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CHAPTER - 5

RESULTS AND DISCUSSION

The present thesis includes a theoretical part which deals with the flow of a fluid of grade two in the annular region between two eccentric cylinders. Moreover, the stress component and the total forces and torques acting on the boundaries are determined. These calculations are represented in the second and third chapters.

Based on these calculations, the eccentric cylinder rheometer is designed, which allows the determination of the second order constant α_1 besides the viscosity coefficient μ. The idea of this device and a detailed description of it, is given in the fourth chapter. A major part of the present thesis, besides the construction of the proto-type of the eccentric cylinder rheometer, is the investigation of the significance and reproducibility of the measurements obtained by this apparatus.

5.1):RESULTS OF THE THEORETICAL CALCULATIONS :

The main results of the theoretical calculations, which are presented in the second and third chapters, can be summerized as follows:-

(a) The stream function which describes the steady state flow of a fluid of grade two in the annular region between two eccentric cylinders has the form:

$$\Psi^{*}=\Omega h(\xi,\theta)\left\{ H\left[\delta(\xi-\xi_2)\mathrm{Sinh}(\xi_1-\xi)-(\xi_1-\xi)\mathrm{Sinh}\,\delta\,\mathrm{Sinh}(\xi-\xi_2)\right]\right.$$

$$+\,Q\left[\frac{\mathrm{Cosh}\,\xi_1}{\delta\,\mathrm{Sinh}\,\delta}(\xi-\xi_2)\mathrm{Sinh}(\xi-\xi_2)\quad +\right.$$

$$\left.\left.\mathrm{Cos}\theta\,\frac{(\xi-\xi_2)\mathrm{Cosh}\,\delta-\mathrm{Sinh}(\xi-\xi_2)\mathrm{Cosh}(\xi_1-\xi)}{\delta\,\mathrm{Cosh}\,\delta-\mathrm{Sinh}\delta}\right]\right\} \qquad 5.1$$

H is a constant given by:

$$H=-\frac{Q\left[\delta\,\mathrm{Cosh}\,\xi_2+\mathrm{Sinh}\,\delta\,\mathrm{Cosh}\,\xi_1\right]\mathrm{Sinh}\xi_1+C\,\delta\,\mathrm{Sinh}\,\delta}{\delta\,\mathrm{Sinh}\,\delta\,\mathrm{Sinh}\xi_1\left(\mathrm{Sinh}^2\,\delta-\delta^2\right)} \qquad 5.1a$$

where $\delta=\xi_1-\xi_2$. The rate of flow per unit length along any line of constant μ in the annular region, as shown from the boundary conditions Eqn.(2.14), is given by:

Rate of flow = Ω Q C. 5.1b

The value of Q as determined from the condition of single valuedness of the pressure, Eqn.(2.29), is given by

$$Q = -\frac{C\left[\delta \text{ Sinh } \xi_2 - \text{Sinh } \delta \text{ Sinh } \xi_1\right]\left[\delta \text{ Cosh } \delta - \text{Sinh } \delta\right]}{q \text{ Sinh } \xi_1} \quad 5.1c$$

and q is given by

$$q = \delta \text{ Sinh } \delta \left[\text{Sinh}^2 \xi_1 + \text{Sinh}^2 \xi_2\right] - 2 \text{ Sinh } \xi_1 \text{Sinh } \xi_2 \text{Sinh } \delta \text{ .}$$

(b) The resultant force F_x along the X-axis, which lies in the plane of the two axes of the cylinders and perpendicular to them is given by the expession

$$F_x = \Omega^2 \alpha_1 \check{F}_x \quad 5.2a$$

where α_1 is a second order material coeficient and $\check{F}_x$ is a geometrical function given by

$$\check{F}_x = -\frac{2\pi R_1 (X-1)\varepsilon}{X q_1^2 \text{ Sinh}^2 \delta}\left[2X^2 D_1^2 + 2XD_1 D_2 \sqrt{\text{Sinh}^2 \delta + (X-1)^2 \varepsilon^2} + (X-1)^2 D_2^2 \varepsilon^2\right] . \quad 5.2b$$

where δ, D_1, D_2 and q_1 constants given by Eqns.(3.54), (3.57a), (3.57b) and (3.57c), respectively.

(c) The resultant force, which acts perpendicular to the X-axis and the plane of the axes of the cylinders, is given by the equation,

$$F_y = \Omega \mu \check{F}_y \text{ ,} \quad 5.3a$$

where $\breve{F}_y$ is a geometrical function determined by the relation,

$$\breve{F}_y = - \frac{4\pi R_1 (X - 1) \varepsilon}{q_1} . \qquad 5.3b$$

(d) The resultant torque produced on the outer cylinder about its owen axis is given by

$$M_z = \Omega \, \mu \, \breve{M} , \qquad 5.4a$$

and $\breve{M}$ is a geometrical function of the form

$$\breve{M} = \frac{4\pi R_1^2 \, X^2 \, D_1}{q_1 \, \mathrm{Sinh}\, \delta} . \qquad 5.4b$$

Since, the aim of the eccentric cylinder rheometer is to measure the forces F_x , F_y , and the torque M_z , the geometrical functions $\breve{F}_x$, $\breve{F}_y$, and $\breve{M}$ are of special significance in the design of the eccentric cylinder rheometer which is described in details in chapter four. Some numerical values which characterizws the present proto-type of the rheometer are tabulated in table (5.1). In this table, ε represents the dimensionless eccentricity which is defined by relation (3.53) and $\breve{\Gamma}$ is a geometrical function given by Eqn.(3.58). The rate of deformation quantity is given by

$$\bar{\dot{\gamma}} = \Omega \, \breve{\Gamma} \qquad 5.5$$

To realize the relations (5.2a), (5.3a) and (5.4a),

5.2) EXPERIMENTAL TESTS :

for the examination of the reliability of results obtained with the new device, measurements were carried out with a suitable Newtonian fluid, namely an 80/20 % glycerine–water mixture. Fig.(5.1) shows that the viscosity values, recived from measurements of the torque M_z and the force component F_y are independent of the effective shear rate $\bar{\dot{\gamma}}$ and coincide very well with one another and with the values determined with a commercial rheometer. This coincidence is, however, not restricted to measurements with small relative eccentricities but can be realized up to values ε = 0.75. As figure (5.2) shows, in the latter case shear rates can be attained down to 1 s^{-1}. In the respective experiments the force component F_x was observed always to disappear, therefore no graphs of the first normal–stress coefficient are added.

Subsequently, a 0.3% solution of high–molecular–weight polyisobutyline in a low–molecular–weight polyisobutylene* was inspected. As figure (5.3) shows this fluid represents

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* : Oppanol B200 and B1, respectively, from BASF AG, FRG.

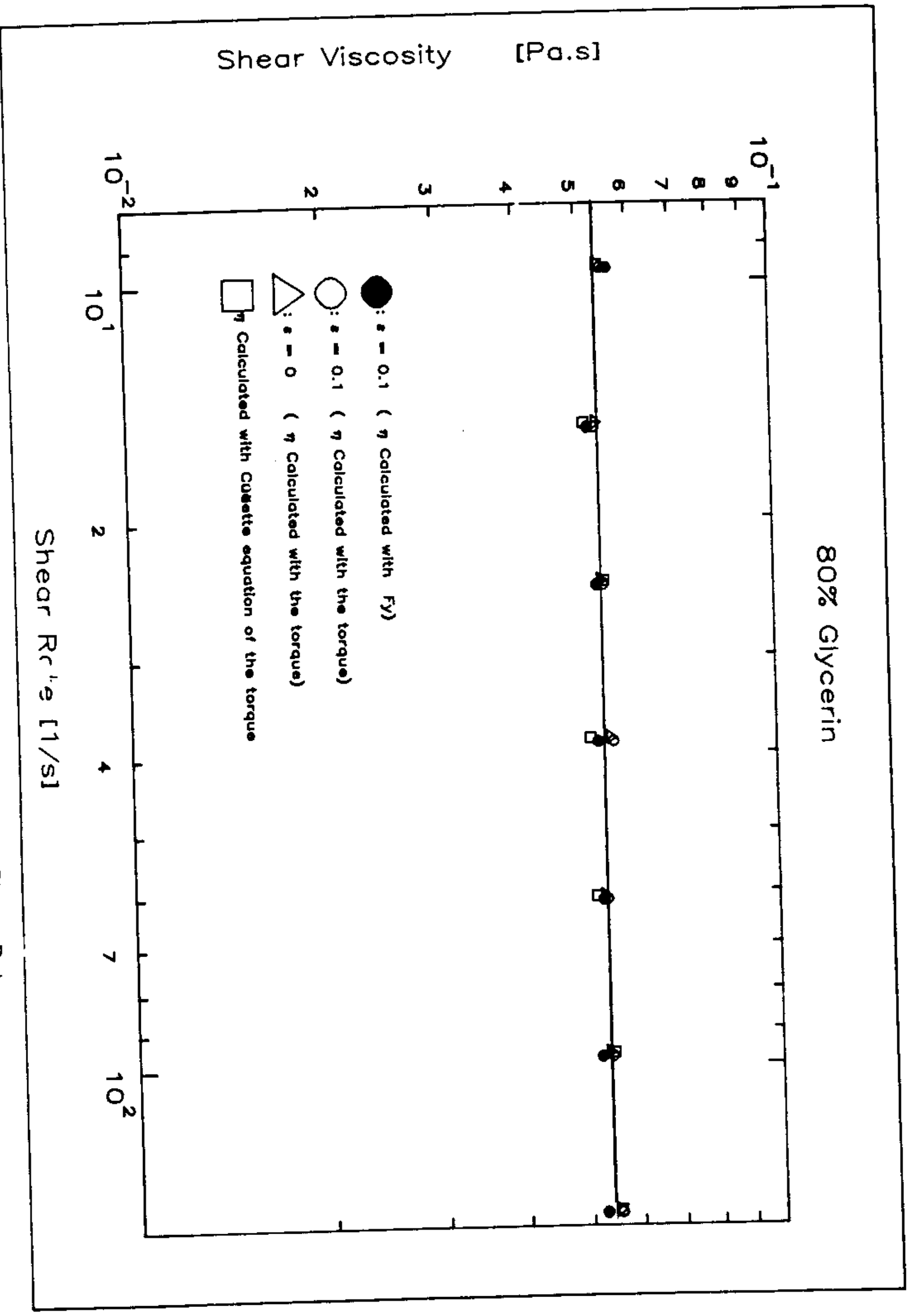

Fig.(5.1):The Shear Viscosity versus the Shear Rate

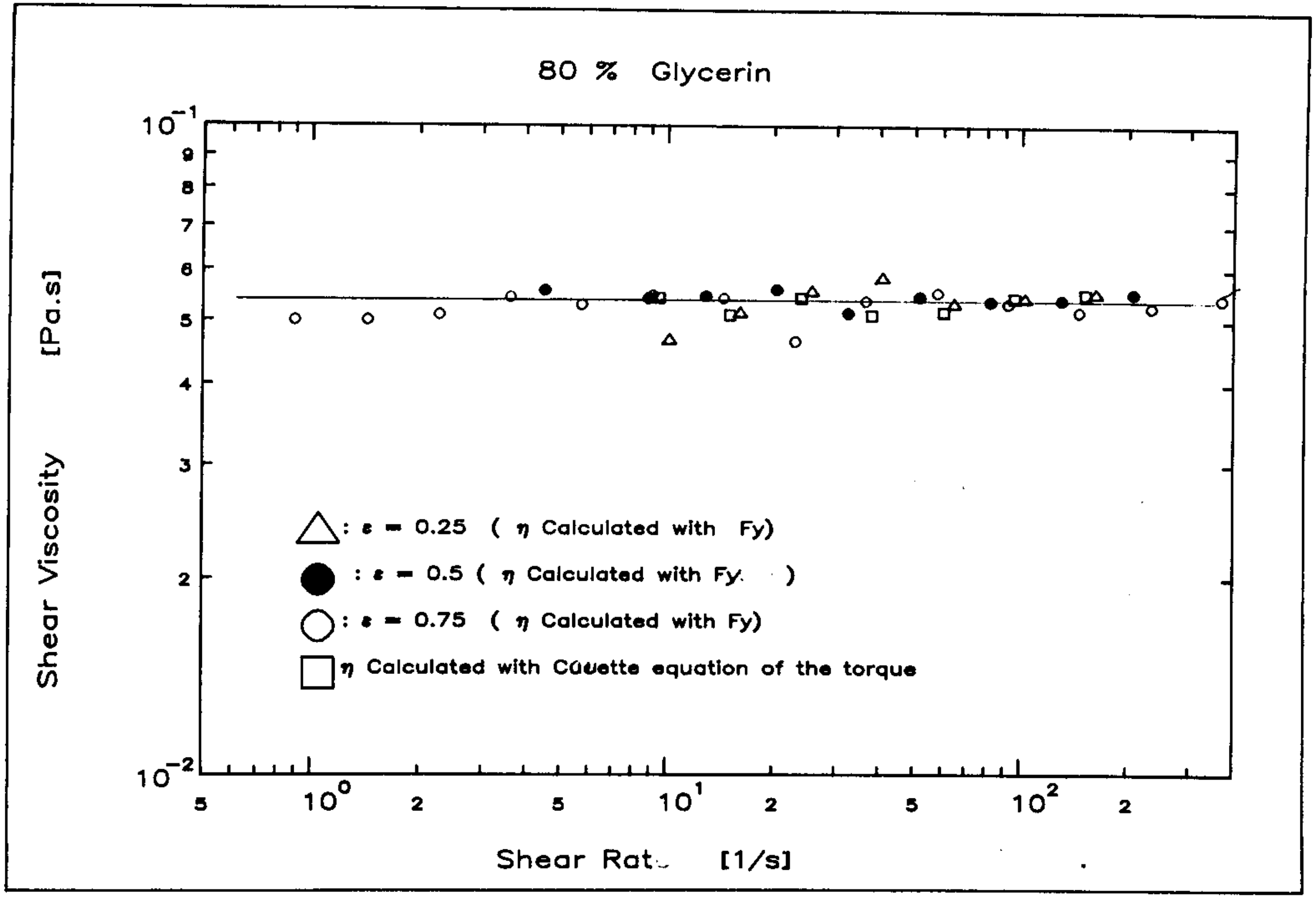

Fig.(5.2):The Shear Viscosity versus the Shear Rate

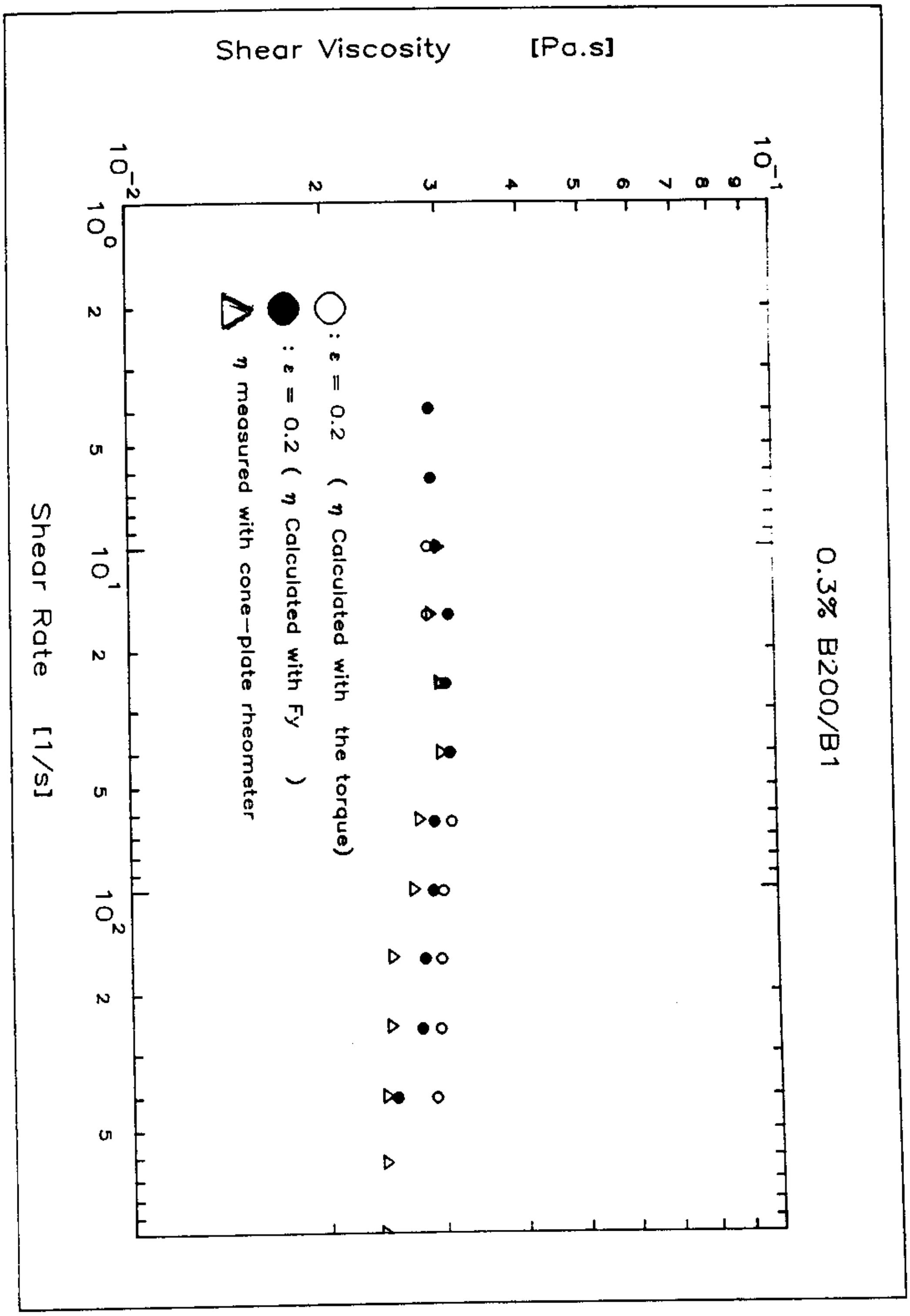

Fig.(5.3):The Shear Viscosity versus the Shear Rate

to a good degre of approximation a so-called Boger fluid. Here also the mviscosity values determined from force and torque measurements are nearly coincident with one another and with the cone-and-plate results. There are, indeed, slight differences for the higher shear rates, but these are not so far outside the estimated range of experimental errors that they must be considered to be real. As is demonstrated in figure (5.4), the determination of the first normal-stress coefficients from F_x and the cone-and-plate rheometer shows also very good agreement for the smaller shear rates and only slight differences for the higher ones. In this case the eccentric- cylinder-rheometer results look more trustworthy than those from the cone-and-plate device which is employed here near the lower limit of sensibility and with the necessity of an essential centrfugal force correction.

Similar results are obtained with a 10% commercial polystyrene solution in decalin with which also higher eccentricities up to ε = 0.5 could be applied, cf figures (5.5) and (5.6), however, no systematic dependence of

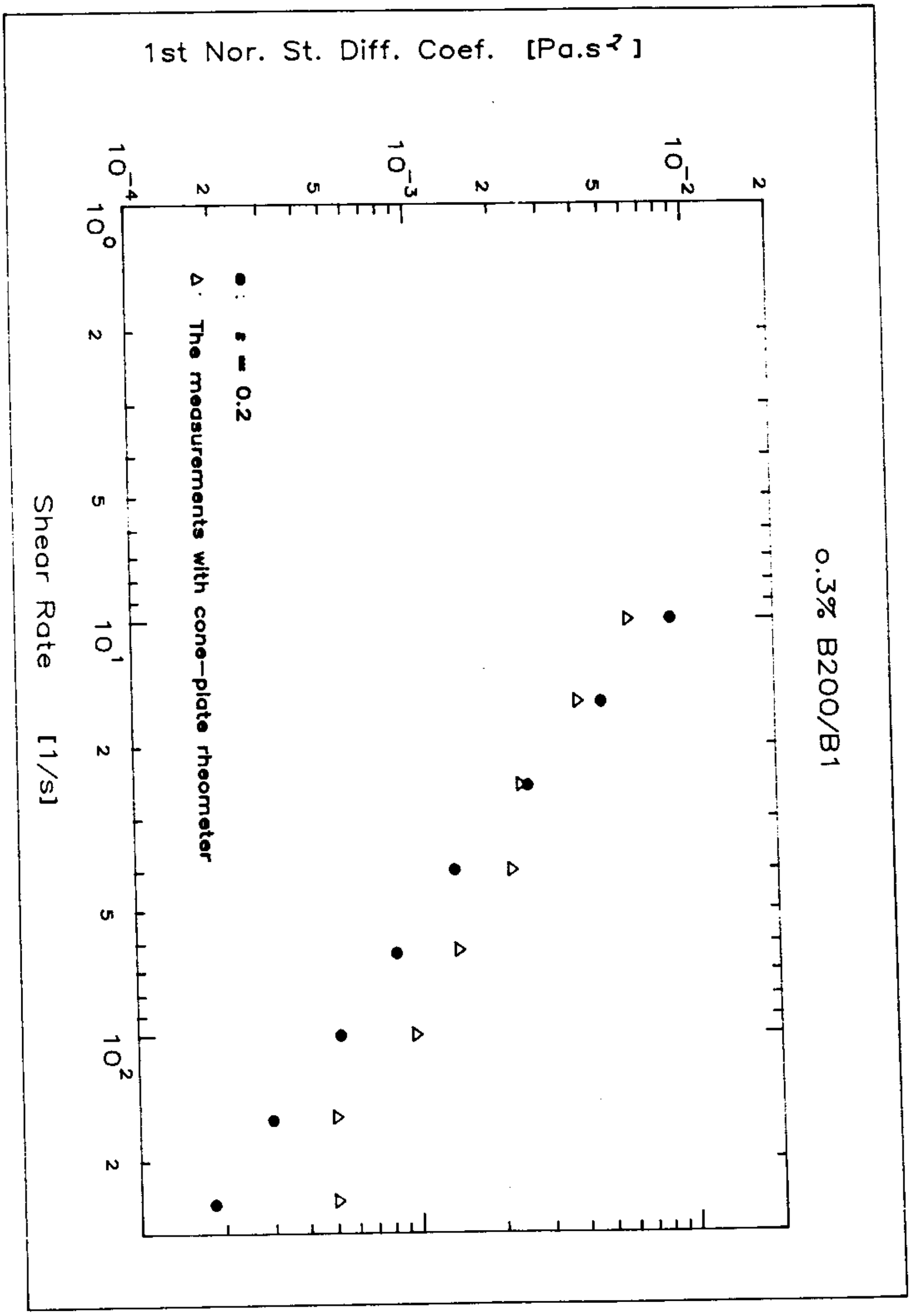

Fig.(5.4):The First Normal Stress Difference Coefficients versus the Shear Rate

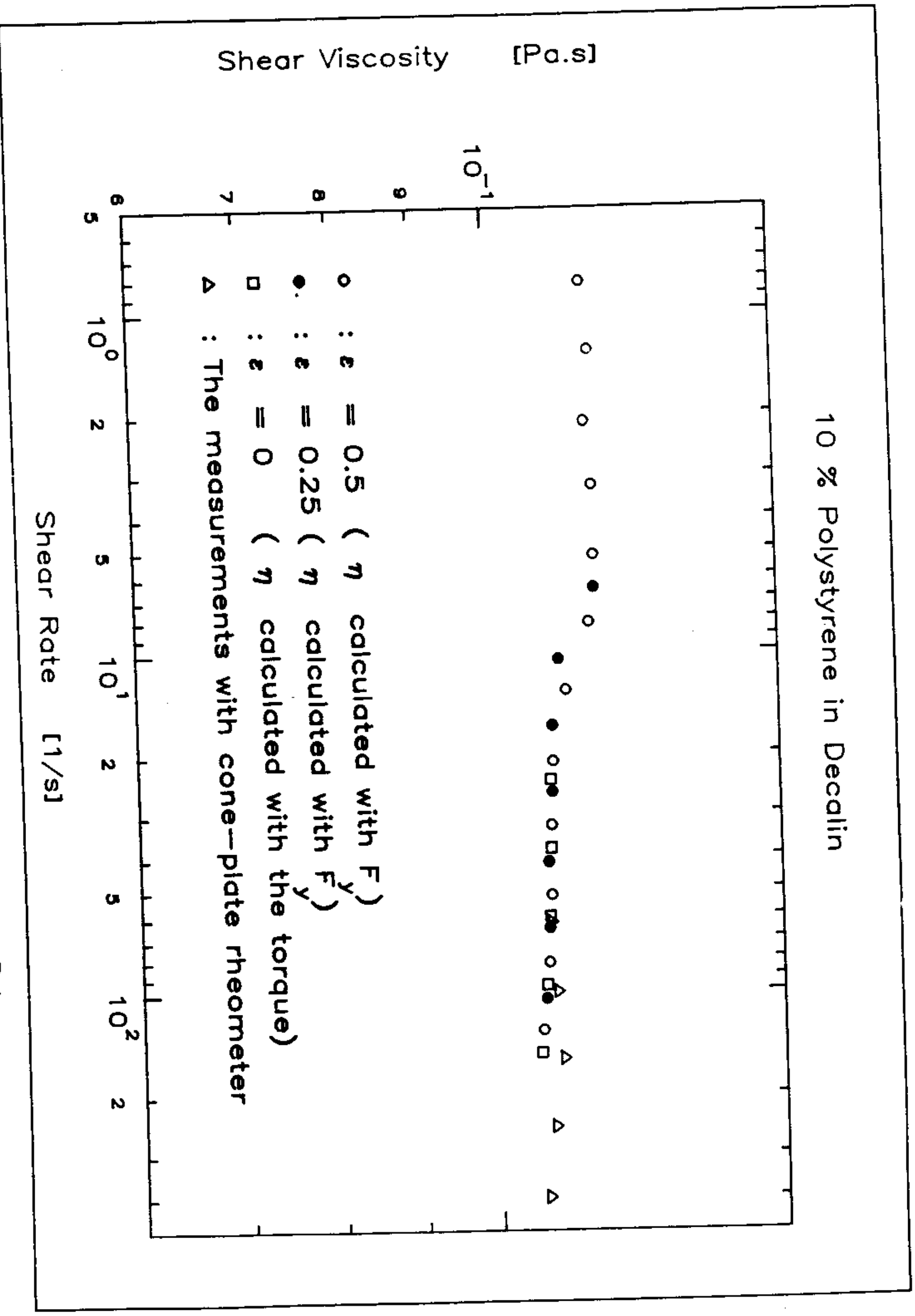

Fig.(5.5) The Shear Viscosity versus the Shear Rate

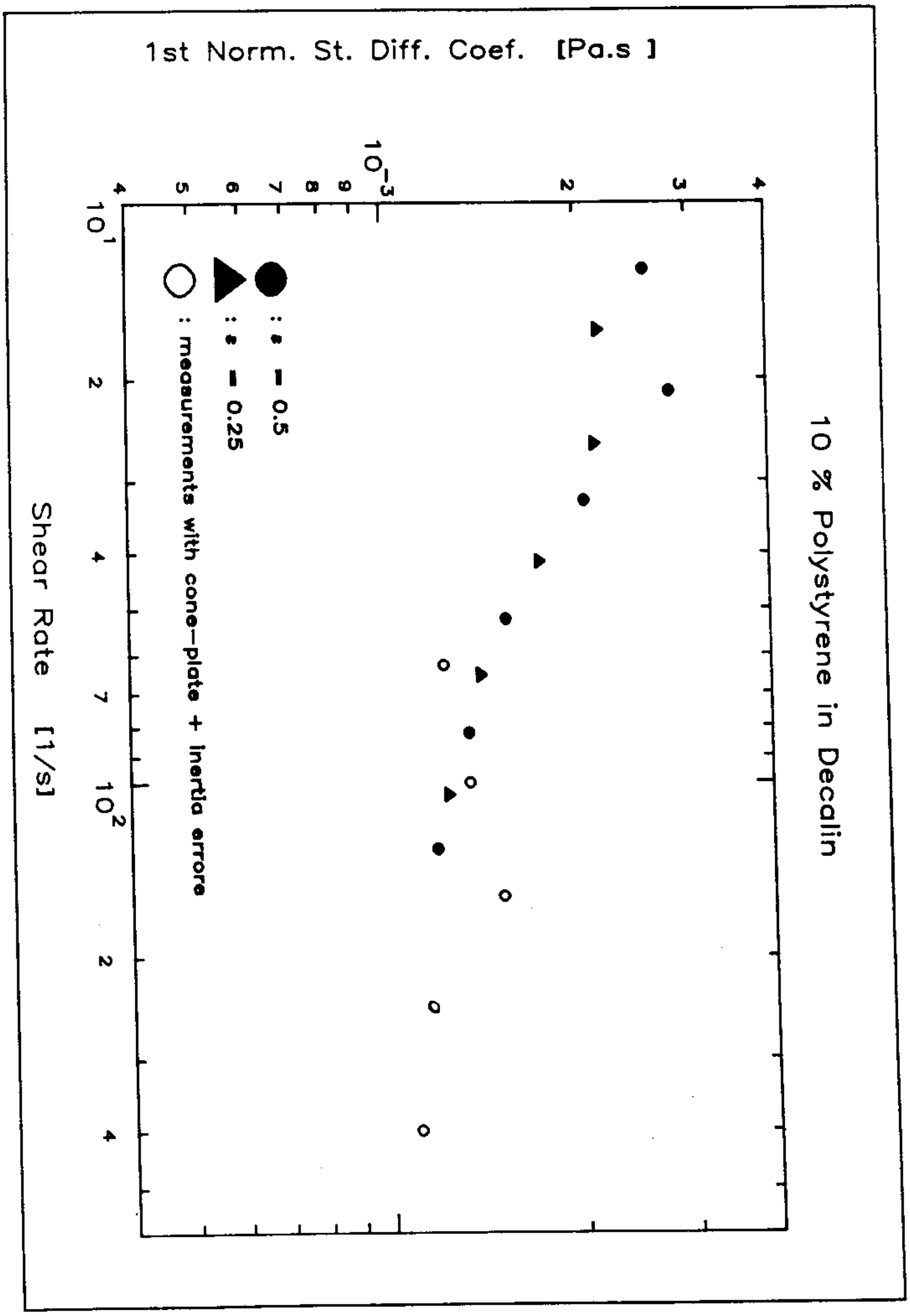

Fig.(5.6):The First Normal Stress Difference Coefficients versus the Shear Rate viscosity and first normal-stress coefficient on the eccentricity can be observed, and also no systematic deviation from cone-and-plate data as far as these can be estimated to be trustworthy.

5.3) ADVANTAGES AND DISADVANTAGES OF THE ECCENTRIC CYLINDER RHEOMETER :

- Due to the use of guard rings the construction of the eccentric-cylinder rheometer, no error in the measurements of the viscosities as well as the measurements of the first normal stress difference coefficients arises from the edge effect as well from the free surface.

- The eccentric-cylinder rheometer does not need any calbiration to measure the viscosities or the first normal-stress coefficients.

- The first normal-stress coefficients determined by the eccentric-cylinder rheometer are more precise than that determined by cone-and-plate rheometer. This is due to an error of about 100% due to the inertial forces.

■ There was only one error which arrises from unacuracy of the determination of the eccentricity, but this errror cauld be avoieded by making two eccentricities in opposite direction and taking the average value.

5.4) CONCLUSION :

The reported experimental tests show that the eccentric-cylinder rheometer gives highly reproducible plots of modified shear viscosity and first normal-stress coefficient which are in agreement with the respective plots obtained from conventional rheometers. The deviations from one another, expected for higher effective shear rates, could, however, not be evidenced unequivocally with the polymer fluids under investigation which had a relatively low viscosity. Because of this it is to be recommended to carry out measurements with solutions exhibiting more pronounced viscoelastic properties, i.e. with higher molecular weight or higher concentration. for this sake, however, some slight modifications at the basic rheometrics instrument have to be accomplished beforehand.