1. INTRODUCTION

One of the most remarkable episodes in the history of astronomy occurred on the evening of March 13th, 1781, when Sir William Herschel, whilst pursuing an ambitious project of examining every section of the northern skies, discovered near the foot of Gemini, a strangely blue object of unusual size. Suspecting that it might be a comet, he carefully noted its position. Further observations on the following evening confirmed its motion and the discovery was promptly conveyed to Greenwich and Oxford.

For a comet, the new object was certainly peculiar. It was so starlike that even experienced observers had difficulty in finding it. Nevil Maskeline, the Astronomer Royal, suspected that the object was a planet and not a comet, an idea quickly confirmed by the Russian astronomer, Andres Johann Lexell, who happened to be visiting England at the time. Lexell's computations showed that the new object moved in a nearly circular orbit, always remaining more distant than Saturn which was the most distant planet then known. The new object was Uranus, the seventh planet belonging to the solar system.

Joseph Jerome Lafrancais de Laland was among the first to prove by calculation of its orbit that Herschel's object was a planet. Because of the smallness of the eccentricity of the then known planets, it was first assumed that the eccentricityof the new planet Uranus was also small, and accordingly the early investigations by Lexell (1781), Laland

(1782) and Laplace (1783) were based on the assumption of a circular orbit. As more observational material became available Laplace (1784), Boscovitch (1785), Fixlmillner (1789) and others made several attempts to determine simple elliptic elements, whilst Oriani (1789), Delambre (1789) and Laplace (1802) took into account the corrections to be made due to the perturbations of the planet by Jupiter and Saturn. As a result of the steadily improving values found for the elements of the orbit, it soon became possible to determine with reasonable accuracy the position of Uranus prior to its discovery.

As Uranus was a little fainter than stars just visible to the naked eye it occurred to the German astronomer Johann Elert Bode that possibly Uranus had been mistaken for a star in the past, in which event its position would be recorded in the contemporary catalogues. A search of the older catalogues proved more successful than had been anticipated, for Uranus had been observed on no fewer than 19 different occasions, the observations most remote in time being those made in 1690 by Flamsteed, the first Astronomer Royal. The observations made between 1690 and 1771 were referred to as 'ancient' observations to distinguish them from the possibly somewhat more reliable observations made subsequent to 1781 and which later were termed the 'modern' observations. The ancient observations had the advantage of covering rather more than one complete revolution of Uranus in its orbit, while the modern observations covered rather less than half a revolution, but were regarded as being more accurate than the ancient ones.

However, even with this wider range of observations, all efforts to produce a satisfactory ephemeris proved only partially successful. For, although the theories appeared to be satisfied by the modern and, where applicable, the ancient observations, the disagreement between observations and theory became significant within a few years of publishing tables of the planet.

In 1821 the French astronomer, Alexis Bouvard, published his tables for the three exterior major planets, Jupiter, Saturn and Uranus, having forty years of modern observations available for the latter. Bouvard had faced a major problem. No orbit could be found that would satisfy both ancient and modern observations. If the two series of observations were utilised, the discrepancies between observation and theory were excessively large, ranging from $-40''.2$ to $+32''.0$ sexagesimal. On the other hand, if the orbit was derived from the modern observations alone the largest discrepancy was reduced to $+10''.5$. Using the ancient observations alone, the largest discrepancy amounted to $73''.8$. As a consequence, Bouvard rejected the ancient observations as unreliable and based his tables solely upon the modern observations, writing in the introduction: "I leave to the future the task of determining whether the difficulty arises from the inaccuracy of the ancient observations or whether it depends upon some strange and unknown cause which may have been acting on the planet."

Laplace spoke of "some extraneous and unknown influence which has acted upon the planet."

Within a few years of the publication of Bouvard's tables, Uranus was seen to deviate more and more from the predicted positions, the discrepancy in longitude increasing alarmingly.

The opinion became widespread among astronomers that there must be an unknown body disturbing the motion of Uranus. In the late eighteen thirties Bessel persuaded one of his students, F. W. Flemming, to attempt a computation of this unknown body from the deviations in the orbit of Uranus. Unfortunately Flemming died just after the work had started.

In 1841 the discrepancy in geocentric longitude reached 70", and it became evident that these discrepancies were almost certainly due to the presence of an unknown planet.

Such was the position in 1841 when John Couch Adams, an undergraduate at St. John's College, Cambridge, penned his celebrated memorandum of July 3rd in which he recorded his determination, as soon as he had taken his degree, to investigate the irregularities in the motion of Uranus on the hypothesis that these arose from the perturbation of an unknown planet. Adams began his investigations, producing between 1843 and 1846 six solutions of ever-increasing accuracy.

In the meantime, the problem was undertaken by Urbain Jean Joseph LeVerrier (1811-1877), quite independently of and unknown to Adams. In the summer of 1845, LeVerrier began a careful study of the nature of the irregularities of the motion of Uranus and the cause of the unexpected inequalities, seeking to discover the direction and magnitude of the disturbing body. There appeared to be three possibilities. Either

(i) the irregularities were caused by a large satellite accompanying Uranus,

or (ii) a comet had suddenly disturbed the motion of Uranus,

or (iii) the irregularities were attributable to the presence of a hitherto unknown planet.

The first possibility was ruled out as the corresponding oscillations produced in the motion of Uranus would be of short period whereas precisely the opposite results from the observations.

As for the cometary theory, LeVerrier was well satisfied with the movement of Uranus between 1781 and 1820 without any recourse to any extraordinary force. However, from about 1826 onward, Uranus began to deviate so much from its predicted position that such a theory became untenable.

There remained the third hypothesis, viz., that of a body continuously acting upon Uranus changing its movement very gradually. From what was known about the solar system at that time, such a body could only be a hitherto unknown planet.

Thus motivated, LeVerrier inquired: "Is it possible that the inequalities of Uranus could be due to the action of a planet situated in the ecliptic at an average distance double that of Uranus? If so, where is this planet actually situated? What is its mass? What are the elements of the orbit which it traverses?"

On 1st June, 1846, LeVerrier presented his first results to the Academy of Sciences, and on 23rd September of that year Neptune was discovered. When details of the orbit of the new planet finally became available it was found that the elements of both Adams and LeVerrier were almost totally erroneous, as demonstrated in the following table:

Orbital Elements	LeVerrier	Adams	Neptune
Semi-major Axis (A.U.)	36.15	37.25	30.07
Eccentricity	0.1076	0.1206	0.0086
Longitude of Perihelion	284° 45'	299° 11'	44°
Mass of sun/Mass of Neptune	9300	6666	19300
True Longitude (at time of discovery)	326° 0'	329° 27'	326° 57'

For this reason it has always seemed desirable that an investigation be conducted into the problems and queries surrounding the theoretical procedures involved in the theories of both Adams and LeVerrier to see to what extent their solutions were graced by good fortune. Whereas the work of Adams has been thoroughly investigated, (Brookes, 1970), that of LeVerrier has not.

The present research is a systematic analysis of LeVerrier's theory and seeks to determine to what extent LeVerrier's solution was correct only at the time of discovery of Neptune. This work is divided into six main sections:

The first section is an examination of the validity of the various stages of LeVerrier's first solution, including re-calculations of the elements of Neptune using the same data and making the same assumptions as LeVerrier. The second and third sections represent analyses of LeVerrier's second and final solutions, respectively. Section Four includes a determination of the elements of Neptune for different values of $\alpha = \frac{a}{a'} =$ 0.49, 0.51 up to $\alpha = 0.60$, to determine to what extent LeVerrier's statement $\alpha < 0.5475$ is true. Section Five contains the derivation of orbital elements determinable at times other than 1845. The final section provides a comparison of the results of LeVerrier, Adams, Brookes and the present author in order to examine the final discrepancies in the orbit of Neptune.