Chapter (I)

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Rheology is the science concerned with the deviations of the properties of materials from the classical behaviors described by Newton's constitutive equation

$$\underline{\underline{t}} = 2\eta \, \underline{\underline{d}} \quad , \quad \underline{\underline{d}} = \tfrac{1}{2}(\nabla \underline{v} + (\nabla \underline{v})^T),$$

where $\underline{\underline{t}}$ is the stress tensor, η is the viscosity of the fluid, $\underline{\underline{d}}$ is the rate of strain, and $\underline{v}$ is the velocity of the fluid.

The second half of the last century has witnessed fundamental research works [1,2] to formulate general constitutive equations which are able to describe basic deviation from the classical behaviors of fluids such as normal stress effects , shear thinning, shear thickening, ...etc.

The constitutive equations for a simple fluid, which relates the stress at present time to the entire history of the motion of the body, is considered as the most general constitutive equation for incompressible, homogenous, and isotropic fluids. However this constitutive equation allows the solution of the equation of the motion for a very limited class of flows.

To make this equation amenable to tackle more complicated flows, the constitutive equation for a simple fluid is expanded in terms of a set of kinematic tensors via the principle of "fading memory" . The main idea of this principle is that the fluid remembers deformations which take place in the recent past in a greater extent than those take place in the remote past. The expansion may be in the form of series of multiple integral called "memory integrals" or in the case of slow flow, in a series of kinematical tensors[1].

In many boundary-value problems, the flow under consideration does not meet the restrictive conditions imposed on the above expansions . In such cases, one is obliged to choose one of the pragmatic constitutive equations which is able to reproduce the behavior of the fluid and at the same time keep the calculations within tractable limits. One of these efficient constitutive equations is the general one derived by Oldroyd.

By including low-order, frame-invariant products of the velocity gradient tensor and the stress tensor, Oldroyd obtained a general differential constitutive equation with eight empirical constants based on continuum arguments alone, without reference to molecular assumptions :

$$\underline{\underline{t}} + \lambda_1 \overset{\nabla}{\underline{\underline{t}}} + \lambda_3(\underline{\underline{t}}.\underline{\underline{d}} + \underline{\underline{d}}.\underline{\underline{t}}) + \lambda_5(\mathrm{tr}\,\underline{\underline{t}})\underline{\underline{d}} + \lambda_6(\underline{\underline{t}}:\underline{\underline{d}})\underline{\underline{I}} = 2\eta_0\left[\underline{\underline{d}} + \lambda_2 \overset{\nabla}{\underline{\underline{d}}} + \lambda_4(\underline{\underline{d}}.\underline{\underline{d}}) + \lambda_7(\underline{\underline{d}}:\underline{\underline{d}})\underline{\underline{I}}\right],$$

where $\underline{\underline{t}}$, $\underline{\underline{d}} = \frac{1}{2}[(\nabla \underline{v}) + (\nabla \underline{v})^T]$ and $\underline{\underline{I}}$ are the extra-stress, the rate of deformation and the unit tensors; respectively. The material constants

η_o , λ_1 and λ_2 are, respectively, the zero-shear rate viscosity, the relaxation and retardation times, while λ_3 to λ_7 are further material time constants. The symbol "∇" over a symmetric tensor $\underline{\underline{G}}$ denotes the upper-convected derivative defined by

$$\overset{\nabla}{\underline{\underline{G}}} = \frac{\partial \underline{\underline{G}}}{\partial t} + \underline{v} \cdot \nabla \underline{\underline{G}} - \underline{\underline{G}} \cdot \nabla \underline{v} - (\underline{\underline{G}} \cdot \nabla \underline{v})^T ,$$

This constitutive equation makes useful qualitative predictions but is not quantitatively accurate.

Oldroyd [3] was able to show that the class of fluid defined by the former constitutive equations, exhibit most of the non-Newtonian properties observed in flow fields of viscoelastic fluids. Such non-Newtonian properties reveal themselves in :

(a) The dependence of the apparent viscosity, in simple shearing, on the rate of strain.

(b) The normal stress phenomena demonstrated by "Weissenberg effects",

(c) The swelling of the fluid at the exit of a tube of circular cross-section.

Therefore, the model defined by the above mentioned equation is appropriate for solving the boundary-value problems which are to be investigated in the present thesis.

In order to reproduce the required behavior of viscoelastic fluids, the Oldroyd model should fulfill the following restrictions :

$$\sigma_i = \lambda_i(\lambda_3 + \lambda_5) + \lambda_{i+2}(\lambda_1 - \lambda_3 - \lambda_5) + \lambda_{i+5}(\lambda_1 - \lambda_3 - \tfrac{3}{2}\lambda_5),$$

with i=1,2 and $\sigma_2 \geq \frac{1}{9}\sigma_1$

where σ_1 and σ_2 are the normal stresses defined in a viscometric flow [3].

In the last few decades, much work has been done on the flow of viscous or viscoelastic fluids about objects of different geometries; of special interest are objects with spherical symmetry. The rotational as well as the translational motions of a sphere in viscoelastic fluids has been investigated extensively because of geometric simplicity and their importance in the problems of sedimentation and dynamical separation processes. These studies are carried out theoretically, experimentally, and partially by numerical methods [4-7].

Leslie [8] studied the translational motion of an Oldroyd 8-constant fluid past a fixed sphere. Neglecting the inertial forces, he calculated the drag forces up to the second-order. Using the retarded motion approximation, Giesekus [9] solved this problem up to the third-order. He calculated the forces and the torques acting on the sphere undergoing simultaneous rotational and transitional creeping motion in a fluid of grade three and related these calculated quantities to the material constants of that fluid. Giesekus [9] hoped to determine the material constants by measuring the torques and forces in an appropriate set up, however, the practical realization did not take place. Ranger [10] introduced an exact solution for the axisymmetric swirling motion of a solid sphere whose speed decays exponentially with time translating and rotating in a viscous fluid; he obtained a formula for the torque and the drag force. The torque and drag force decay also with time in the same sense as the velocity fields. Omitting the inertial terms Fosdick et al. [11] studied the rotation of a sphere in a third order fluid by using the retarded motion expansion model. Walters et al. [12-14] computed solutions for a fluid of third-order via some perturbation methods. In their calculations, these authors did not neglect the inertial term. The expressions representing the torque in the present work stand in accordance with that determined by Walters et al. [12-14] so that we can compare the constants for the two model, .i.e., the retarded motion expansion and the general Oldroyd model and construct some relations between them . Using another expansion parameter, Thomas et al. [15] studied, without neglecting the inertial term, the motion of an Oldroyd-B fluid up to the second-order due to the rotation of a sphere about its diameter. It was shown that the streamline projections containing the axis of rotation are strongly dependent on the parameters of the fluid. They showed that the inertia adds positively to the viscous torque, while the viscoelasticity adds negatively to it . Proudman et al.[16] used the perturbation theory to study the translation of a sphere in a viscous fluid taking into account the inertial term. Their results show an increase of the drag force due to inertia. Nowak [17] presented abstract results of an experimental investigation of the flow of a highly dilute cationic surfactant solution around a rotating sphere. He found that the buildup of the shear-induced structures occurs only above a critical shear rate.

Watanabe et al.[18] investigated experimentally the drag on a sphere in dilute solutions at high Reynolds number range. It was shown that the fall velocity is larger than that in Newtonian fluids within the critical Reynolds number range. It was concluded that the drag in dilute solution is smaller than that in Newtonian fluids. The maximum drag reduction occurs at Weissenberg number within the range of We=3~10. The results obtained by Nowak [17] and Watanabe [18] confirm those of Proudman[16].

Bodart et al. [19] and Zheng et al. [20] applied numerical studies for the falling of a sphere in a cylindrical tube under gravity in a viscoelastic fluid. They studied the effect of Weissenberg number and the ratio of the sphere to the cylinder radius on the velocity field and the drag. They found that the viscoelastic effects are damped when the radius of the cylinder becomes much larger than the radius of the sphere. The velocity overshoot decreases when the Weissenberg number increases.

Arigo et al.[21] presented the first quantitative experimental measurements of the transient motion of a sphere as it accelerates from rest along the axis of a tube containing a highly viscoelastic, shear-thinning liquid. The results show the dependence of the shear rate on the Deborah number and the ratio of sphere to tube radius.

Harlen [22] studied the phenomenon of the negative wake behind a sphere sedimenting through a viscoelastic fluid. It is shown that there are two competing viscoelastic forces at work in this flow. The relaxation downstream of shear stresses generated near the side of the sphere drives a flow directed away from the sphere, giving rise to a negative wake. This force is opposed by the extensional stresses generated in the extensional flow at the rear of the sphere, which drives a flow towards the sphere producing an extended wake. The parameter controlling the balance between these forces is the extensibility of the polymer, with high extensibility producing an extended wake and smaller values giving negative wake. Fan [23] employed the Galerkin/least-square *hp* finite element method to solve the problem of steady flows of a sphere falling in a tube filled with viscoelastic fluids, the diameter ratio being 2.0. The computations show that for a finitely extendable nonlinear elastic spring fluid with low extensibility of the molecular chain can easily exceed the limiting Debora number of the upper convected Maxwell fluid, but with the high extensibility the solution exhibits the same characteristics as in the upper convected Maxwell fluid case. Chen et al. [24] investigated experimentally the flow past a sphere falling at its terminal velocity through a column of a wormlike micelle solution, where the ratio of the tube to sphere radius have been taken 0.0625 and 0.125. The investigations were performed over a wide range of Deborah number. The drag on the sphere was found to decrease initially with increasing Deborah number because of shear thinning effects. As the Deborah number is increased, the establishment of a strong extensional flow in the wake of the sphere causes the drag to increase to a value larger than that of a Newtonian fluid with the same viscosity.

In a series of important papers, Yamaguchi et al. [25-27] studied the viscoelastic flow between concentric spheres experimentally where polyacrylamide (PAA)-water solutions are used as a model viscoelastic fluid and Giesekus model was applied to interpret the obtained results.

Abu-El Hassan [28,29] studied the flow fields of Oldroyd 8-constant fluid between two concentric spheres. He solved the problem analytically and calculated the flow field and the torque up to the third-order approximation.

The present thesis deals with two boundary-value problems [30-32] ; namely, (a) The steady state flow about a sphere which rotates in an infinite general Oldroyd fluid and (b) The combined rotational and translational motion of the sphere in the same fluid.

In the first boundary-value problem a sphere of radius "R" rotates with uniform angular velocity Ω about the z-axis located at its center. The inertial term in the momentum equation is assumed negligibly small relative to the viscoelastic term. The velocity, stress and pressure fields are expanded in powers of the dimensionless relaxation paramter λ . Hence, a set of successive partial differential equations is obtained. The solutions of these equations approximate these fields successively. The velocity field, the streamlines, the torque as well as the drag fields on the sphere are calculated and studied up to the third-order explaining the effect of material parameters of the fluid η_0, λ_1, λ_2, λ_5, λ_6 and λ_7 on these fields.

In the second boundary-value problem the linear translating flow of the general Oldroyd fluid past a rotating sphere that rotates with an angular velocity Ω about an axis in the direction of translational motion. The fluid translates in the direction of that axis of rotation with a linear velocity $\underline{V}_0$. As in the first boundary-value problem, it is assumed that the viscoelasticity of the fluid dominates its inertial forces such that the latter can be neglected in the momentum equation. An analytical solution up to the second order is obtained through a perturbation technique where the dimensionless λ is used as a perturbation parameter. The velocity field, stream function, torque and drag fields are calculated up to the second order explaining the effect of the material parameters of the fluid η_0, λ_1, λ_2, λ_5, λ_6 and λ_7 on these fields .

The present thesis includes five chapters, the introduction being the first one.

The definition of the first boundary value problem, which is concerned with the rotational flow only, as well as the formulation of the problem, the perturbation method, the solution of the problem up to the third order are included in chapter two.

Chapter three includes the second boundary value problem, where both the rotational and translational flows about the sphere are considered. The formulation of the problem and its solution up to the second order are performed in this chapter.

Chapter four contains the analysis and discussion of the velocity field, stream-function, stresses and the torque acting on the sphere for the two boundary-value problems. The drag field for the second boundary-value problem is also discussed in this chapter.

The final results and discussion of the present work are outlined in chapter five.