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In the present thesis, we consider the flow field about a sphere of radius "R" which rotates with a uniform angular velocity Ω about its axis in an infinite general Oldroyd-fluid . Moreover, the problem is extended to the case when a uniform translational motion in the direction of the axis of rotation is superimposed onto the rotational motion of the sphere .

Throughout the present investigation, the viscoelasticity of the fluid is assumed to dominate the inertia such that the latter can be neglected in the momentum equation .

The velocity, stress and pressure fields are expanded in powers of a dimensionless retardation parameter, $\lambda = \lambda_2 \Omega$, which for moderate rate of rotation ($\Omega \approx 10^2$) is of order $\lambda \approx 10^{-2}$. Hence, a set of successive partial differential equations are obtained which govern the successive approximations of the velocity field determined by the ϕ -component $W(r, \theta)$ and the stream function $\Psi(r, \theta)$.

In the first boundary value problem which concerns the rotational flow only, the leading term, $W^{(0)}(r, \theta)$, is the Newtonian flow. The first-order approximation produces a stream function, $\Psi^{(1)}(r, \theta)$, which describes a secondary flow in the plane $\phi = \text{constant}$. This includes the material constants of the Oldroyd-B fluid such that the general fluid up to this order is approximated by the special fluid. The second-order

approximation leads to a viscoelastic contribution $W^{(2)}(r,\theta)$ which depends on all the material parameters of the general model . The contribution $W^{(2)}(r,\theta)$, which is superimposed onto the Newtonian flow $W^{(0)}(r,\theta)$, can either enhance or oppose the flow $W^{(0)}(r,\theta)$ according to the values of the material parameters .

The stream function $\Psi^{(3)}(r,\theta)$, which is obtained from the third-order approximation, includes, as expected, all the material parameters of the general fluid. Due to the complicated form of this term, it is discussed for some special cases as upper-convected Maxwell fluid and Oldroyd-B fluids besides the general Oldroyd fluid under consideration.

The torque calculated up to third-order approximation is composed of a viscoelastic contribution \tilde{M}_2 added to the viscous contribution \tilde{M}_0 due to the primary flow. The \tilde{M}_2 field may enhance or oppose the \tilde{M}_0 field depending on the values of the material parameters. However, if Oldroyd-B fluid is considered, the viscoelastic contribution adds negatively to the viscous contribution .

In the second part of this thesis, where the fluid is assumed to translate with velocity \underline{V}_0 in the direction of the axis of rotation of the sphere, the approximation procedure leads to more elaborate results. The zero-order approximation produces Newtonian flow, i.e. the ϕ -component $W^{(0)}(r,\theta)$ and the stream function $\Psi^{(0)}(r,\theta)$ which agrees

with results calculated for Newtonian fluids. The first-order term is composed of $W^{(1)}(r,\theta)$ and $\Psi^{(1)}(r,\theta)$, which are superposed, respectively, onto $W^{(0)}(r,\theta)$ and $\Psi^{(0)}(r,\theta)$. The two functions $W^{(1)}(r,\theta)$ and $\Psi^{(1)}(r,\theta)$ depend on the material parameters of the Oldroyd-B fluid only. The second-order solution leads to a viscoelastic contribution $W^{(2)}(r,\theta)$, as well as the stream function $\Psi^{(2)}(r,\theta)$, where both fields depend on all the material parameters of the general Oldroyd fluid. Further steps of approximation are practically intractable.

Each of the torque and drag fields due to combined rotational and translational flows, when calculated up to the second-order, are composed of the sum of a viscous (or Newtonian) term and an elastic term. The elastic contribution in both fields, which depends on all the material parameters of the general Oldroyd model, may add positively or negatively to the viscous term according to the numerical values of the material parameters of the fluid.

On the basis of these theoretical calculations a rheometer may be designed in order to determine the material parameters η_0 , λ_1 , λ_2 , λ_3 , λ_6 and λ_7 .