For a very general class of non-Newtonian fluids, the relation between M and Ω in steady Couette flow of a particular fluid is determined once the relation between Q and f is known for steady Poiseuille flow of the fluid. Both relations M=M (Ω) and Q=Q (f) can be calculated from a material function $\eta(\dot{\gamma})$ which is called shear-dependent viscosity, where $\dot{\gamma}$ is the rate of shear.

1.1.2 Normal stress effects:

Reconsidering the steady Couette flow for a Navier-Stokes fluid, it comes out that the difference $\Delta T^{(rr)}$ between the normal traction per unit area on the outer and inner cylinders should be given by

$$\Delta T^{(rr)} = T^{(rr)}(R_2) - T^{(rr)}(R_1) = -\int_{R_1}^{R_2} \rho r[\omega(r)]^2 \, dr \tag{1.5}$$

where ρ is the mass density and $\omega(r)$ is the angular velocity of the fluid at radial distance r from the axis of the cylinders. Equation (1.5) reveals that the difference in normal thrust on the bounding cylinders should be due solely to the centrifugal force on the fluid. It follows from (1.5) that $\Delta T^{(rr)} < o$;i.e.,

$$T^{(rr)}(R_2) < T^{(rr)}(R_1). \tag{1.6}$$

This mean that the pressure $-T^{(rr)}(R_2)$ on the outer cylinder should be greater than the pressure $-T^{(rr)}(R_1)$ on the inner cylinder. Experiments easily confirm the inequality (1.6) for water and other Navier-Stokes fluids.

In Fig. (1.3) [3], however, we see that for certain fluids, such as polymer solutions, the thrust on the inner cylinder can be greater than the thrust on the outer cylinder, an experimental result which indeed shows the principal deviation from the Navier-Stokes theory.

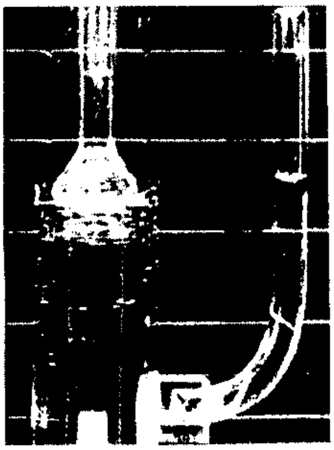

Fig.1.3.

The difference in normal thrusts on the bounding cylinder in the steady Couette flow of an 8.5% solution of polyisobutylene in decalin. The thrust on the inner (rotating) cylinder is indicated by the level of a surface of the fluid inside the inner tube. The thrust on the outer (stationary) cylinder is indicated by the level of the fluid in the side arm.

The phenomenon illustrated in Fig(1.3.) [3], that is the failure of the inequality (1.6) and hence equation (1.5), belongs to a class of phenomena called normal stress effects. Later on, it will be shown that these normal stress effects are governed by two material functions σ_1 and σ_2 , which are not determined by the viscosity function η which controls departures from equations (1.3) and (1.4).

In a series of striking demonstrations, Weissenberg [4] demonstrated the normal stress effects which is often called "The Weissenberg effects". One of his diagrams [3] shown in Fig (1.4), illustrates the normal stress effects in tortional flow.

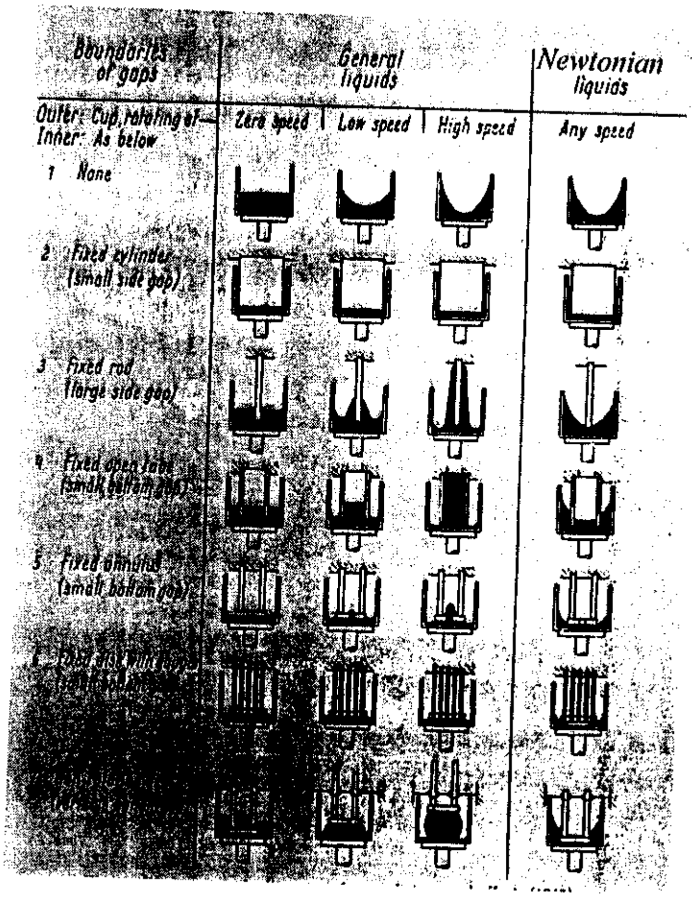

Fig. 4
Weissenberg's diagrams of normal-stress end effects (1947)

Another phenomenon that demonstrates normal stress effects is the swelling of a non-Newtonian fluid which issues from a circular pipe into an atmosphere prescribed by constant pressure P_0. The swelling effect was first reported by Markovitz [3]; Fig.(1.5) includes his photograph that demonstrates this effect.