## **SUMMARY**

Hydrosystems is originally coined by V.T. Chow to collectively describe the technical areas of hydrology, hydraulics and water resources.

Here we are concerned with the mathematical modelling of problems in water project design, analysis, operations, and management.

The development of mathematical simulation models in the early 1970 [22] provided groundwater planners with quantitative techniques for analyzing alternative groundwater pumping or recharge schedules.

Although simulation models provide the resource planner with important tools for managing the groundwater system, the predictive models do not identify the optimal groundwater development, design, or operational policies for an aquifer system. Instead, the simulation models provide only localized information regarding the response of the groundwater system to pumping and/or artificail recharge. In contrast, groundwater optimization models can identify the optimal groundwater planning or design alternatives in the context of the system's objectives and constraints. Optimization modelling was originally introduced in the early 1974 by E. Aguado[2]. In subsequent works, optimization models has been developed by several authors [19,20,21,33,36,44,45,48].

This thesis aims to introduce a mathematical model to deduce the optimal dynamic utilization of groundwater in Egypt. It consists of five chapters.

Chapter 1 is an introduction to groundwater systems, model building, and the groundwater in Egypt.

Chapter 2 introduced a Safe-Yield Model using Inventory Theory. In this chapter we deduced the Safe-Yield Abstraction of the four regions in Egypt under the assumption that the demand is a continuous random variable.

Chapter 3 introduced a mathematical model which gives the optimal dynamic utilization of the Nile Delta aquifer. In this chapter we assumed that the aquifer has a transient condition and it is assumed to be hetergenious, anisotropic and semiconfined aquifer. We deduced the response equations from its governing equation by using the finite difference method [40], and built the optimization model using the optimal control approach [22], in which the response equations become a subset of the constraints. The optimization model that is solved numerically is a linear system maximization problem.

Chapter 4 introduced a mathematical model which gives the optimal control of the Nile Delta aquifer. In this chapter we extend Ladon's model [21] to the time-dependent leaky aquifer. The resulting optimization model is a nonlinear optimization problem.

Chapter 5 presented a mathematical model which gives the optimal control of the South Western region to the Nile Delta. In this chapter we assumed that the aquifer has a transient, isotropic and unconfined condition. The resulting optimization model is also a nonlinear optimization problem.

Notations

The following notations are used in what follows:

Notation	Its meaning
RIGW	Research Institute of Groundwater.
L.E.	Egyptian Pound.
$G_{ m l}^{f k}$	Water table in grid 1 and period k.
h <mark>k</mark>	Head in grid I and period k.
$M_1$	Thickness of aquitard cap in grid 1.
$Q_{\mathbf{l}}^{\mathbf{k}}$	Pumping in grid 1 and period k.
$Q_{max}^{k}$	Upper bound of pumping in period k.
$Q_{min}^{\mathbf{k}}$	Lower bound of pumping in period k.
y <sup>r</sup>	Safe yield abstraction in period k.
h <sup>k</sup> min	Lower bound of heads in period k.
h <sup>k</sup> max	Upper bound of heads in period k.
S	Storativity coefficient of the Nile Delta aquifer.
Т	Transmissivity of the Nile Delta aquifer.
K	Hydraulic conductivity of the Nile Delta aquifer.
Kz	Vertical hydraulic conductivity of the Nile Delta aquifer
$\tilde{Q}_{l}^{k}$	Pumping in control node I and period k.
$\tilde{\mathbf{h}}_{l}^{k}$	Head in control node l and period k.
ξ <sup>k</sup>	Water demand in period k.