Chapter(I)
Introduction

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The theoretical and experimental studies concerning the flow of viscous or viscoelastic fluids in the annular geometrical regions, or specially in the narrow gaps , between two rotating bodies are very interesting boundary value problems in rheology. These problems represent the keystone in the high developing today industries and technology such as the flow in rotation turbo machinery, in ball-bearing (or journal-bearing) lubrication, or of synovial fluid in ball, socket joints , petroleum and food industries and so on. On the other hand ,and from a practical point of view, they give us information about the mechanical properties of solutions and melts . This information is very important in the processing of these materials in almost all branches of industries

The last few decades of the last century has seen an immense development of highly sophisticated theories which describe the non-linear (non-Newtonian) behaviors of such types of materials . Some of these theories are based on microscopic models [1,2] , where special types of statistics , termed conformation statistics [3] , are employed in order to determine the macroscopic mechanical properties . Other theories are based on phenomenological equations of state [4] . In the present work , the constitutive equation based on the retarded motion approximation up to the fluid of grade three is used .

Within the frame of non linear theories of constitutive equation , whether macroscopic or microscopic , these properties are designated by a set of parameters known as material constants . The fluid of grade three is characterized by six parameters ; namely the coefficient of viscosity , μ, and the two elastic constants α_1 and α_2 which are related to the two normal stress differences $(S_{11} - S_{22})$ and $(S_{11} - S_{33})$,and the third-order parameters β_1 ,β_2 and β_3 where $(\beta_2 + \beta_3)$ are related to shear viscosity. The determination of these material parameters is done using proper devices , known as rheometers and the branch which is concerned with such measurements is termed rheometry .

In general , the rheometer is based on the solution of a specific boundary value problem which allow a number of experimental measurements sufficient to determine a specific set of material parameters. The conventional rheometers based on the realization of either steady or periodic viscometric flow are not capable to determine the non-Newtonian properties of fluids . For this reason , one of the major purposes of rheology is to investigate further types of flow other than viscometric flow, which allows the determination of some of the non-Newtonian properties of fluids .

It is worthy to mention that about 30 years ago [5], the force and the torque acting on a sphere which undergoes simultaneous rotational and translational creeping motion in a fluid delivers nearly complete characterization of an incompressible fluid , at least up to third order .

The pioneering work on two concentric cylinders includes Taylor [6], and in the case of two eccentric cylinders Ballal et. al. [7], Zidan and Zidan et.al. [8,9], and Abdel Wahab et.al. [10] . A boundary value problem concerning the Taylor vortex flow between conical cylinders is investigated experimentally by Wimmer [11] .

For two concentric spheres the studies are carried out , say by Wimmer [12], and Yamaguchi et. al.[13,14,15] . A large number of theoretical and experimental works are done on the viscous flow between two eccentric spheres .Jeffery [16], Stimson and et- al. [17], Majumdar [18], Munson [19], Menguturk et-al. [20]. Recently, Abu-El Hassan et .al. studied the flow of a viscoelastic second-order fluid between two eccentric spheres [21] .

Hoffmann et al. [22], investigated the boundary value problem of two coaxial rotating cones theoretically. Flow through rotating curved ducts is investigated numerically by M. Selmi et. al. [23] .

As a result for all symmetric problems (concentric cylinders , concentric spheres , coaxial cones) there is no force that arise on the outer body or on the inner body. If we rotate the inner body the torque on the outer can be determined and vise versa .The only coefficient which can be determined, up to the first-order approximation, in this case is the coefficient of viscosity μ .The secondary flow in these problems may arise in the third or fourth-order .

On the other hand for all anti-symmetric problems (eccentric cylinders , eccentric spheres ,non-coaxial cones) if we rotate one of them there is a resultant force and a resultant torque which take place on the other body. The coefficients which can be determined, up to the second-order approximation, in many of the aforementioned problems are the coefficient of viscosity μ as well as the two elastic constants α_1 and α_2 . Therefore, the secondary flow in these problems arise mainly in the second-order approximation.

The present work is concerned with the solution of a new boundary value problem which, to the best of our knowledge , is not investigated in the literature .The problem under consideration , is the steady state isochoric flow of a fluid of grade two i.e. second- order fluid; in the annular region between two rotating confocal ellipsoids.

The solution of the problem is carried out within the frame of retarded motion approximation . The obtained velocity field up to a second-order is being a superposition of a first-order primary flow distributed uniformly around the axis of rotation of the two ellipsoids and a secondary flow which is every where perpendicular to it. The stresses, forces and torque acting on the outer ellipsoid, when it is kept at rest, had been calculated.

The pioneering work on rigid ellipsoids includes Jeffery [24], Burgers [25], Peterlin[26], Saito [27], Scheraga[28], and Giesekus [29]. A theoretical analysis of the behaviour of a suspension of ellipsoidal particles was first undertaken by Jeffery. The results obtained later were generalized by Goldsmith et .al.[30], and subsequently by Brenner [31]. Jeffery calculated the moment of force acting on arbitrary ellipsoidal particle immersed in creeping shear flow. He deduced for inertialess particle that in the simple shear flow the motion is being periodic . Gierszewski et.al [32], executes numerical examples, on the basis of Jeffery's results, for ellipsoidal inertialess particle in the simple shear flow. Hinch et.al. [33], considered the same problem in much more detail and showed that the motion of triaxial ellipsoid with comparable axis is doubly periodic . They also showed double periodicity for a nearly spheroidal ellipsoid with an arbitrary axis ratio .

Recently and in the same treatment Yarin et .al. [34], found that the rotation of a small non-Brownian triaxial ellipsoidal particles immersed in a simple shear flow may become chaotic . The rotation of axisymmetric spherical particles in two dimensional recirculating flows (e.g. in a four-roll mill) have been studied by Szeri et.al. [35], and Szeri [36] .Their studies showed that in some cases non-periodic rotation of an axisymmetric sets is due to the time-dependent character of flow. In three dimensional flows the rotation of axisymmetry particles becomes quasi-periodic ; Szeri et.al. [37] .

In Giesekus publication the first normal stress coefficients are obtained for rigid ellipsoids . For both oblate and prolate ellipsoids the second normal stress difference is negative, with the negative second normal stress difference to the first normal stress difference ratio being 2/7 for extremely flat ellipsoids (oblate) and 1/7 for extremely long ellipsoids (prolate) .The shear flow behavior of ellipsoid suspensions at high shear rates has been presented by Leal et.al. [38].

The present thesis includes four chapters where the introduction is being as the first chapter , besides appendix at the end of this thesis .

The definition of the boundary value problem as well as the formulation of the equations of motion , the method of approximation for solving the system of partial differential equations that govern the flow fields are discussed and solved in terms of the prolate spheroidal system of coordinates . This part is presented in chapter two .

The distribution of surface tractions , forces and torques on the boundaries up to the third -order are evaluated in chapter three .

The last chapter includes the discussion and the final results.

The system of the prolate spheroidal coordinates are described analytically in the Appendix at the end of this thesis.