INTRODUCTION

The main object of this thesis is to obtain further properties of the number m of integrable-square solution of formally symmetric differential equation of fourth order:

$$My = (p_o(x)y'')'' - (p_1(x)y')' + p_2(x)y$$

for $0 \leq x < b \leq \infty$.

It is known that, for complex λ, the number m of $L^2[0,\infty)$ solutions of $My = \lambda y$ satisfies $2 \leq m \leq 4$, and is also independent of λ. One of our aims in this thesis is to obtain criteria for $m = 2$. In analogy with the Wyle-classification, we shall call this "Limit Point" case, more precisely lim-2.

This problem has attracted a great deal of attention during recent years and many strong results are known. In many of these investigations, the method of approach is based on the same basic idea.

The method used here is an extension of those ideas from the books [28] and [31].

Only the even order case is considered here since this offers the simplification of separated boundary conditions. The even order case with real coefficients which is included in the analysis given here has been considered by [18] and [29] using linear operator methods.

In first chapter, we introduce the main terminology and the main concept of limit-point of the second order differential equation $M_o y = \lambda y$ where $M_o y = -Dp_o Dy + p_1 y$ as an introduction study for the main problem and to be used in subsequent chapters, more precisely in Chapter II and Chapter III. Also we give a short review of some of the previously known order differential equation. Further we will comment on these known results in a set of a remarks.

In Chapter II we shall study the main problem of fourth order differential operators M conditions are given on $p_o(x)$, $p_1(x)$ and $p_2(x)$ and their derivatives to ensure that M is in limit-point or precisely the equation $My = \lambda y$ has exactly two linearly independent solutions which are in the integrable square space $L^2(0,\infty)$ for all complex number λ with $\mathrm{Im}\lambda \neq 0$. The main object of this chapter just to explain the method, with some special treatment, which will be used in Chapter III.

In Chapter III we use the same method, developed in Chapter II, without any constraints throughout our treatment. This generalization leads to a more general result than that of Chapter II. This result, of the present chapter covers many of those known results which are concerned with the present problem of the concept of

limit point case at infinity of fourth order symmetric differential operators. The results obtained are applied to the case when the coefficients $p_k(x)$, $k = 0,1,2$ are powere of x in $[0,\infty)$.