SUMMARY

A topological vector space X is, roughly speaking, a set which carries two structures : a structure of topological space; a structure of vector space.

Furthermore, some kind of compatibility condition must relates these two structures on X. one of the most important classes of topological vector spaces is the class of locally convex spaces. In fact, the theory of locally convex spaces is significantly richer in results than the theory of topological vector spaces, chiefly because there are always plenty of continuous linear functionals on an locally convex space. Moreover, almost all the concrete spaces that occur in functional analysis are locally convex. In a sense, semi--norms and duality play a vital role in the theory of locally convex spaces : In fact, to introduce a topology in a linear space of infinite dimension suitable for applications to classical and modern analysis, it is sometimes necessary to make use of a system of an infinite number of semi-norms. If the system reduces to a single semi-norm, the corresponding linear space is called a normed space. If further more, the space is complete with respect to the topology defined by this semi-norm, it is called a Banach space. Also, duality is what makes this theory powerful because it establishes a tool to translate a problem on the space (where it may appear to be difficult) into one concerning its linear forms (which may happen to be much easier to handle). Duality also admits the replacement of the original topology by simpler ones when dealing with problems involving boundedness convexity, continuity, etc. One of the most important of these topologies is the weak topology on a given locally convex space.

This thesis is devoted to study some cocepts in locally convex spaces, viz., weakly boundedness, duality, some linear topologies (β-topologies, Mackey topologies, τ_{pc} topologies , etc.), equicontinuous sets and compactologies, bilinear forms and topological tensor products.