INTRODUCTION

Many physical systems Mechanical, electrical, ecological, economical, and even in case of cancer researches, the mathematical analysis leads to the study of an ordinary differential equation, or a system of ordinary differential equations with constant or variable coefficients. Such equations, may be linear or weakly nonlinear, or strongly nonlinear. An important part of modern engineering is the analysis and predictions of the dynamic behavior of such physical systems. The famous type of dynamical behavior is the oscillatory motion or simply the oscillation, in which the system oscillates about certain equilibrium position. The famous methods of studying a differential equation could be classified into three general classes. The first class is qualitative which can be termed as a topological method giving the information about the solution, without finding it. The second class is quantitative which may be termed analytical methods and is based on the process of integration and constructing of an explicit expression of solution. The third class is that termed as the numerical methods it is a step by step process giving solutions as label corresponding values of independent and dependent variables. Periodic solutions of a differential equation (the response of the physical systems) have been the subject of a great amount of study in the field of ordinary differential equations. Physical systems can be classified according to two distinct types, namely discrete system and continuous system.

Discrete system, possess a finite number of degrees of freedom, where as continuous system possess an infinite number of degrees of freedom. The number of degrees of freedom is defined, as the number of independent coordinates required to describe its motion completely.

Many physical systems are subject to excitations that induce vibration in the system. When the excitation appears in an inhomogeneous term in the governing equations it is called external excitation. When the excitataion appears as atime dependent coefficient in the governing equations,it is called parametric excitation. The forced response of non-linear physical systems having one-degree or multi-degree of freedom, subject to external excitation or parametric excitation or both is investigated by many authers [1-16].

These systems of equations which have been investigated by Nayfeh and Mook [1], Nayfeh [2], Dugundji and Mukhopadhyay [3], Mukhopadhyay [4], and Nayfeh [5, 6] have single and multi-degrees of freedom. Although problems involving single-frequency excitations have received considerable attention [7-9], limited problems involving multi-frequency excitations have been studied in references [10-12] for the case of one-degree-of-freedom systems and in references [13-15] for the case of multi-degree-of- freedom systems. Tiwari and Subramanian [16] studied the case in which the ratio of two excitation frequencies is an integer and determined the necessary conditions for sub-harmonic resonance and the conditions under which the response of the primary resonance can be enhanced, reduced, or even quenched by a super-harmonic resonance.

Yamamoto et al. [17,18] used the method of harmonic balance to determine the response of one-degree-of-freedom system with quadratic and different types of non-linearities. Yamamoto et al. [19] examined the response of single-degree-of-freedom system with cubic non-linearities to combination resonance. Nayfeh [11] showed that the large response of single-degree-of-freedom system, with quadratic and cubic non-linearities to a primary resonance can be enhanced, reduced, or quenched by the addition of a load involving two frequencies that produce a combination resonance of the additive or difference type, depending on the amplitudes and phase of excitation components.

For such systems a mono-frequency excitation can generate responses over a range of frequencies when combination resonance are present. Similarly a fractional relationship between the excitation frequency and system natural frequencies can produce sub-harmonic and super harmonic effects and regions of multi-valued solutions where the response depends on the initial conditions.

In physical systems the external excitations can be in the form of initial displacement or initial velocity or both. However, the excitation can also be in the form of forces, which persist for an extended period of time. The response of such forces is called forced response or forced vibration.

Many authors [1, 19- 26], investigate the forced response of nonlinear systems. Wu and Chine [27] used the method of harmonic balance for a quite general non–linear oscillations problem subjected to a periodic force. The non-linear forced vibration of plates is investigated by Kong and Gheung [28], using an implicit time integration scheme and a reduction method.

Nayfeh and Balachandran [29] have studied the nonlinear response of two-degree-of-freedom systems with quadratic non-linearities to external excitation in the presence of two-to-one internal resonance. Van dooren [30, 31] studied the response of two-degree-of-freedom system with quadratic and cubic-non-linearities to combination resonance of the additive or difference type, of the form $\omega_i \simeq \Omega_1 \pm \Omega_2$, where ω_i (i = 1, 2) are the natural frequencies and Ω_1 and Ω_2 are the excitation frequencies. Efstathades [61] examined the response of two-degree-of-freedom systems with cubic non-linearities to a combination resonance of one mode. Asfar et al. [14] investigated the response of two-degree-freedom self-excited oscillators to multi-frequency excitations. Goodier and Mclvor [20] and Mclvor and Lovell [32], analyzed the response of cylindrical and spherical shells taking into account the coupling of breathing and flexular modes when their frequencies are in the ratio of two–to–one Nayfeh and Raouf [33- 35] used a combination of the Galerkin procedure and the method of multiple scales to analyze the non–linear forced response of infinitely long circular cylindrical shells in the presence of two-to-one internal resonance, Yasuda and Kushide [22] investigated theoretically and experimentally the response of a shallow spherical shell to a harmonic excitation.

Mook and Co-workers [24, 36], investigated the forced response of systems with quadratic and cubic non-linearities to a sub-harmonic and combination resonance. Yamamoto and Co-workers [10], investigated the forced response of systems with quadratic and cubic non-linearities to a harmonic excitation.

Nayfeh et. al.[35] studied parametric responses of distributed parameter systems with quadratic non-linearities and two-to-one auto-parametric resonance. Nayfeh [37], Gu and Sethna [57], and. Also the nonlinear parametric response of systems with quadratic and cubic nonlinearities. Nayfeh and Zavodney [52], studied parametrically excited two-degrees-of-freedom system with quadratic nonlinearities and one-to-one auto-parametric resonances were studied by Kamel [15] and Nayfeh [37].

Plaut and Limam [39], El-Naggar and El-Bassiouny [40] are used multiple scale perturbation technique to analyze principal parametric resonance and other different resonance cases that investigated from the systems of two degrees of freedom. El-Halafawy and Eissa [41] investigated the response of two-degrees- of-freedom system with quadratic and cubic nonlinearities to harmonic excitations. Timoshenko [42] showed that beams