Results and Discussion

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The present thesis is concerning with the formulation of constitutive equations which are based on continuum model . Hence, the stress tensor can be written as a general functional of the strain history as a series of integrals of ever increasing dimensionally, i.e., in terms of the memory integral expansion. Herein, the results of the present thesis are presented as the follows:

In chapter one it is stated that the constitutive equation for viscous fluids is given by Newton's linear relation of stress and rate of strain tensors . Moreover, the mechanical behavior of many real fluids, especially those of low molecular weight, appear to be accurately described by the theory of Navier-Stokes fluids. Many real substances which are definitely "fluid-like" and incompressible under moderate pressure changes are not at all described by these two aforementioned models. Among them are Viscoelastic fluids where the main problem with formulation of their constitutive equations lies in the need to describe substantial non-linear effects which occur in the flow of these liquids due to a combination of their high elasticity and viscous effects. Several attempts have been made over the years by scholars from different scientific fields to address this problem; specially those based upon continuum physics and molecular theory.

Chapter two presents the general principles governing the mechanical behavior of materials. These are three: the principle of material frame in difference, the principle of determinism and the local action. Moreover, the concept of body, the dynamical quantities associated with its motion, the kinematical definitions and formula needed to describe the deformation and flow of material are outlined. Next, an opening with the general constitutive equation, which defines mathematically the class of ideal materials obey these basic principles, is introduced and discussed. Each such material is characterized by a particular response functional. Emphasis is placed on simple materials, in which the collection of the kinematical facts needed to determine the stress reduces to the history of the deformation gradient alone. The theory so obtained, while special with respect to the three principles, is still a very broad one. It includes are not only the theories of finite elastic strain and of non-linear viscosity but also theories of viscoelasticity and stress relaxations. Qualities distinguishing one kind of material from another are then defined by invariant properties of the response functionals; the term "materially uniform" and homogeneous' are defined in terms of these functionals. It is shown that if the response functional of a simple material is sufficiently smooth in a certain sense , then the Boltzmann's equations of linear viscoelasticity(memory integral expansion) result as an approximation in motion, the histories of which are nearly constant. Finally, the principle of fading memory as a fourth principle for special kinds of materials is stated.

Chapter three is concerned with the expansion of the constitutive equation for simple fluids which is a formal relationship between stress and deformation history defined by the functional relation,

$$\mathbf{T}(t) = -p\mathbf{I} + \underset{s=0}{\overset{\infty}{\mathrm{H}}} (\mathbf{G}(s)); \qquad \mathbf{G}(s) = \mathbf{C}^{(t)}(s) - \mathbf{I}, \tag{3-1}$$

$$\mathrm{T}_E(t) = \mathbf{T}(t) + p\mathbf{I} = \mathrm{tr}\ \underset{s=0}{\overset{\infty}{\mathrm{H}}} (\mathbf{G}(s)); \quad p = -\frac{1}{3}\mathrm{tr}\mathbf{T}(t) \quad . \tag{3-2}$$

H is an isotropic tensor-valued functional of the entire history of the symmetric stretching tensor **G**. A theory based on this assumption alone can scarcely have predictive value, for the entire history of a body can never be known. The "principle of fading memory" is assumed that to be appropriate under suitable circumstances for most isotropic simple fluids. In order to put this principle in a mathematical form, an influence function $h(s)$ is introduced such that: $h(s)$ is defined for $0 \le s \le \infty$ and has positive real values, normalized and decays monotonically in such a way that:

$$\lim_{s \to 0} s^r h(s) = 0, \quad \text{for } r \succ \frac{1}{2}. \tag{3.3}$$

The history of stretching $\underline{\mathbf{G}}$ accommodates the fading memory through the introduction of a Hilbert-space norm,

$$\|\mathbf{G}(t)\|_n = \left[\int_0^\infty \{h(s)|\mathbf{G}(t)|\}^2 ds \right]^{\frac{1}{2}}, \qquad |\mathbf{G}| = \left[tr\mathbf{G}.\mathbf{G}^T \right]^{\frac{1}{2}} \quad , \tag{3.4}$$

where $\mathbf{G}(s)$ vanishes at the neighborhood of the material point if there is no deformation in the past. The magnitude of the norm (3.4) indicates how much the deformation history of the material element differs from the rest history for which $\|\mathbf{G}(t)\|$ vanishes.

Coleman and Noll [26] assumed further that the response functional $\overset{\infty}{\underset{s=0}{\mathrm{H}}}(\mathbf{G}(s))$ is defined and continuous for histories $\mathbf{G}(s)$ in the neighborhood of the zero history $s = 0$ (or the present time) in the Hilbert space whose elements are the histories $\mathbf{G}(s)$. This continuity implies that, two histories differs little in norm if their values are close to each other for small s (recent past), though they may be far apart for large s (remote past). This is due to the fact that the norm is weighted with the decaying influence function $h(s)$. By making use of Freshet differentiation, we get:

$$\delta^k \overset{\infty}{\underset{s=0}{H}} (\tilde{\mathbf{G}}(s);\mathbf{G}(s)) \quad \text{of degree} \quad k = 0,1,.....,n \quad \text{in } \underline{\mathbf{G}}(s)$$

such that

$$\overset{\infty}{\underset{s=0}{H}} (\tilde{\mathbf{G}}(s) + \mathbf{G}(s)) = \sum_{k=1}^{n} \frac{1}{k!} \delta^k \overset{\infty}{\underset{s=0}{H}} (\tilde{\mathbf{G}}(s) + \mathbf{G}(s)) + o(\|\mathbf{G}(s)\|_h^n)$$

In the present case, where $\tilde{\mathbf{G}}(s) = \mathbf{G}(0) = \mathbf{0}$, the last two equations reduce to

$$\delta^k \overset{\infty}{\underset{s=0}{H}} (\mathbf{0},\mathbf{G}(s)) = \delta^k \overset{\infty}{\underset{s=0}{H}} (\mathbf{G}(s)) \quad \text{of degree} \quad k = 0,1,.....,n \quad \text{in } \underline{\mathbf{G}}(s) \tag{3.5}$$

and $$\overset{\infty}{\underset{s=0}{H}} (\mathbf{G}(s)) = \sum_{k=1}^{n} \frac{1}{k!} \delta^k \overset{\infty}{\underset{s=0}{H}} (\mathbf{G}(s)) + o(\|\mathbf{G}(s)\|^n) \tag{3.6}$$

Integral representation of lower order approximations:

When $n = 1$, (3.6) reduces, in the case of incompressible fluids, to

$$\overset{\infty}{\underset{s=0}{H}} = \int_0^\infty \mathrm{M}_1(s)\mathbf{G}(s)ds\,; \quad \mathbf{T}(t) = -p\mathbf{I} + \int_0^\infty \mathrm{M}_1(s)\mathbf{G}(s)ds \quad , \tag{3.7}$$

where $M_1(s)$ in a material is the time-dependent shear modulus of the fluid which describes stress-relaxation. The leading term of the expansion (3.6) as given by (3.7) is identical with the results of the "theory of linear viscoelasticity" which describes only the shear-dependent viscosity but not the normal stress differences.

Under certain smoothness assumptions, the second and third approximations of (3-6) can be written as double and triple integrals, called "***memory integrals***", in the form