CHAPTER (I)

INTRODUCTION

1.1 Birefringence

Many crystalline substances (i.e. solids whose atoms are arranged in some sort of regular repetitive array) are optically anisotropic. In other words, their optical properties are not the same in all directions within any given sample. If the crystals lattice atoms were not completely symmetrically arrayed, the binding forces on the electrons would be anisotropic. Fig.1 represents the isotropic oscillator using simple mechanical model of a spherical charged shell bounded by identical springs to a fixed point. This was a fitting representation for optically isotropic substances (amorphous solids, such as glass and plastic, are usually but not always isotropic). Fig.2 shows another charged shell, this one bounded by springs of differing stiffness (i.e. having different spring constants). An electron that is displaced from equilibrium along a direction parallel to one set of "springs" will evidently oscillate with a different characteristic frequency than it would were it displaced in some other direction. Light propagates through a transparent substance by exciting the electrons within the medium. The electrons are driven by the E-field and they reradiate; these secondary wavelets recombine and the resultant refracted wave moves on. The speed of the wave, and therefore the index of refraction is determined by the difference between the frequency of the E-field and the natural or characteristic frequency of the electrons. An anisotropy in the

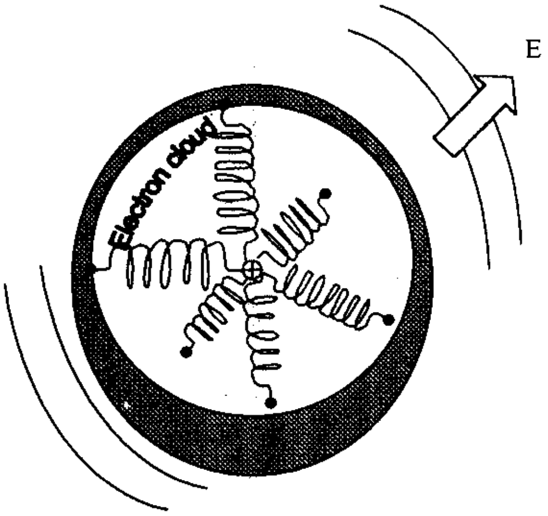


Fig. 1: The mechanical oscillator model for an isotropic medium-all the springs are the same, and the oscillator can vibrate equally in all directions.

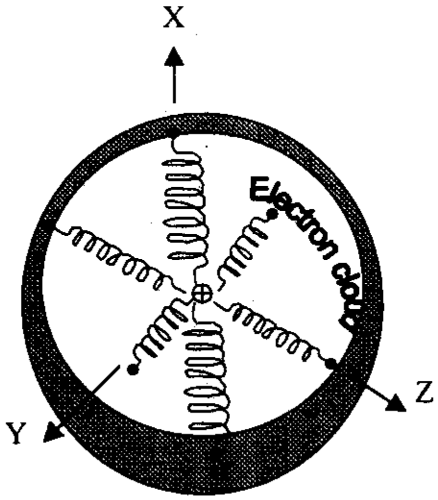


Fig. 2: Mechanical model depicting a negatively charged shell bound to a positive nucleus by pair of springs having different stiffness.

binding force will therefore be manifested in an anisotropy in the refractive index. For example, if plane polarized (P-state) light were to move through some hypothetical crystal so that it encountered electrons that could be represented by fig.2, its speed would be governed by the orientation of E. If E was parallel to the stiff springs, that is, in a direction of strong binding, here along the x-axis, the electron's natural frequency would be high (proportional to the square root of the spring constant).In contrast, with E along the Z-axis, where the binding force is weaker, the natural frequency would be somewhat lower. A material of this sort, which displays two different indices of refraction, is said to be birefringent. Clearly a birefringent material that absorbs one of the orthogonal P-states, passing on the other, is in fact dichroic. Furthermore, suppose that the crystal symmetry is such that the binding forces in the Y and Z-directions are identical, in other words, each of these springs has the same natural frequency and they are equally loss. The x-axis now defines the direction of the optic axis. In as much as a crystal can be represented by an array of these oriented anisotropic charged oscillators, the optic axis is actually a direction and not merely a single line. The model works rather nicely for dichroic crystals, since if light were to propagate along the optic axis (E in the yz-plane), it would be strongly absorbed, and if it moved normal to that axis, it would emerge linearly polarized. Often the characteristic frequencies of birefringent crystals are above the optical range, and they appear colorless.

1.2 Birefringent crystals

Cubic crystals, such as sodium chloride (i.e., common salt), have their atoms arranged in a relatively simple and highly symmetric form. (There are four 3-fold symmetry axes, each running from one corner to an opposite corner, unlike calcite, which has one such axis). Light emanating from a point source within such a crystal will propagate uniformly in all directions as a spherical wave. As with amorphous solids, there will be no preferred directions in the material. It will have a single index of refraction and be optically isotropic. In that case all the springs in the oscillator model will evidently be identical. Crystals belonging to the hexagonal, tetragonal, and trigonal systems have their atoms arranged so that light propagating in some general direction will encounter an asymmetric structure. Such substances are optically anisotropic and birefringent. The optic axis corresponds to a direction about which the atoms are arranged. Crystals like these, for which there is only one such direction, are known as uniaxial. The remaining crystallographic systems, namely, orthorhombic, monoclinic, and triclinic, have two optic axes and are therefore said to be biaxial. Such substances, for example, mica [$KH_2AL_3(SO_4)_3$] have three different principal indices of refraction. Each set of springs in the oscillator model would then be different. The birefringence of biaxial crystals is measured as the numerical difference between the largest and smallest of these indices [1].

1.3 Propagation of plane waves in uniaxial crystals

When a beam of ordinary unpolarized light is incident on a calcite or quartz crystal normal to a cleavage plane, there will be, in addition to the reflected beam, two refracted beams in place of the usual single one observed, for example, in glass. This phenomenon, shown in fig.3 for calcite, is called double refraction or birefringence. This effect can be easily demonstrated as follows. Place a black dot on a white paper and cover it with a calcite crystal, The image of the dot will appear as consisting of two gray dots (black where they overlap). Upon rotating the crystal we notice that one of the dots will remain stationary while the other will appear to move in a circle about it, following the motion of the crystal. The rays forming the fixed dot behave as if they are passing through a glass plate. These rays are known as the ordinary or o-rays. The rays coming from the other dot, which behave in such unusual manner, are known as the extraordinary or e-rays. If the crystal is now examined though an analyzer,

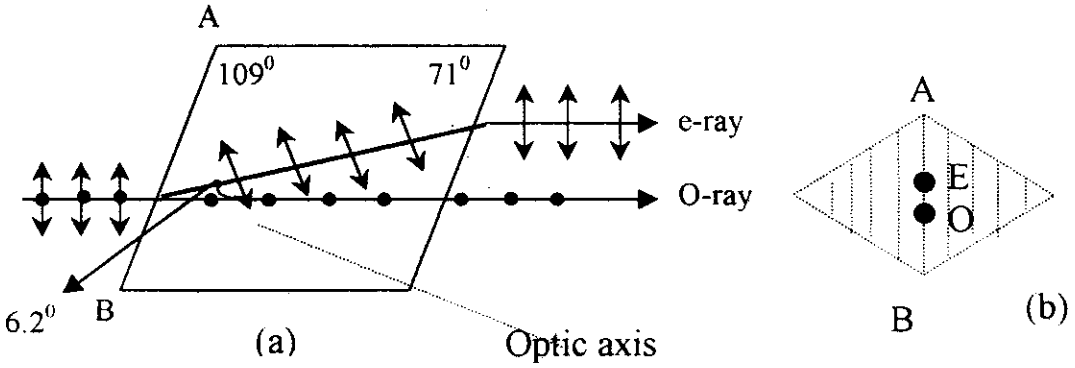


Fig. 3: A light beam with two orthogonal field components traversing a calcite principal section.(a) Cross-section of a principal plane. (b) End view.

it will be found that the ordinary and extraordinary rays are linearly polarized as in fig.3. Moreover, the two emerging P-states are orthogonal [1]. A point source of natural light imbedded within one of these specimens gives rise to spherical o-wavelets and ellipsoidal e-wavelets. It is the orientation of the field with respect to the optic axis that determines the speeds with which these wavelets expand. The E-field of the o-wave is everywhere normal to the optic axis, so it moves at a speed $v_\perp$ in all directions. Similarly the e-wave has a speed $v_\perp$ only in the direction of the optic axis as in fig.4, along which it is always tangent to the o-wave. Normal to this direction, E is parallel to the optic axis, and that portion of the wavelet expands at a speed $v_\parallel$. Uniaxial materials have two principal indices of refraction, $[n_o \equiv c/v_\perp]$, and $[n_e \equiv c/v_\parallel]$. The difference $\Delta n = (n_e - n_o)$ is a measure of the birefringence. In calcite $v_\parallel > v_\perp$, $(n_e - n_o)$ is (-0.172) and it is said to be negative uniaxial. In comparison, there are other crystals, such as quartz (crystallized silicon dioxide) and ice, for which, $v_\perp > v_\parallel$. Consequently, the ellipsoidal e-wavelets are enclosed within the spherical o-wavelets, as shown in fig.5. (Quartz is optically active and therefore actually a bit more complicated.) In that case $(n_e - n_o)$ is positive, and the crystal is said to be positive uniaxial [1]. The origin of the double refraction of light at a crystal surface is readily explained in terms of the wave

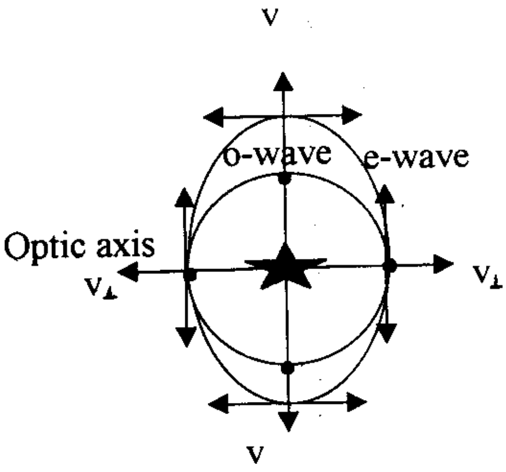


Fig 4: Wavelets in a negative uniaxial crystal.

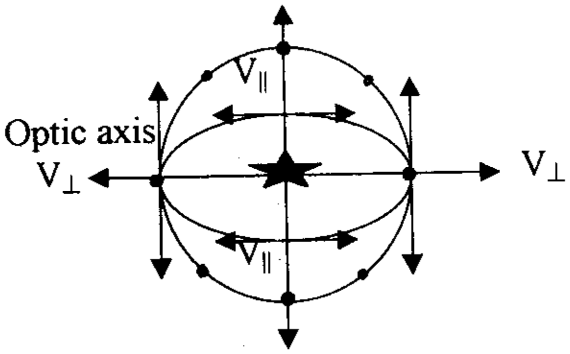


Fig 5: Wavelets in a positive uniaxial crystal.

surfaces just described. This is accomplished by the use of Huygens principle of secondary wavelets. Consider, for example, a beam of parallel light incident normally on the surface of a crystal like calcite, whose optic axis makes some arbitrary angle with the crystal surface. The optic axis has the direction shown by the broken lines. According to Huygen's principle, we may now choose points anywhere along the wave front as new point sources of light. Here A, B and C are chosen angle with the crystal surface as in fig.6,7just as the wave strikes the crystal boundary. After short time interval the Huygens wavelets entering the crystal from these points will have the form shown in the figure. If one now proceeds to find the common tangents to these secondary wavelets, The result is the two plane waves labeled OO' and EE' in the figure. Since the first is tangent to spherical wavelets, it behaves like a wave in an isotropic substance with a velocity proportional to AA', BB' and CC'. We have seen that the tangent to the ellipsoidal wavelets represents the wave front for the E vibrations, which takes place in the principal section. The E rays, connecting the origins of the wavelets with the points of tangency, differs from the O rays and are no longer perpendicular to the wave front. They represent the direction in which a narrow beam of light would be reflected, which is the direction in which the energy of the E vibrations is transmitted. Its velocity, proportional to Aa, Bb or Cc, is called the ray velocity. This is greater than the normal velocity, measured by Aa', Bb', or Cc', the velocity with which the wave advances through the crystal in a direction normal to its own plane. In constructing fig.6 the optic axis was assumed to be in the plane of the page. In case the optic axis is not in the plane of the page, a plane drawn tangent to the ellipsoidal wavelets will make contact at points in front of or in back of the plane of the page. If the optic axis is either parallel or normal to the surface of the crystal, the situation is especially simple. Fig.7 illustrates Huygens construction in these important cases, where the crystal face is cut (1) parallel to the optic axis as in a and b, and (2) perpendicular to the optic axis as in c. In both cases the ray velocities are equal to the normal velocities and there is no double refraction. In case (1), however, the e-wave travels faster than the o-wave. The o-wave, which vibrates everywhere perpendicular to optic axis, has the same velocity in every direction. The vibrations of the e-wave make a different ray. In particular for the ray drawn along the optic axis, the vibrations of which are perpendicular to the axis, the velocity becomes equal to that of the o-ray, which is also vibrating perpendicular to the axis. These facts suggest that the velocity of light is for some reason dependent on the angle of inclination between the vibrations and the optic axis. In terms of the elastic–solid theory, this could be explained by assuming two different coefficients of elasticity for vibrations parallel and perpendicular to the

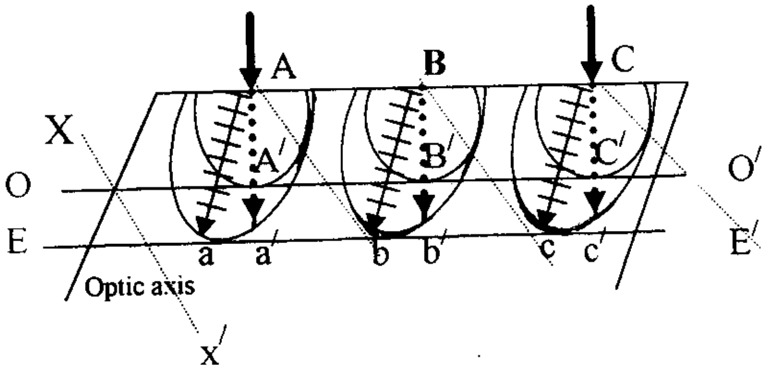
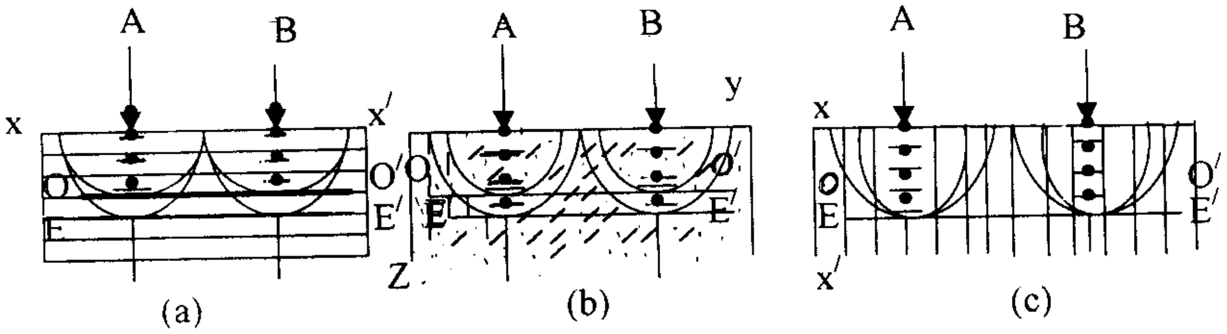


Fig. 6: Wave surfaces and normal-velocity surfaces for uniaxial crystals.



(a) (b) (c)

Fig.7: Propagation of normally incident plane waves through calcite crystals cut parallel and perpendicular to the optic axis.