III-1 Results and Discussion

The first application of Breit equation in case of positronium system was given by Crater and Alstine [74], using the following interaction,

$$V(r) = -\frac{e^2}{r}\left[1 - a\alpha^{(1)}\alpha^{(2)} - b\alpha^{(1)}r\alpha^{(2)}r\right] \tag{3-1}$$

In that case they have shown that, a pole singularity appears in the ${}^1S_0$-states of both $a=\frac{1}{2}$ and $b=\frac{1}{2}$ (corresponding to Breit interaction), and $a=1$ and $b=0$ (corresponding to Eddington and Gaunt interaction). This means that the ${}^1S_0$ equations are based on interactions which are both not acceptable [75-77]. So, they concluded that Breit interaction must be treated as perturbative term in that case.

So in positronium case, a non-perturbative treatment of Breit equation by using Breit potential will lead to a wrong result [70] in spite of the interaction between two Dirac particles is described covariantly by the exchange of virtual photons, and this interaction can be taken as Coulomb plus Breit potential in the non-relativistic limit. This interaction agrees with the retarded interaction of two classical particles if one replace the classical velocity $\alpha^i \to v_i/c$ by the Dirac matrices. In addition, Breit equation with Breit interaction is just an approximation of Bethe-Salpeter equation within ladder approximation by using a Coulomb gauge [70].

Tisibidis [28], applied Breit equation in case of strong force (quarkonia problem). He used Breit interaction beside the scalar resonable agreement between theoretical and experimental values in some of $c\bar{c}$ and $b\bar{b}$ states.

In the peresent work, Breit equation is solved by taking Breit interaction under consideration without using the perturbation theory. Then we reduced Breit equation with Breit interaction into a set of sixteen radial equations which, in turn, can be classified in accordance to the parity and spin quantum number of the different states. We applied this method to QCD problem, in which we used the following potential :- Breit interaction, strong vector Coulomb-like potential and scalar confinement. Thus the result of these procedures is a three categories of differential equations (2-61 to 2-64). From these equations we can see that, the singularity in eq. (2-64) is due to the terms $(E - kr + \frac{4}{3}\frac{\alpha_s}{r})$ and $(E - kr)$, also in eq. (2-62 and 2-63) the singularity comes due to the term $(E - kr)$ only. We can see in these two systems of equations the appearance of a new singularity $r_0 = \frac{E}{k}$, which is due to Breit interaction. But this singularity exists in position of the same order of that given by eq.(3-2). For the lowest energy levels, these singularities will not cause any problem where it lies out of the QCD range at which the wave function will be vanished. But the problem arises in eq. (2-61) due to Breit interaction. There is a singularity due to the term $(E - kr - \frac{4}{3}\frac{\alpha_s}{r})$, the pole at

$$r_0 = \frac{1}{2}\frac{E}{k} - \sqrt{\frac{1}{4}\left(\frac{E}{k}\right)^2 - \frac{4}{3}\frac{\alpha_s}{k}} \tag{3-3}$$

From this equation we can see that, Breit interaction produces a new singularity inside the range of interaction. This singularity does not appear in the initial Breit equation or Breit potential, but it appears due to the reduction procedures with the presence of Breit potential. This singularity appears at $r_0 \prec 0.2 GeV^{-1}$ only for the states with $J = L$ and $S = 0$.

Our next aim, is to determine the parameters α_s and k that appear in the potential and the mass parameters m_c and m_b. This is done by solving the equations (2-62 to 2-64) numerically and compare our results with the corresponding experimental data. The experimental states for the two systems $c\bar{c}$ and $b\bar{b}$ which are described by this equations are enough for this aim, and there is only one experimental energy state that can be described by eq. (2-61). In the present work we will avoid eq. (2-61) and restrict our attention to the other equations which can be solved easily.

III-2 Application to charmonium and bottomonium systems

For the light quarks u, d and s of masses $2 \le m_u \le 8$, $5 \le m_d \le 15$ and $100 \le m_s \le 300$ MeV [28]. The use of the non-relativistic potential and non-relativistic methods to study the spectroscopy of the mesons composed of these quarks are no longer satisfied, since, the speed of these systems is too large to allow using approximated relativistic equations. So a fully relativistic treatment is required to describe that systems.

wave functions and compare our results with the corresponding published values. The best parameters that produce exactly the experimental data are given in tables (3-1) and (3-2).

STATE		$k(GeV^2)$	α_s	$mass(GeV)$
$c\bar{c}$	$J/\psi(1S)$	0.128	0.478	1.180
	$\psi(2S)$	0.129	0.478	1.341
	$\psi(3S)$	0.128	0.478	1.262
	$\psi(4S)$	0.130	0.477	1.288
	$\psi(5S)$	0.128	0.477	1.260
	$\psi(6S)$	0.130	0.478	1.288
	$\chi_{c0}(1P)$	0.130	0.477	1.471
	$\chi_{c1}(1P)$	0.128	0.478	1.544
	$\chi_{c2}(1P)$	0.130	0.477	1.330

Table (3-1): The best parameters which produce the experimental energy levels of $c\bar{c}$ system.

STATE		$k(GeV^2)$	α_s	$mass(GeV)$
$b\bar{b}$	$\gamma(1S)$	0.128	0.478	4.530
	$\gamma(2S)$	0.130	0.478	4.699
	$\gamma(3S)$	0.130	0.478	4.765
	$\gamma(4S)$	0.130	0.478	4.789
	$\gamma(5S)$	0.130	0.478	4.852
	$\gamma(6S)$	0.130	0.478	4.852
	$\chi_{b0}(1P)$	0.130	0.478	4.852
	$\chi_{b1}(1P)$	0.128	0.478	4.882
	$\chi_{b2}(1P)$	0.130	0.478	4.673
	$\chi_{b0}(2P)$	1.290	0.478	4.892
	$\chi_{b1}(2P)$	0.130	0.478	4.911
	$\chi_{b2}(2P)$	0.130	0.478	4.753

Table (3-2): The best parameters which produce the experimental energy levels of $b\bar{b}$ system.

From tables (3-1) and (3-2), one can see that, in case of $c\bar{c}$; the calculations give a satisfactory results if we consider the states $\psi(3770), \psi(4040), \psi(4160)$ and $\psi(4415)$ assigned as $\psi(3S)$, $\psi(4S)$, $\psi(5S)$ and $\psi(6S)$, respectively. Also the related mass to that system have a relatively large error, this is due to the states $J/\psi(1S)$ and $\chi_{c1}(1P)$, but the mass of the $b\bar{b}$ system has a relatively small error, which reflect the fact that Breit interaction give a good results in the case of a heaviest mass systems which ensure our assumptions. From the behavior of the wave function versus r, one can see that the size of the quarkonium system increases as the resonance masses increased. See for example figures (3-1) and (3-2).

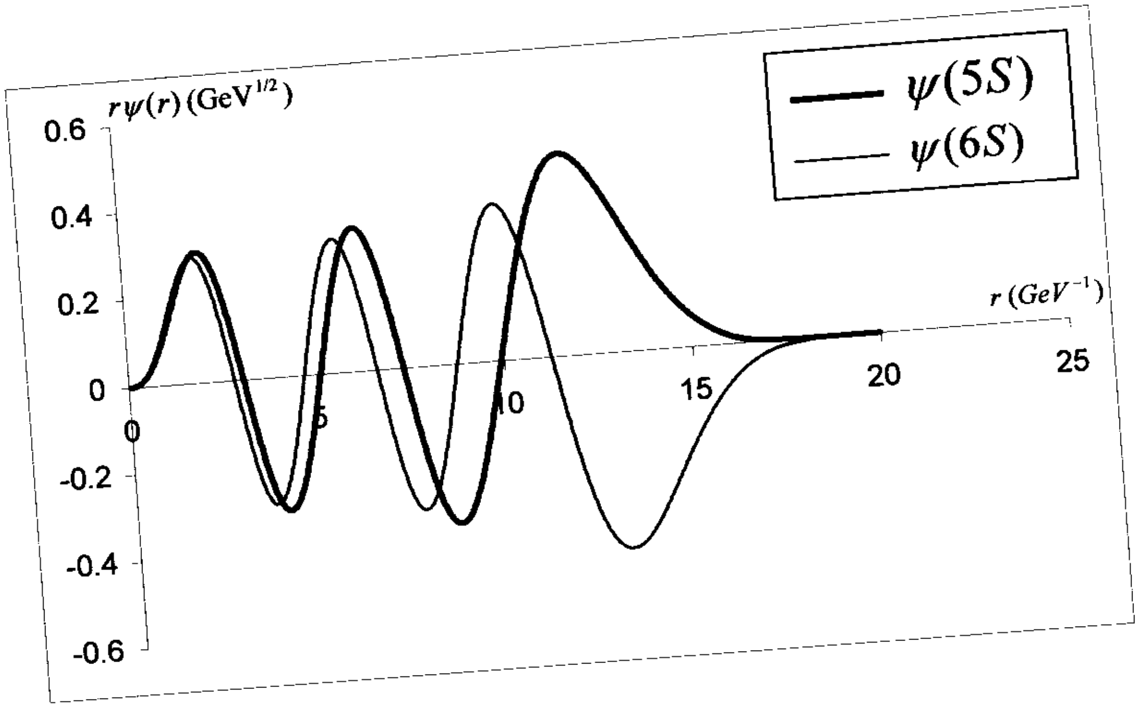

Figure (3-1): Wave functions of states $\psi(5S)$ and $\psi(6S)$.

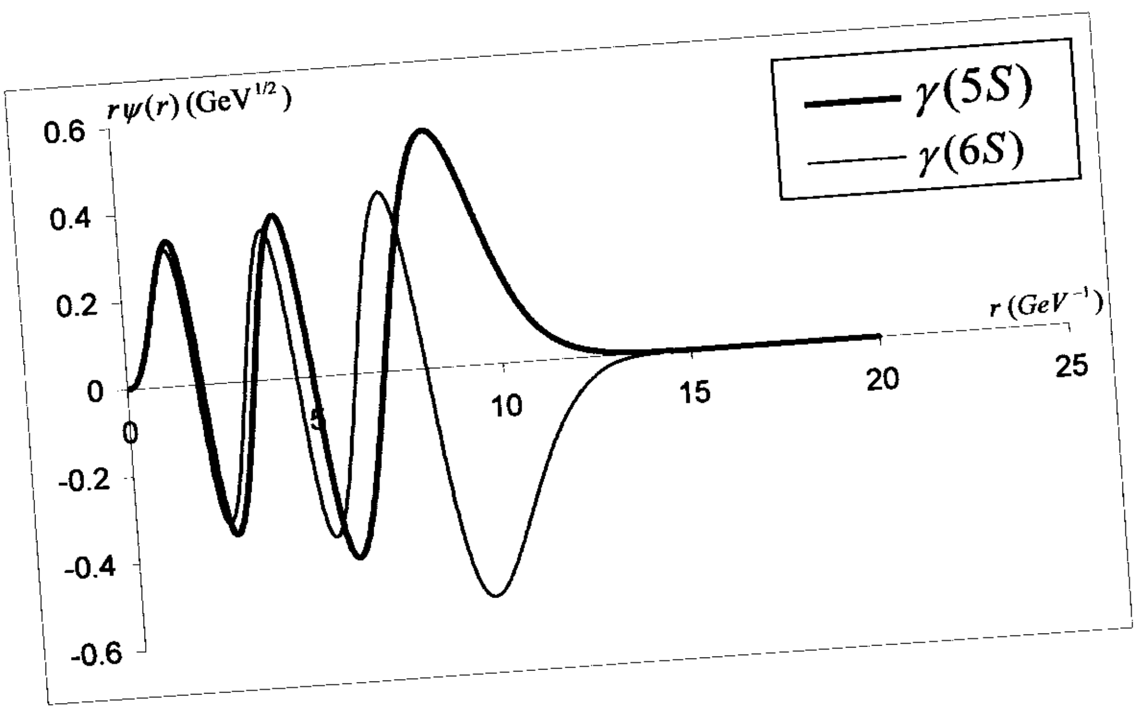

Figure (3-2): Wave functions of states $\gamma(5S)$ and $\gamma(6S)$.