 we have neglected the diffusion fluxes parallel to the surface in the bulk phases adjacent to the surface element. By means of the two-dimensional version of Gauss's theorem we can transform the circulation integral in eq. (1.18) to a surface integral, i.e.

$$\frac{dQ}{dt} = \int_{S(t)} \left[(\vec{J}_1 - \vec{J}_2).\vec{n} - \mathrm{div}_s\, \vec{J}_s \right] ds \ . \qquad (1.19)$$

The accumulation of the third component in the selected volume element may also be found by taking the substantial time derivative of the volume integrals of the bulk concentration and the surface integral of the surface excess concentration of the third component (adsorption Γ)

$$\frac{dQ}{dt} = \frac{d}{dt}\left[\int_{-d_1}^{o} C^{(1)}\, dV + \int_{s} \Gamma\, dS + \int_{o}^{d_2} C^{(2)}\, dV \right] . \qquad (1.20)$$

When d_1 and d_2 go to zero, we can neglect the two volume integrals in comparison to the surface integral, because the mass of the third component on the interface is finite. We then have

$$\frac{dQ}{dt} = \frac{d}{dt} \int_{S(t)} \Gamma \, dS \quad ; \tag{1.21}$$

Now, we have for the line element (here and in the following the Einstein summation convection for repeated indices is used)

$$ds^2 = a_{\alpha\beta} \, du^\alpha \, du^\beta \quad , \tag{1.22}$$

where $a_{\alpha\beta}$ is the metric tensor with elements which are functions of the coordinates u^α and u^β ($\alpha,\beta = 1,2$) and time. For an element of area dS we have

$$dS = a^{1/2} \, du^\alpha \, du^\beta \quad , \tag{1.23}$$

a being the time dependent determinant of the metric tensor $a_{\alpha\beta}$. We also introduce another set of surface coordinates u^γ($\gamma = 1,2$) which we call the convected coordinates. We have the following transformation

$$du^\alpha \, du^\beta = J \, du^\gamma \, du^\Delta \quad , \tag{1.24}$$

J being the Jacobian of the transformation from convected to fixed coordinates. The Jacobian J will then satisfy the two-dimensional analogue of Euler's expansion formula (Aris [2])

$$\frac{dJ}{dt} = J \, \mathrm{div}_s \vec{V} = J \, V^\alpha_{,\alpha} \quad , \tag{1.25}$$

where $V^\alpha = \left.\frac{du^\alpha}{dt}\right|_{u^\gamma, u^\Delta}$ and the comma notation is conventionally applied to denote differentiation with respect to the coordinate given by the subsequent index.

Applying the formulae (1.23),(1.24) and (1.25) we are able to perform the substantial time derivation in eq. (1.21), we obtain

$$\frac{dQ}{dt} = \frac{d}{dt}\int_{S(t)} \Gamma \, dS = \int_{\Sigma} \frac{d}{dt}(\Gamma .\ a^{1/2} .\ J) du^{\gamma}\, du^{\Delta}$$

$$= \int_{S(t)} \left(\frac{d\Gamma}{dt} + \frac{\dot{a}}{2a}\Gamma + \Gamma \, div_s \vec{V}\right) dS \ . \qquad (1.26)$$

Here, Σ denotes the form which S(t) is originally convected.

Comparing eq.(1.19) and (1.26) we have

$$\frac{\partial\Gamma}{\partial t} + \frac{\dot{a}}{2a}\Gamma + div_s(\Gamma\vec{V}) + div_s \vec{J}_s = (\vec{J}_1 - \vec{J}_2).\vec{n} \quad . \qquad (1.27)$$

As constitutive relations for diffusion we shall use the simplest possible, i.e., Fick's first law of diffusion with constant diffusion coefficients [42].

$$\vec{J}_s = - D_s \nabla_s \Gamma \ ; \ \vec{J}_i = - D_i \nabla C^{(i)} (i = 1,2) \quad . \qquad (1.28)$$

Inserting (1.28) in (1.27) we then obtain

$$\frac{\partial\Gamma}{\partial t} + \frac{\dot{a}}{2a}\Gamma + div_s(\Gamma\vec{V}) - D_s \nabla_s^2 \Gamma = [\![D\nabla C]\!] .\vec{n} \quad . \qquad (1.29)$$

Here and in the following we use the abbreviation $[\![X]\!]$ for X(fluid 2)-X(fluid 1) when the surface discontinuity is passed.

The simplest possible constitutive relations between viscous stress and rate of strain, namely the linear relations valid for Newtonian bulk and surface fluids. In particular, for a surface fluid we have the rate of strain tensor

$$S_{\alpha\beta} = \frac{1}{2} a_{\alpha\beta} + \frac{1}{2}(V_{\alpha,\beta} + V_{\beta,\alpha}) \quad . \qquad (1.30)$$

interface and the liquid adjacent to it and F^i is the force per unit area of interface due to external fields and to forces from the adjacent liquid layers due to differences in pressure and differences in the viscous pressure tensor. We have for Newtonian bulk liquids

$$F^i = - \delta_{i3} \Gamma g + [\![(-P + \lambda w^k_{,k}) n^i + \mu (w^{i,j} + w^{j,i}) n_j]\!] \cdot \quad (1.35)$$

In the last equation, g is the acceleration due to the homogeneous gravitational field, P is the hydrostatic pressure, λ the dilatation viscosity of the bulk liquid and μ the shear viscosity in the bulk.

Although the term $(- \delta_{i3} \Gamma g)$ in (1.35) is taken into account by [99] in the linearised normal momentum balance for the interface, other authors have meglected this term which would not be expected to be important in view of the very small interfacial mass densities found in usual systems of surface chemistry. In the famous book by Levich [64], the dynamics of interfaces in connection with capillary waves in presence of surfactants is treated negligible as well as the interfacial weight term $(g\Gamma)$ and the interfacial inertial terms $\Gamma \frac{dv_z}{dt}$.

More important, he also ignored interfacial viscosity in the tangential boundary conditions, while Bellman and Pennington [8] discussed the effects of interfacial tension and viscosity on Taylor instability.

We proceed with the explanation of symbols in eq. (1.34). If we write for the deformed surface in three-dimensional Euclidean space

$$x^3 = f(u^1 , u^2) = f(x^1 , x^2) \quad , \tag{1.36}$$

we link the surface and space coordinates by the hybrid tensor $t^i_\alpha = \frac{\partial x^i}{\partial u^\alpha}$ with the components

$$t^3_\alpha = \frac{\partial f}{\partial u^\alpha} \quad , \; t^j_\alpha = \delta^j_\alpha \quad (j = 1,2) \quad . \tag{1.37}$$

We shall use the notation f_α for $\frac{\partial f}{\partial u^\alpha}$ and $f_{\alpha\beta}$ for $\partial^2 f/\partial u^\alpha \partial u^\beta$. Since the surface metric tensor is defined as

$$a_{\alpha\beta} = t^i_\alpha \; t^i_\beta \quad . \tag{1.38}$$

The surface metric determinant is

$$a = 1 + f^2_1 + f^2_2 \; . \tag{1.39}$$

The covariant tensor $b_{\alpha\beta}$ is the tensor which defines the second fundamental form of the surface

$$b_{\alpha\beta} = \left[\frac{\partial^2 x^i}{\partial u^\alpha \partial u^\beta} + \left\{ {i \atop j\,k} \right\} \frac{\partial x^j}{\partial u^\alpha} \; \frac{\partial x^k}{\partial u^\beta} \right] n_i \quad , \tag{1.40}$$

where n_i is the covariant component of the surface normal $\vec{n}$ (see fig. 1.4) and $\left\{ {i \atop j\,k} \right\}$ is one of the Christoffel symbols. With the definition (1.36) of the surface one obtains $b_{\alpha\beta} = f_{\alpha\beta} \; a^{-1/2}$.

If b denotes the determinant of the tensor $b_{\alpha\beta}$ we have for the total curvature K of the surface, which is the product of the local principal curvatures:

$$K = b/a \quad . \tag{1.41}$$

For all developable surfaces K=0 and for a sphere $K = R^{-2}$.

The mean curvature H of the surface is (Hennenberg et al [42])

$$2H = a^{\alpha\beta} b_{\alpha\beta} = a^{-3/2}\left[f_{11}(1+ f_2^2) - 2f_{12}f_1f_2 + f_{22}(1+f_1^2)\right]. \quad (1.42)$$

The equations (1.29) and (1.34) are very important to study the adsorption.

The theoritical and experimental work of eqs. (1.29) and (1.34) has been pursued in [11], [22], [39], [49], [50], [83], [93], [96], [99] and [100].

(1.3) The concept of "Marangoni instability"

The interfacial tension of an interface between two immiscible fluids is generally a monotonically decreasing function of adsorption. Unless opposed by effects of contamination, say, any adsorption gradient imposed along the interface will produce a corresponding gradient in interfacial tension and hence a bulk fluid motion. This type of motion is called thermocapillary of Marangoni convection [98].

The Marangoni instability with the driving force of heat or mass transfer across fluid interfaces causes a self-amplification and self organisation of movements at a fluid interface which develop a spectrum of multiform dissipative structures. There exist manifold substructured spatial periodic systems of electrohydrodynamic kind with a time depending behavior which can degenerate to a typical relaxation oscillation. The latter differs completely from the spatial and temporal period structure of a two-parameter-oscillation with a wave like behaviour. These instabilities lead to a nonlinear acceleration in the mass and heat transfer [47]. Even the effect of Marangoni stability is provable as a damping of the eddies of the forced surface renewal.

Under the condition of the balance between the shear stress and the surface tension gradient at a surface [99],which is covered with adsorbed surface active agents, there exists at low viscous surfaces a more turbulence-analogic hydrodynamic instability with stationary spatial periodic and instationary spatial and temporal periodic dissipative structures.

(1.4.a) The Rayleigh-Taylor model

The instability of the equilibrium formed when a heavy fluid is supported through hydrostatic pressure by a light fluid characterized a variety of physical situations . Hence, this Rayleigh-Taylor type of instability appears in the literature of not only hydrodynamics but in that of hydromagnetics (Chandrasekhar [19]), plasma dynamics and electrohydrodynamics [71] and [73] as well.

This problem was first adressed by Rayleigh (1916) and has been extensively studied since. For example,Chandrasekhar [19] studied this system for a full account of the linearized problem. The dispersion relation which was obtained by Chandrasekhar [19] is

$$\omega^2 = \frac{k}{\rho_1+\rho_2}\left[g(\rho_1 - \rho_2)+ \sigma k^2 \right] , \tag{1.43}$$

where ρ_1 and ρ_2 are the density of the lower and upper fluids respectively, k is the wavenumber, g is the acceleration normal to the interface and σ is the surface (interfacial) tension.

Melcher [71] has also done work (both theoretical and experimental) on electrohydrodynamic surface waves, and deduced the dispersion relation which takes the form

$$\omega^2 = \frac{k}{\rho_1+\rho_2}\left[g(\rho_1-\rho_2) - \frac{(\tilde{\varepsilon}_2-\tilde{\varepsilon}_1)^2 E_0^{(1)} E_0^{(2)}}{(\tilde{\varepsilon}_2+\tilde{\varepsilon}_1)} k + \sigma k^2 \right] , \qquad (1.44)$$

where there are no surface charges present; i.e. $\tilde{\varepsilon}_2 E_0^{(2)} = \tilde{\varepsilon}_1 E_0^{(1)}$, where $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ are the lower and upper dielectric constants respectively,

The effect of a tangential electric field on the linear stability is of a stabilizing nature according to the relation [71]

$$\omega^2 = \frac{k}{\rho_1+\rho_2}\left[g(\rho_1-\rho_2) + \frac{(\tilde{\varepsilon}_2-\tilde{\varepsilon}_1)^2 E_0^2}{(\tilde{\varepsilon}_2+\tilde{\varepsilon}_1)} k + \sigma k^2 \right] . \qquad (1.45)$$

In recent years many authors have tried to understand more about the nonlinear aspects of the instability as a prelude to an understanding of thermal turbulance. Chapman and Procter [20] studied nonlinear Rayleigh-Bénard convection between poorly conducting boundaries. Baker et al [6] described very efficient numerical methods to study Rayleigh-Taylor instability in inviscid, incompressible, layer flows. A nonlinear analysis of the Rayleigh-Taylor stability of two thermally conducting fluids,mass transfer across the interface due to the evaporation and condensation is explicitly studied by Hsieh and Ho [47]. Also,Rayleigh-Taylor instability in an atmosphere in which the density increases exponentially with height and the gravitational acceleration varies with height, $g(z)= g_1+g_2(\pm kz)$ is considered as illustrative by Baker [7]. But Mohamed and El-Shehawy [75] studied nonlinear electrohydrodynamic Rayleigh-Taylor instability with perpendicular field in the absence of surface charges,and deduced for the linear case that the dispersion relation which takes the form

$$\omega^2 = \frac{k}{1+\rho} \left(1-\rho - \delta\, E_o^{(1)} E_o^{(2)} k + k^2\right) , \tag{1.46}$$

where $\rho = \rho_2/\rho_1$, and $\delta = (\tilde{\varepsilon}_2 - \tilde{\varepsilon}_1)^2/(\tilde{\varepsilon}_2 + \tilde{\varepsilon}_1)$.

All analytic work on nonlinear effects has perforce to be accomplished by means of a perturbation expansion in the amplitude, since the fully nonlinear problem usually defines analytical solution. Other studies have investigated the stability of steady solutions at all amplitudes using numerical techniques [6] .

Marangoni instability and Rayleigh-Taylor instability (intermingled) are studied by Hennenberg et al [42] and Sørensen [99] . They also studied the stability of a plane interface with a perpendicular, linear concentration gradient with respect to deformation investigated by means of linear, hydrodynamic stability theory . Both authors obtained the compatibility condition

$$\det (M) = 0 , \tag{1.47}$$

i.e. the determinant of the M_{ij} matrix should vanish, where the matrix elements are given by

$$M_{11} = \omega_o + k^2 D_s + \sum_{i=1}^{2} \frac{r_i}{K_i} D_i \ ,$$

$$M_{12} = \frac{\beta_1 D_1}{\omega_o} \ \Sigma \ \frac{q_i + k}{q_i + r_i} \ ,$$

$$M_{13} = \Gamma_o - \frac{\beta_1 D_1}{\omega_o} \left[\!\left[\frac{q+k}{q+r} \right]\!\right] \ ,$$

$$M_{21} = g \ ,$$

$$M_{22} = \frac{1}{k}\left(\Gamma_o \omega_o - \frac{g}{\omega_o} [\![\rho]\!] + \frac{k^2 \sigma_o}{\omega_o}\right) + \Sigma \ \eta_i \left(1 + \frac{q_i}{k}\right) + \frac{\omega_o}{k^2} \ \Sigma \ \rho_i \ ,$$

$$M_{23} = \left[\!\left[\eta\left(1 - \frac{q}{k}\right) \right]\!\right] \ ,$$

$$M_{31} = \alpha k \ ,$$

$$M_{32} = M_{23} \ ,$$

and

$$M_{33} = -\frac{1}{k}\left(\Gamma_o \omega_o + k^2 \eta_o\right) - \Sigma \eta_i \left(1 + \frac{q_i}{k}\right) \ ,$$

where

$$q_i = \sqrt{k^2 + \omega_o/\nu_i} \qquad , \ r_i = \sqrt{k^2 + \omega_o/D_i} \ ,$$

ν_i is the kinematic viscosity, η_s is the sum of the surface intrinsic viscosity, η_i is the shear viscosity, β_1 and α are constants and K_i is the concentration constants (The ratio K_2/K_1 equals to the ordinary Nernstian distribution coefficient).

where U_1 and U_2 are the streaming velocities of the lower and upper fluids respectively and $k = \sqrt{k_x^2 + k_y^2}$.

Lyon [67] added the effect of compressibility and an applied electric field on the linear electrohydrodynamic Kelvin-Helmholtz instability, but neglected the interfacial tension. His dispersion relation is

$$\omega = \frac{(\rho_1 U_1 + \rho_2 U_2)k}{(\rho_1+\rho_2)} \pm \left[\frac{gk(\rho_1 - \rho_2)}{(\rho_1+\rho_2)} - \frac{\rho_1\rho_2 k^2 (U_1-U_2)^2}{(\rho_1+\rho_2)} + \right.$$
$$\left. + \frac{(\tilde{\varepsilon}_2-\tilde{\varepsilon}_1)^2 E_0^2\, k^2}{(\tilde{\varepsilon}_2+\tilde{\varepsilon}_1)(\rho_1+\rho_2)} \right]^{1/2} , \qquad (1.49)$$

where all the symbols are defined in above.

Maslowe and Kelly [69] analysed, to second order, finite amplitude periodic surface waves for the case of zero interfacial tension without an electric field. Their results do not exhibit the dependence of the velocity on the amplitude except near the linear neutrally stable wavenumber.

The nonlinear development of the Kelvin-Helmholtz instability has been studied previously by Drazin [30] and Nayfeh and Saric [78] for the case where the amplitude of an unstable wave is uniform in space and growing only in time. The dispersion relation by the latter is

$$\omega = \frac{k(U_1 + U_2)}{1+\rho} \pm \left[\frac{k(1-\rho)}{1+\rho} - \frac{\rho k^2 (U_1-U_2)^2}{(1+\rho)^2} + \frac{k^3}{1+\rho} \right]^{1/2} , \qquad (1.50)$$

where $\rho = \rho_2/\rho_1$.

In recent years, the instability of two layers of immiscible inviscid and incompressible fluids in relative motion is treated again by Weissman [112]. He studied the problem with allowance for small, but finite, disturbances and for spatial as well as temporal development. He discussed the constant interfacial tension effect in the dispersion relation

$$\omega = U\rho_2 k_x \pm (gk - \rho_1\rho_2 U^2 k_x^2 + \sigma k^2)^{1/2} \quad , \tag{1.51}$$

where use had been made of the identity $\rho_1 + \rho_2 = 1$, and U is the velocity of the upper fluid.

The expansion of Nayfeh and Saric [78] was criticised by Weissman in the sense that the linear dispersion relation which is presented by a parabolic curve requires that the expansion should be in the form $U = U_c + \Delta$, $0 < \Delta << 1$. This expansion accounts for all regions of the parametric spaces, while Nayfeh and Saric's expansion is only valid for stable regions in the parametric space away from the critical curve.

Klaassen and Peltier [58] tested two-dimensional finite amplitude Kelvin-Helmholtz waves for stability against three-dimensional infinitesimal disturbances and have found that they are most unstable against longitudinal modes which correspond to convective activity in the statically unstable regions induced by the roll-up of the nonlinear wave.

Finally, Kant and Malik [53] studied the hydromagnetic nonlinear Kelvin-Helmholtz stabilities. Also El-Shehawey [34] studied the nonlinear electrohydrodynamic Kelvin-Helmholtz instability for a normal field in the absence of surface charges. Both authors analyses were only valid in stable regions of the parametric space.