

Chapter (IV)
Discussion Of Results
and Conclusions

IV-1. The energy levels :

At the beginning of our discussion let us state clearly the four parameters of the rotation - vibration model which their initial values are taken from experiments. Thus four parameters are :-

- 1) the moment of inertia $\mathcal{I} = \frac{\hbar^2}{2J_0} \simeq \frac{E_{\text{rot}}}{3}$, which is taken from the experimental energy of the first rotational states,
- 2) the equilibrium deformation β_0 of the nucleus, which is obtained from the $B(E2)$ -value for the first rotational states,
- 3) the γ -vibrational energy E_γ , which is fitted to the energy of the second excited 2^+ state,
- 4) the β -vibrational energy E_β , which is taken from the first excited 0^+ -state.

In fig.1 one can see a pictorial illustration of how the initial estimated values of these parameters are determined⁸⁵⁾.

The energies and eigen vectors have been obtained through the diagonalization of the Hamiltonian of the rotation vibration model (RVM). The parameters E_{rot} , E_γ and E_β , in equ. II-71, have adjusted to reproduce quite good, as can as possible, the experimental values of the lowest energy levels, especially these levels which associated with g.s.-, γ - and β -bands. The justification of the parameters : E_{rot} , E_γ and E_β , have been done by iteration the RVM-programe several times with a different values for these parameters. In other words, these

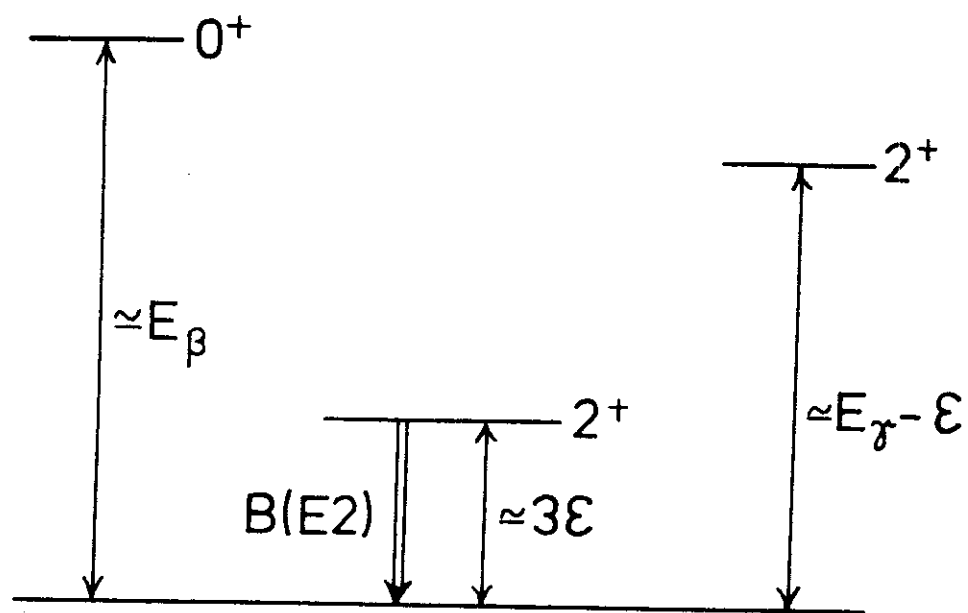


Fig. 1

parameters are considered as input parameters in the RVM - programme. The best values of these parameters are tabulated in table 1.

The energy level's schemes of ^{150}Nd , $^{152,154}\text{Sm}$, $^{154-160}\text{Gd}$, ^{160}Dy , $^{172-174}\text{Yb}$, $^{174-180}\text{Hf}$ and $^{230,232}\text{Th}$ isotopes are given in figures 2 $\rightarrow$ 9. From these figures one may notice that there is a good fitting between the calculated energy - levels and the corresponding experimental data¹¹⁵⁾, especially, for the $I = 0, 2, 4, 6$ levels in the g.s.-band, the $I = 2, 4$ levels in the γ -band and the $I = 0, 2$ in the β -band. This is for all considered nuclei. At the same time one can observe that as I increases, the agreement, between the calculated energy levels and the corresponding experimental values, decreases.

The energy levels values for ^{154}Gd - isotope, as calculated by the RVM, are given in table (2) where they are compared with the available experimental data^{36,115)} and other theoretical calculations^{45,53,55,116)}. The comparison shows that the agreement between the RVM-results and the corresponding experimental data^{36,115)} is better than those which associated with DPPQ - and DNSB-results³⁶⁾, especially, in the lowest bands. At the same time one can observe the superiority of the RVM - results over the IBA - results, especially, those results which related to Gastanos et al¹¹⁶⁾ and van Isacker et al.⁵³⁾ versions. There is a pictorial description for this comparison in fig. 10. Where the experimental values have been taken from table 2.

Table (1)

Isotope	$E_{\text{rot}} (3\epsilon)$ (MeV)	E_{β} (MeV)	E_{γ} (MeV)
^{150}Nd	0.1007	0.6740	1.0110
^{152}Sm	0.0946	0.6805	1.0350
^{154}Sm	0.0702	1.0930	1.4066
^{154}Gd	0.0898	0.6710	0.9466
^{156}Gd	0.0720	1.0400	1.1147
^{158}Gd	0.0699	1.1961	1.1597
^{160}Gd	0.0650	0.9460	0.9621
^{160}Dy	0.0750	1.2627	0.9072
^{166}Er	0.0700	1.4599	0.7310
^{172}Yb	0.0661	1.0400	1.4350
^{174}Yb	0.0678	1.4820	1.5990
^{174}Hf	0.0769	0.8283	1.1945
^{176}Hf	0.0770	1.1499	1.3092
^{178}Hf	0.0762	1.1880	1.1386
^{180}Hf	0.0802	1.1072	1.1730
^{230}Th	0.0460	0.6350	0.7620
^{232}Th	0.0439	0.7100	0.7668

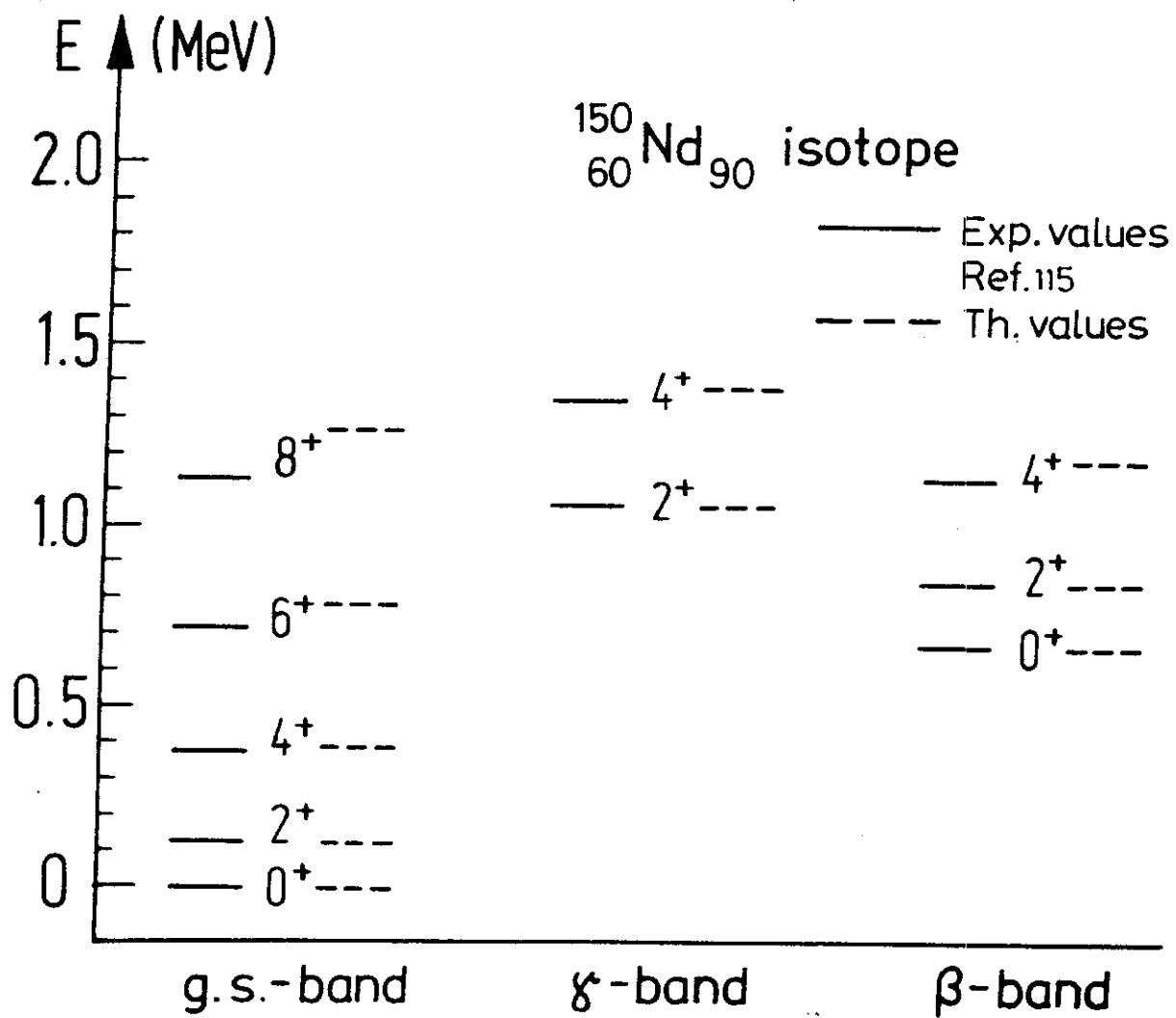


Fig. 2

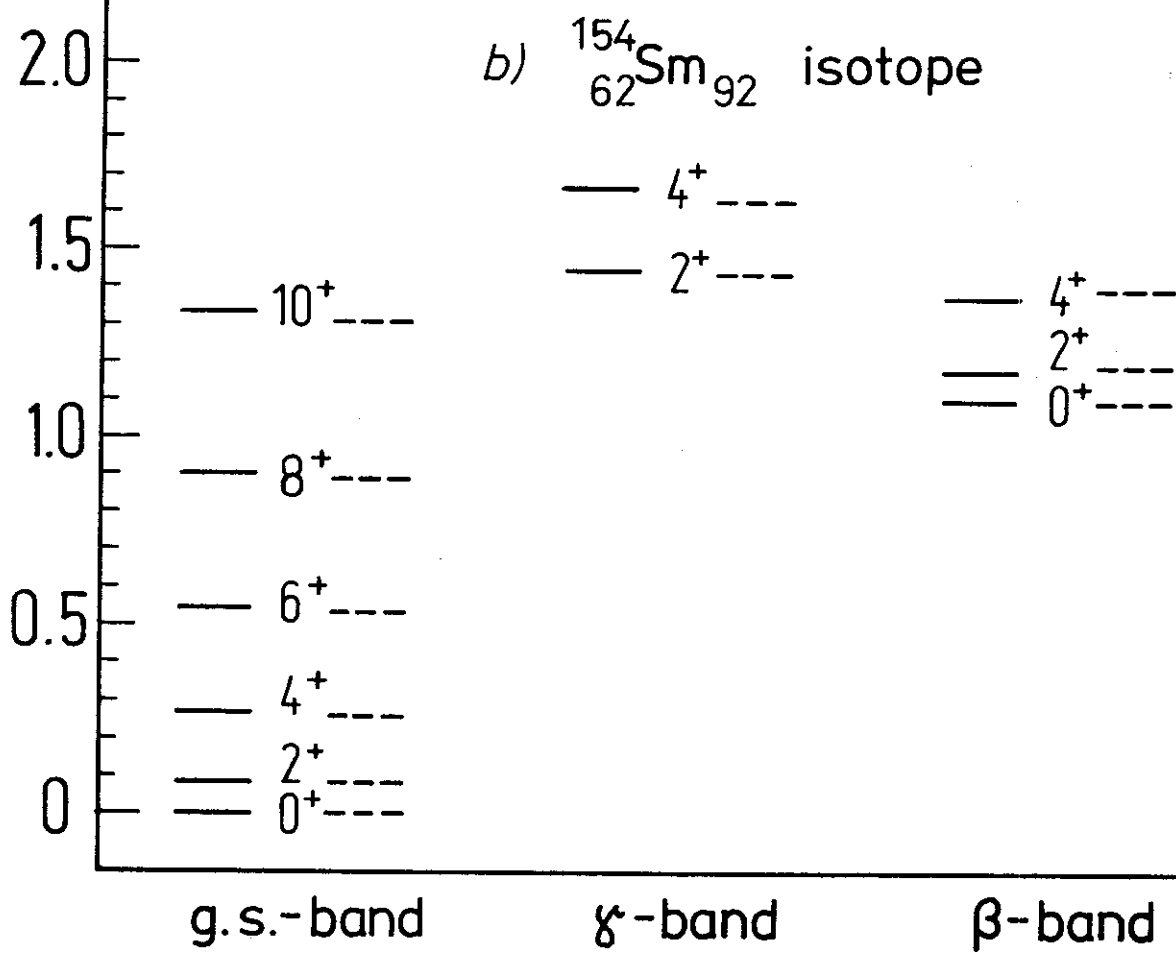
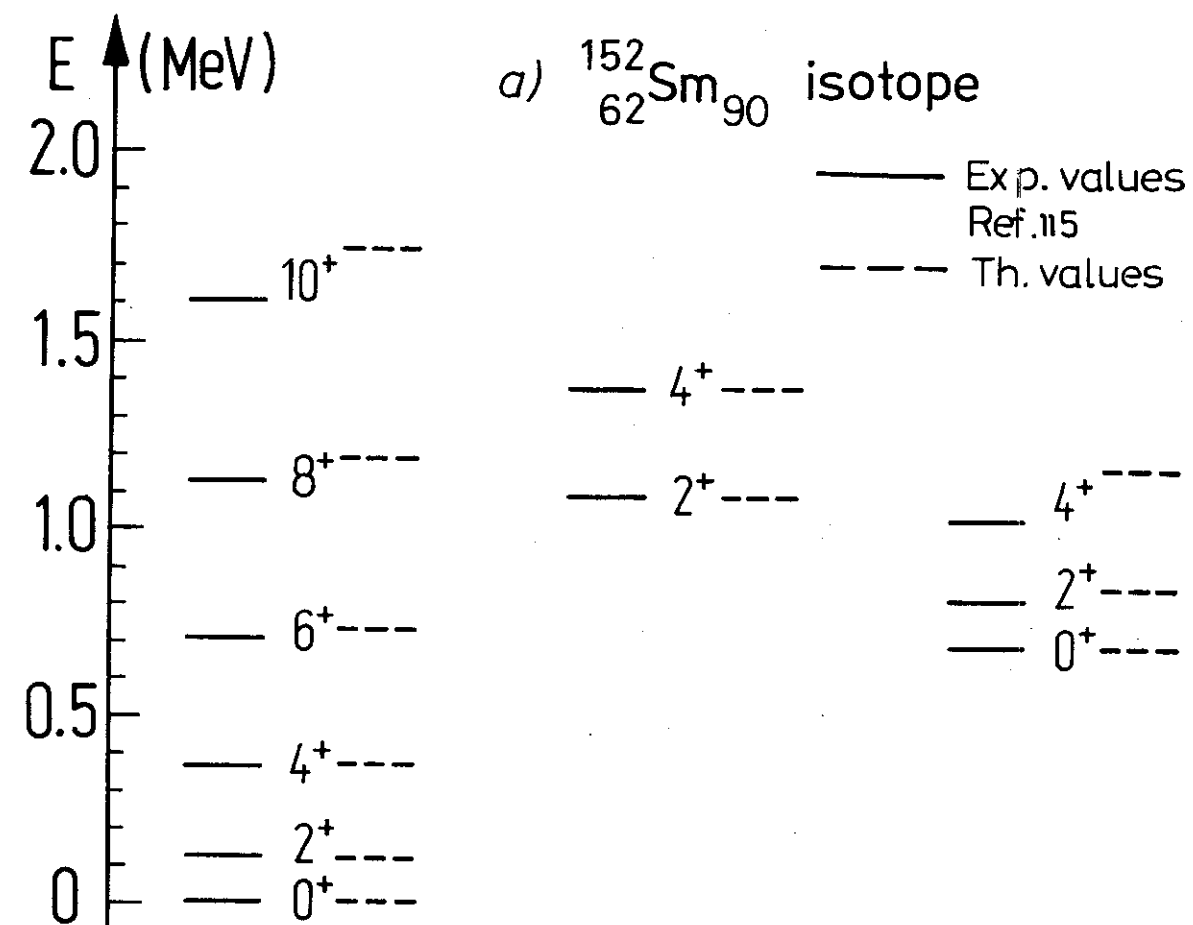


Fig. 3

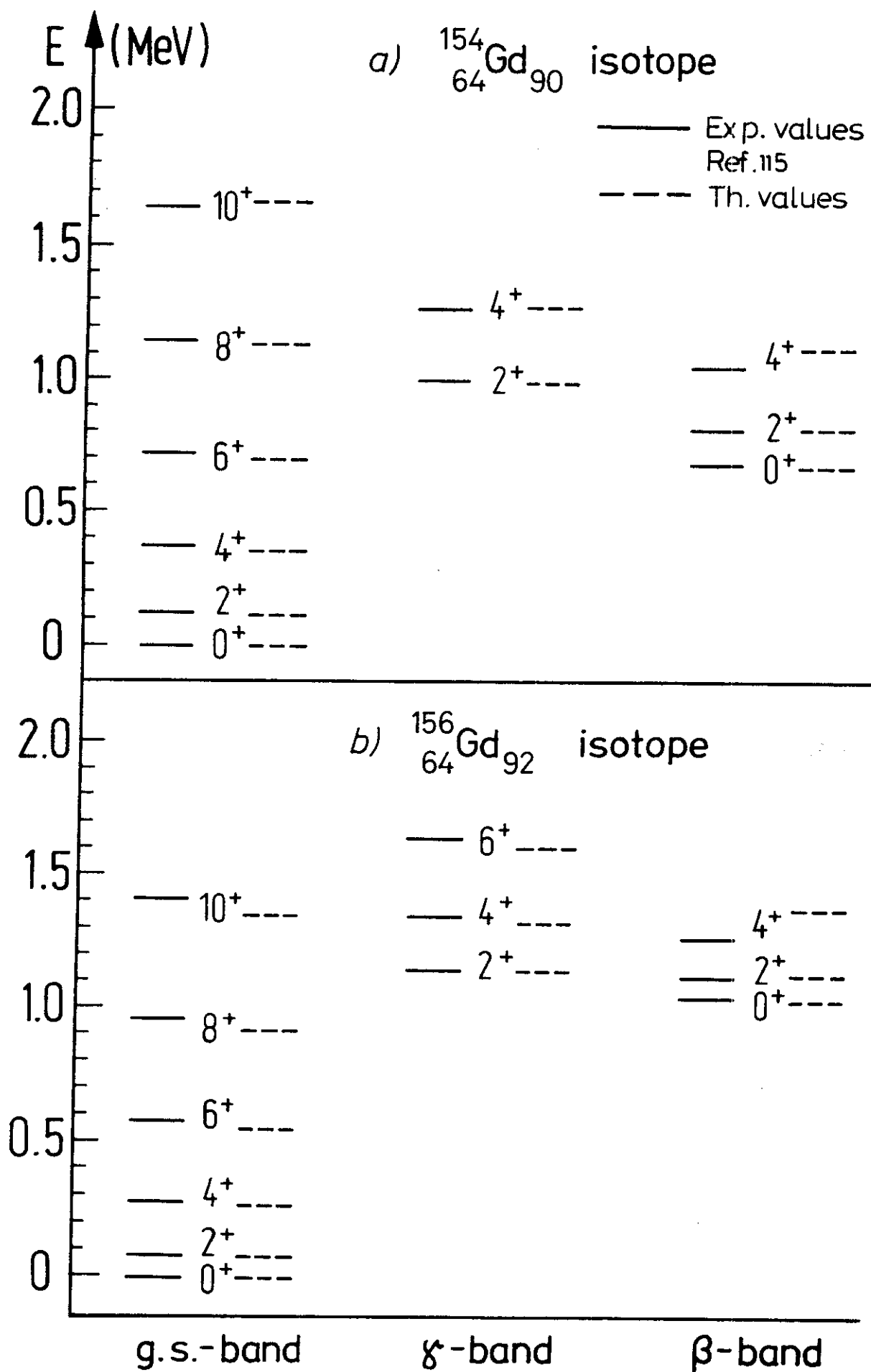


Fig. 4

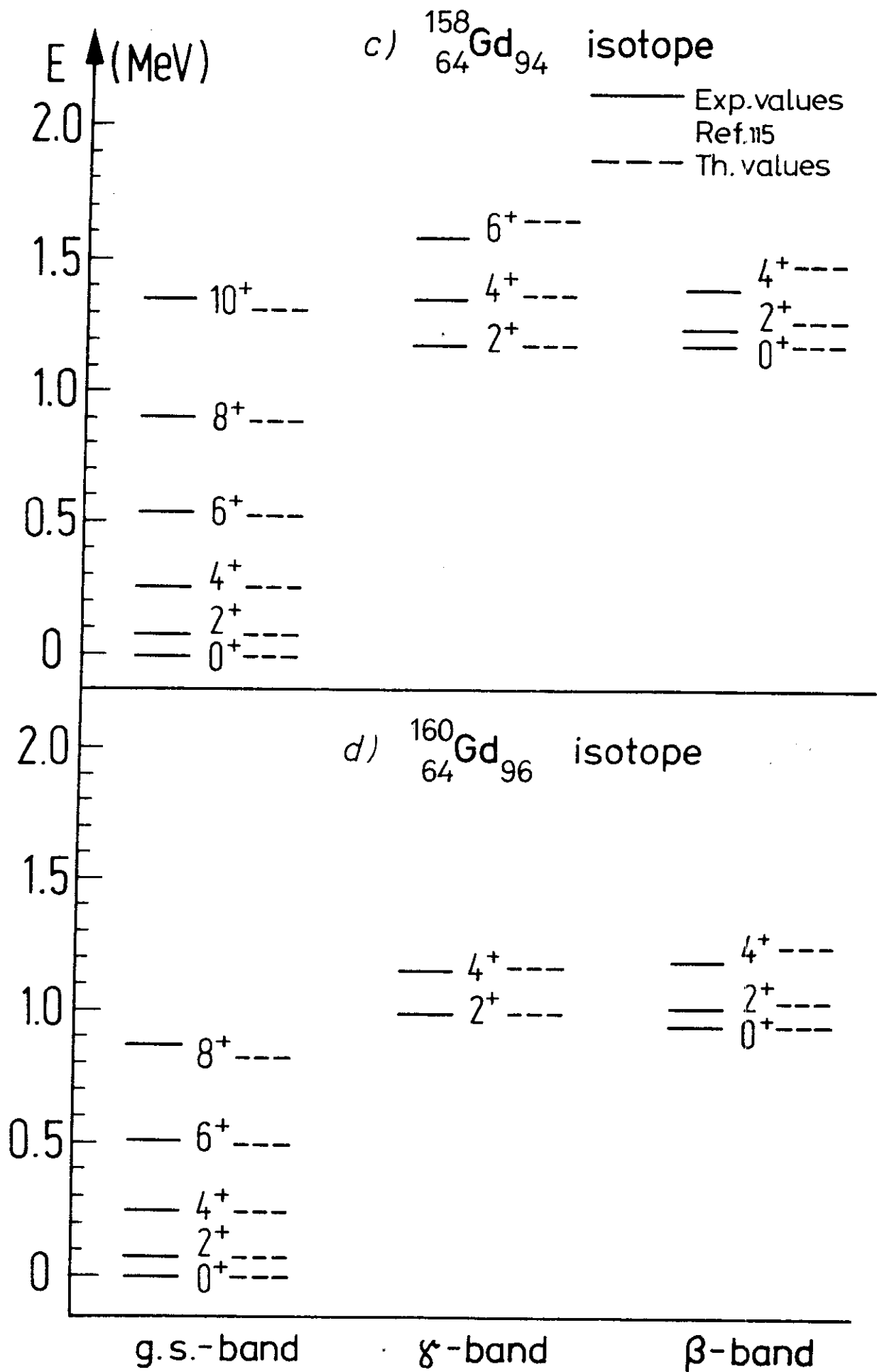


Fig. 4

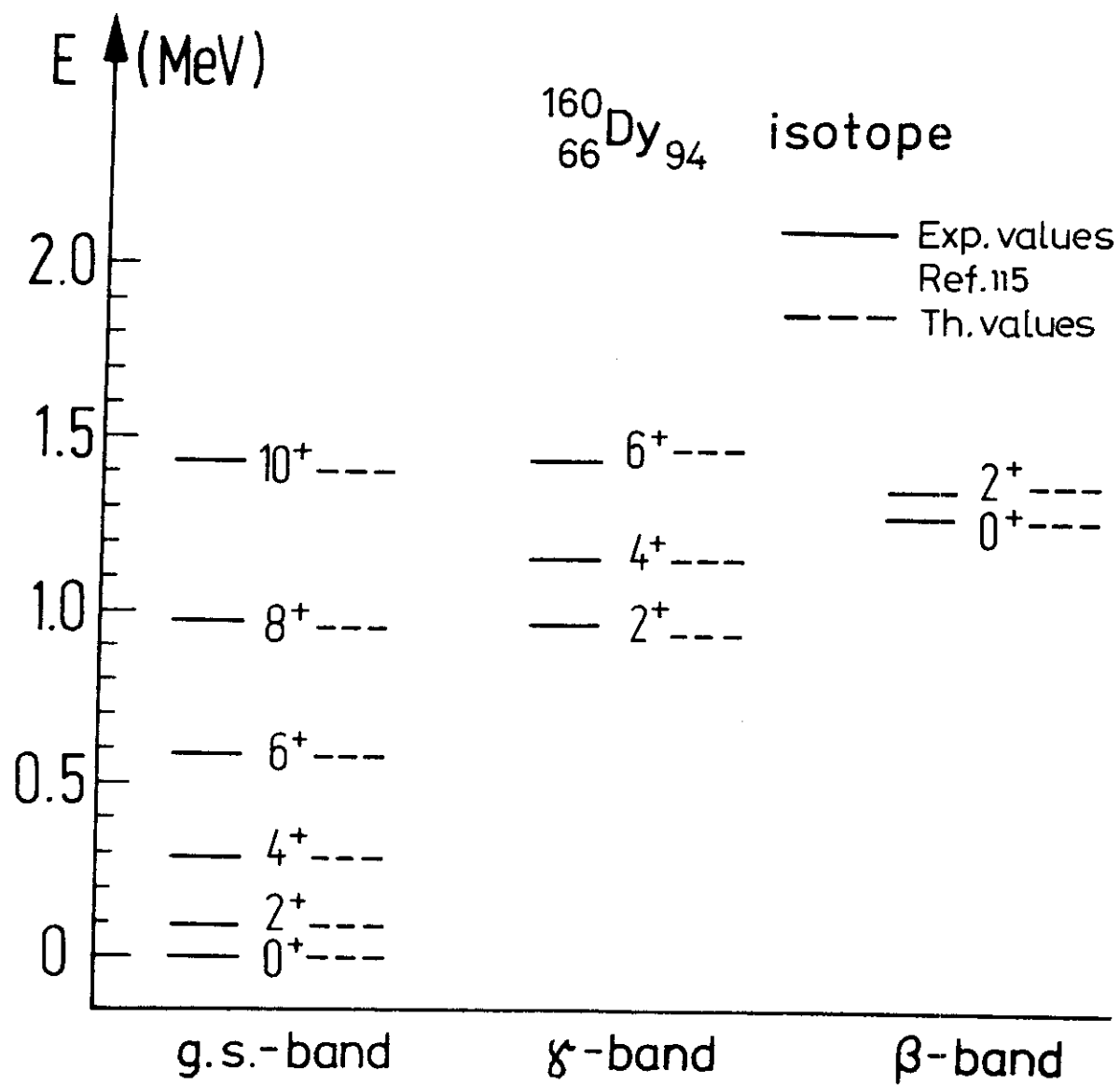


Fig. 5

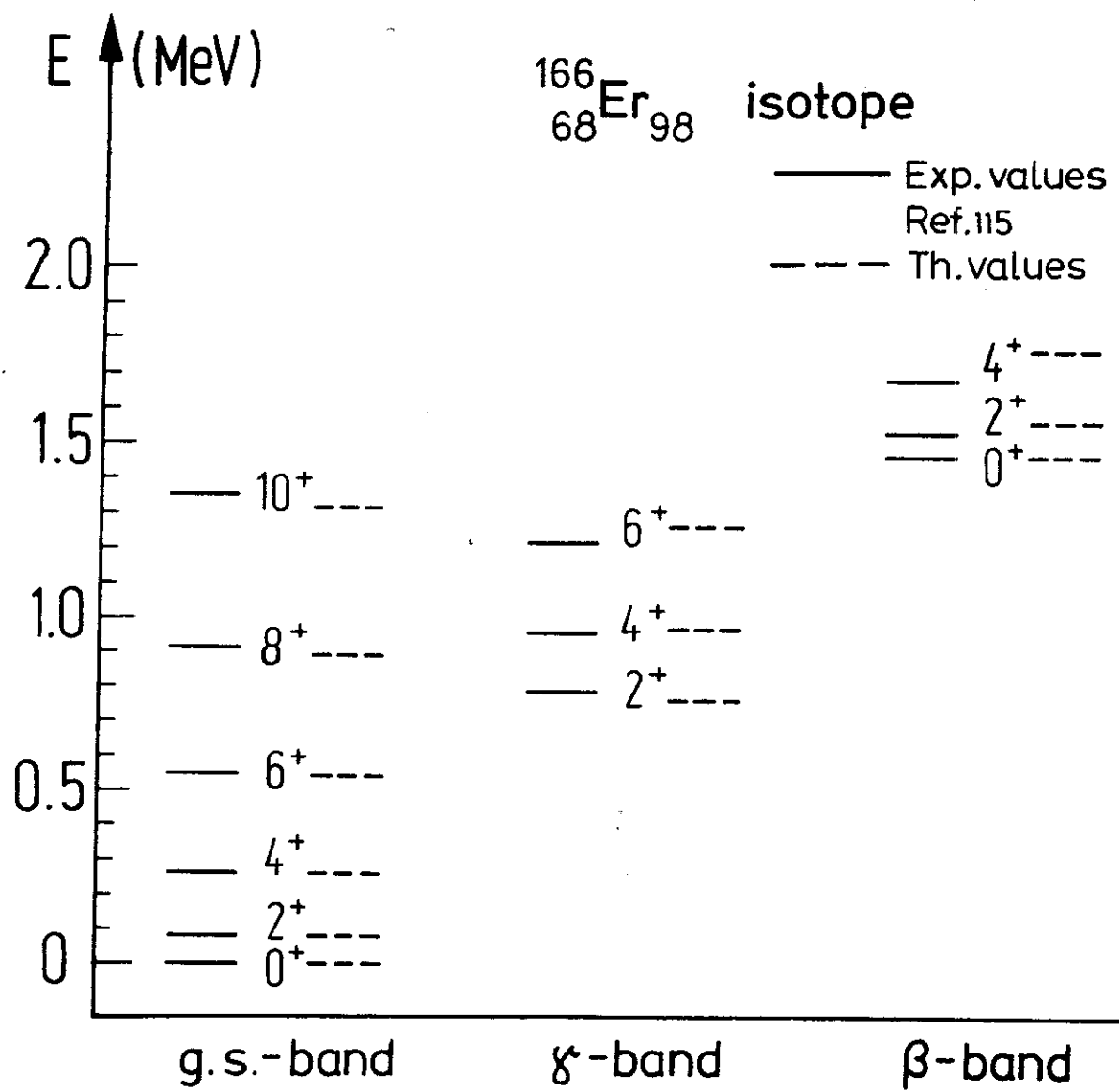


Fig.6

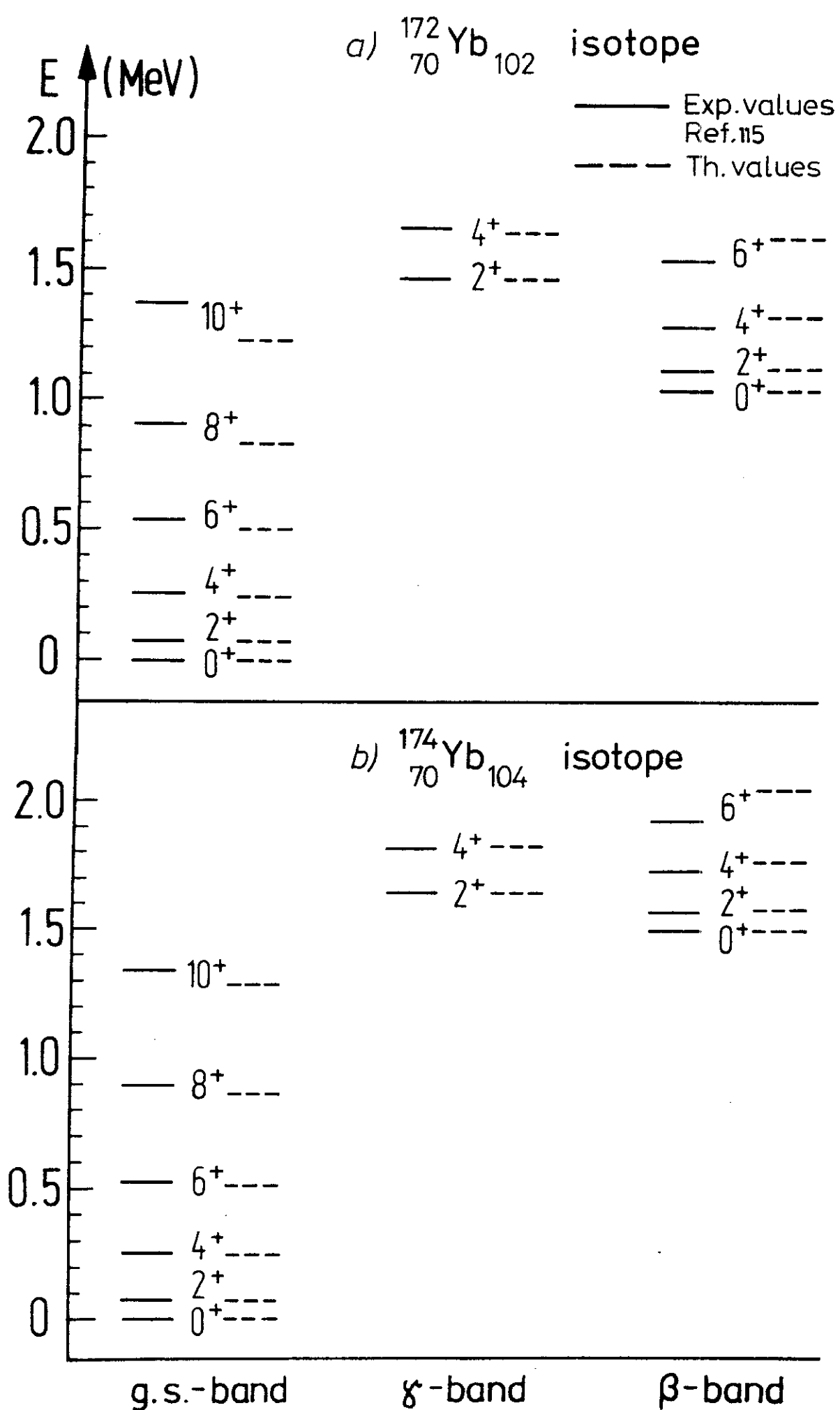


Fig. 7

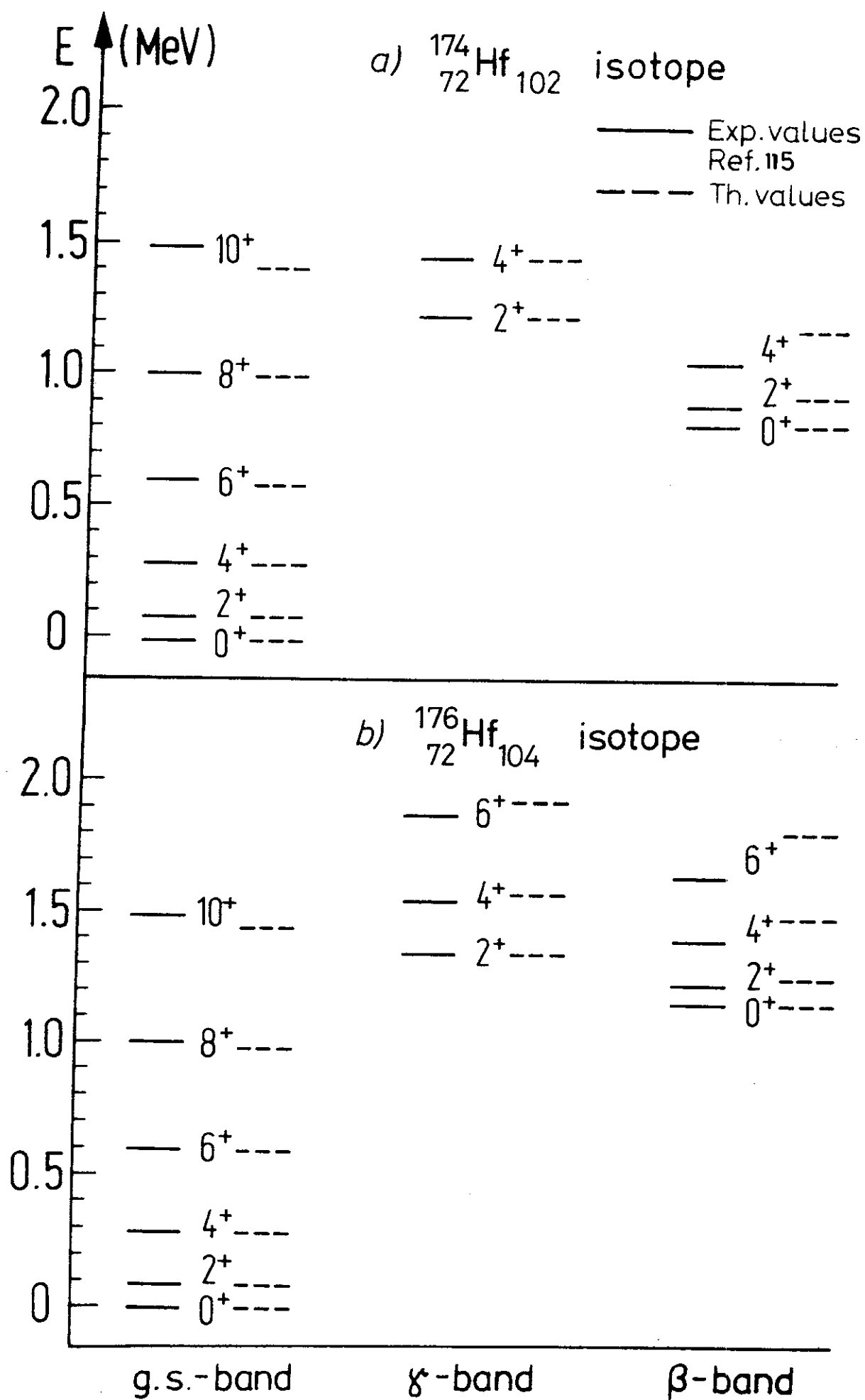


Fig. 8

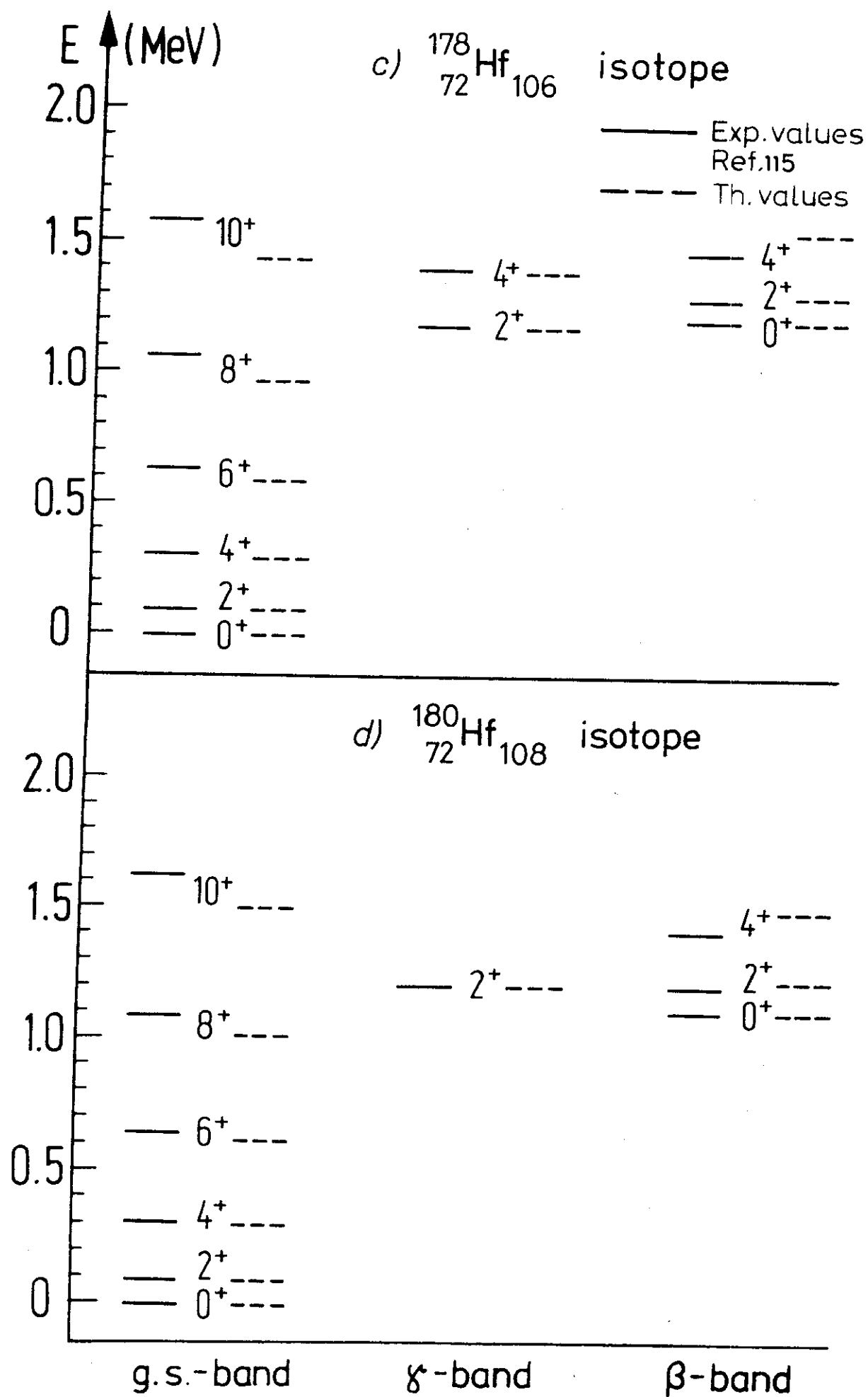


Fig. 8

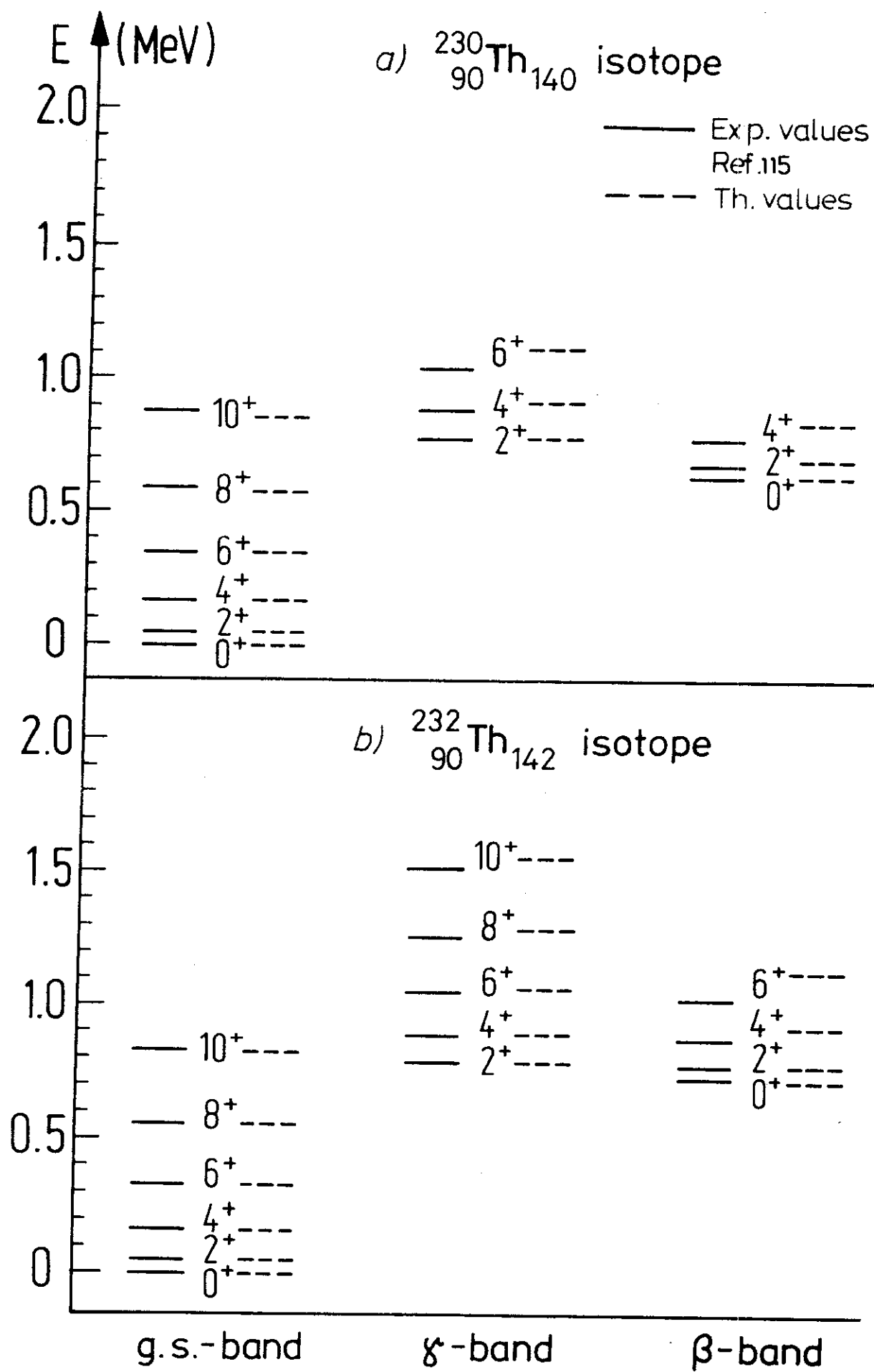


Fig.9

Table(2):- (¹⁵⁴Gd-isotope)

Classification			Energy levels (MeV)							
Band	K	I	Exp.*	Present work (RVM)	IBA				DPPQ ▲ e	DNSB ▼ e
					Castanos et al. a	von Isak- ker et al. b	Lioas et al. c	C.Girits et al. d		
g.s.	0	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		2	0.123	0.113	0.104	0.095	0.103	0.100	0.126	0.167
		4	0.371	0.352	0.337	0.315	0.343	0.334	0.313	0.450
		6	0.718	0.694	0.688	0.650	0.719	0.699	0.585	0.821
		8	1.145	1.129	1.145	1.110	1.232	1.195	—	—
		10	1.637	1.653	1.700	1.580	1.881	1.819	—	—
3	0	0	0.680	0.671	0.748	0.650	0.679	0.683	0.985	0.528
		2	0.815	0.818	0.885	0.780	0.783	0.797	1.180	0.725
		4	1.047	1.117	1.148	1.020	1.026	1.055	1.391	1.155
		6	1.366	—	1.524	1.420	1.408	1.451	1.680	1.705
		8	1.757	2.326	2.001	1.915	1.928	1.980	—	—
γ	2	2	0.996	0.989	1.050	1.000	1.008	1.004	1.506	0.604
		4	1.264	1.276	1.344	1.260	1.251	1.256	1.776	0.888
		6	1.607	—	1.742	1.640	1.631	1.640	2.176	1.275
2β	0	0	1.295	1.342	1.338	1.300	—	1.199	1.842	1.093
		2	1.418	1.533	1.530	1.400	—	1.366	2.156	1.463
		4	1.698	1.929	1.832	1.620	—	1.671	2.490	2.035
3γ	2	2	1.531	2.890	1.668	—	—	1.500	2.522	1.210
		4	1.790	3.300	1.975	—	—	1.828	3.082	1.624
2γ	4	4	1.640	1.990	—	—	—	—	2.843	1.398
		6	1.912	2.504	—	—	—	—	3.398	1.870
2γ	0	0	—	1.893	—	—	—	—	2.723	1.434
		2	2.081	2.024	—	—	—	—	3.042	1.635
		4	2.230	2.313	—	—	—	—	3.367	1.941
3β	0	0	—	3.786	—	—	—	—	2.969	2.233
		2	2.277	3.938	—	—	—	—	3.317	2.762
		4	—	4.300	—	—	—	—	3.620	3.441
γ+2β	2	2	—	2.355	—	—	—	—	3.903	2.096
		4	—	2.857	—	—	—	—	4.585	2.852
2γ+β	4	4	—	2.665	—	—	—	—	4.263	—

a Ref. 116 b Ref.53 c Ref. 55 d Ref.45 e Ref. 36

▲ DPPQ is the abbreviation of the dynamic pairing quadrupole model

▼ DNSB is the abbreviation of the dynamic Nilsson, Strutinsky and Belyaer model

*Ref 36 and 115

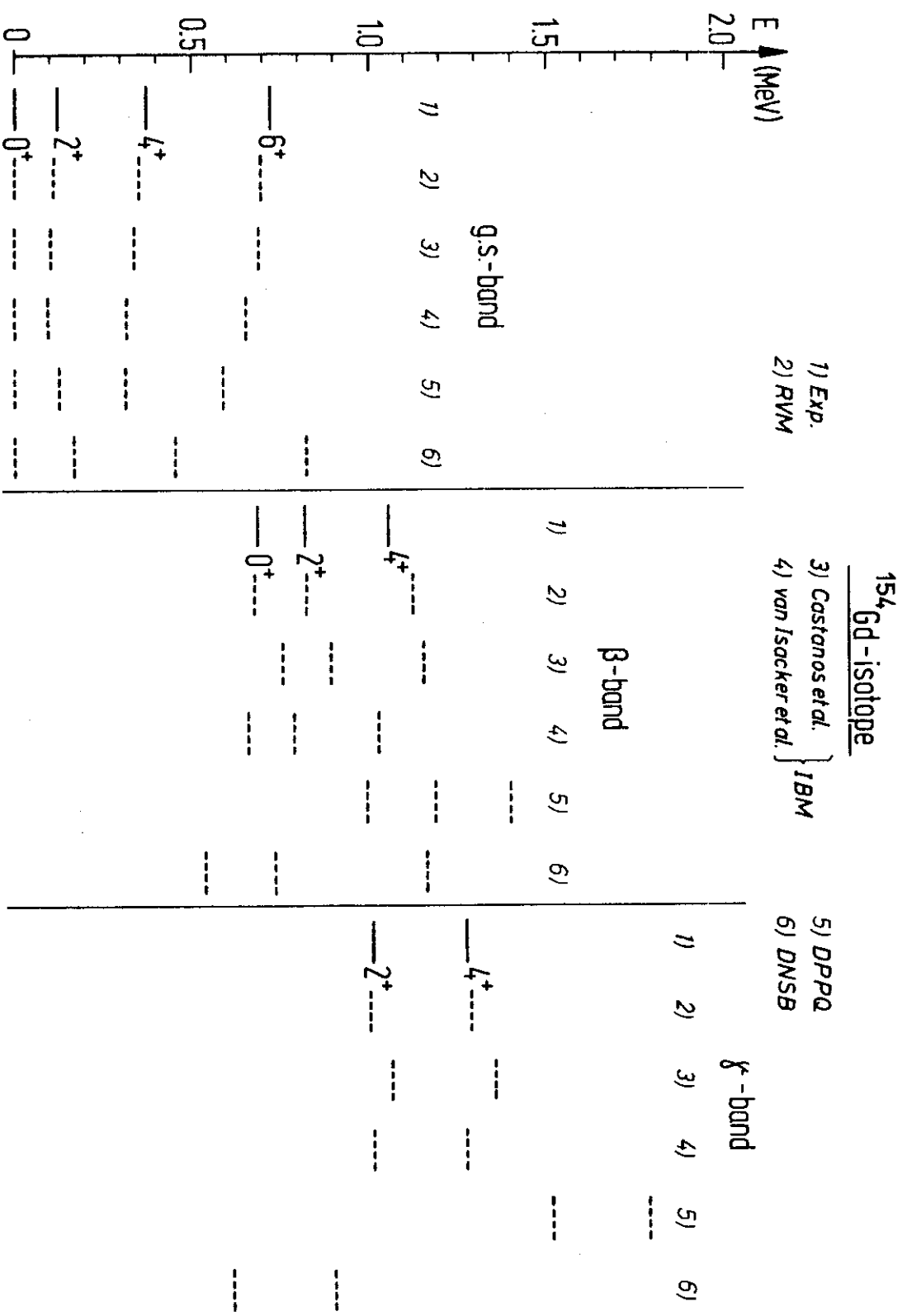


Fig. 10

Table 3 represent a comparison between a different theoretical results and the corresponding experimental data¹¹⁵⁾ of the energy levels of the $^{154-160}\text{Gd}$ - isotopes. One can observe that the projection model (anisotropic version)⁵⁵⁾-, the RVM (present work)-, the IBA-1⁵⁵⁾ and the projection model (isotropic version)⁵⁵⁾ show, respectively, an adequate agreement with the corresponding experimental data¹⁵⁵⁾. At the same time and with some exceptions, it is obvious that the calculated energies of the γ - and β - bands, in case of the projection model (isotropic version), are lower than those which have been calculated by the other versions. In addition the calculated energy levels of the β -band, for all versions, tends to be higher than the corresponding experimental data, especially for $I \geq 6$.

It is easy to observe that from table 4 the preferable theoretical calculations of the energy levels of the ^{172}Yb - isotope are those which associated with the IBA-1⁶⁷⁾. These calculations, with a minor exceptions, give an agreement with the corresponding experimental data^{155,67)} better than those obtained by RVM. At the same time the energy levels of the higher bands : $\beta\beta$ -, $\gamma+\gamma$, $\gamma-\gamma$ and $\beta\gamma$ bands, tends to be too high compartive to the corresponding experimental data^{115,67)}. This is true for both the RVM - and IBA-1⁶⁷⁾ results.

Tables 5 and 6 show a comparison between the theoretical results, which associated with the rotation - vibration model and the coherent states model⁷⁵⁾, and the corresponding

Table(3): (Gd -isotopes)

Isotope	Classification			Energy levels (MeV)				
				Exp.*	Present work (RVM)	Projection model **		IBA-1**
	Band	K	I			Isotr.	Anisotr.	
^{154}Gd	g.s.	0	0	0.0	0.0	0.0	0.0	0.0
			2	0.123	0.113	0.123	0.123	0.103
			4	0.371	0.352	0.371	0.369	0.343
			6	0.718	0.694	0.704	0.702	0.719
			8	1.145	1.129	1.097	1.099	1.232
			10	1.637	1.653	1.529	1.543	1.881
	B	0	0	0.681	0.671	0.533	0.681	0.679
			2	0.816	0.818	0.741	0.876	0.783
			4	1.048	1.117	1.077	1.195	1.026
			6	1.366	—	1.484	1.591	1.408
			8	1.757	2.336	1.934	2.040	1.928
			10	2.195	3.099	2.411	2.528	2.585
	Y	2	2	0.996	0.989	0.603	0.995	1.008
			4	1.264	1.276	0.895	1.293	1.251
			6	1.607	—	1.258	1.654	1.631
^{156}Gd	g.s.	0	0	0.0	0.0	0.0	0.0	0.0
			2	0.089	0.084	0.089	0.089	0.084
			4	0.288	0.274	0.288	0.293	0.281
			6	0.585	0.557	0.582	0.606	0.590
			8	0.965	0.921	0.955	1.018	1.012
			10	1.416	1.357	1.389	1.519	1.546
	B	0	0	1.049	1.040	0.717	1.047	1.035
			2	1.129	1.135	0.835	1.153	1.120
			4	1.298	1.385	1.088	1.396	1.320
			6	1.540	1.738	1.444	1.763	1.633
			8	1.848	2.193	1.879	2.238	2.060
			10	2.220	2.742	2.371	2.805	2.599
	Y	2	2	1.154	1.150	0.757	1.155	1.157
			4	1.355	1.329	0.976	1.369	1.356
			6	1.644	1.615	1.292	1.694	1.669
^{158}Gd	g.s.	0	0	0.0	0.0	0.0	0.0	0.0
			2	0.080	0.080	0.080	0.080	0.076
			4	0.261	0.260	0.261	0.263	0.254
			6	0.539	0.532	0.538	0.546	0.533
			8	0.904	0.885	0.900	0.921	0.915
			10	1.351	1.310	1.335	1.382	1.398
	B	0	0	1.196	1.196	0.902	1.176	1.200
			2	1.260	1.288	0.998	1.268	1.278
			4	1.407	1.501	1.216	1.478	1.458
			6	—	1.826	1.540	1.800	1.743
			8	—	2.256	1.956	2.224	2.131
			10	—	2.769	2.448	2.738	2.624
	Y	2	2	1.187	1.187	0.934	1.207	1.191
			4	1.358	1.376	1.129	1.400	1.370
			6	—	1.655	1.422	1.696	1.652

Table(3): (continued)

Isotope	Classification	Energy levels (mev)				
		Exp.*	Present work (RVM)	Projection model **		IBA-1 **
				Isotr.	Anisotr.	
¹⁶⁰ Gd	g.s. 0 0 2 2 4 4 6 6 8 8 10 10	0.0	0.0	0.0	0.0	0.0
		0.075	0.075	0.075	0.075	0.073
		0.248	0.245	0.247	0.248	0.243
		0.514	0.497	0.507	0.513	0.511
		0.868	0.822	0.844	0.862	0.876
		—	1.212	1.248	1.285	1.338
		—	—	—	—	—
		—	—	—	—	—
		—	—	—	—	—
		—	—	—	—	—
	β 0 0 2 2 4 4 6 6 8 8 10 10	—	0.946	0.791	0.920	0.920
		—	1.036	0.883	1.010	0.993
		1.185	1.243	1.091	1.216	1.164
		—	1.556	1.399	1.526	1.433
		—	1.961	1.788	1.929	1.798
		—	2.451	2.245	2.410	2.261
		—	—	—	—	—
		—	—	—	—	—
		—	—	—	—	—
		—	—	—	—	—
	γ 2 2 4 4 6 6	0.988	0.988	0.822	0.988	0.995
		1.148	1.163	1.007	1.170	1.169
		—	1.556	1.283	1.447	1.441

*Ref. 115

**Ref. 55

Table(4):- (^{172}Yb -isotope)

Classification			Energy levels (MeV)		
Band	K	I	Exp.*	Present work (RVM)	IBA-1**
g.s.	0	0	0.0	0.0	0.0
		2	0.079	0.075	0.075
		4	0.260	0.244	0.248
		6	0.540	0.500	0.516
		8	0.912	0.831	0.875
		10	1.370	1.230	1.320
		12	1.907	1.692	1.845
		14	2.518	2.213	2.446
		16	3.198	3.792	3.197
β	0	0	1.043	1.040	1.023
		2	1.118	1.127	1.099
		4	1.287	1.325	1.272
		6	1.538	1.620	1.540
		8	1.854	1.995	1.890
		10	2.213	2.438	2.333
		12	2.607	2.941	2.843
γ	2	2	1.466	1.461	1.466
		4	1.658	1.641	1.658
$\beta\beta$	0	0	1.405	2.080	1.907
		2	1.477	2.182	2.010
		4	1.633	2.423	2.165
$\gamma\gamma$	4	4	2.073	2.925	2.772
γ - γ		0	1.794	2.870	2.559
		2	1.850	2.951	2.737
		4	1.975	3.136	2.969
		6	2.156	3.417	3.257
$\beta\gamma$	2	2	1.609	4.333	2.268
		4	1.803	4.534	2.463

* Ref. 115 , Ref. 67 , ** Ref. 67

Table (5),
(HP-isotopes)

Isotope	Classification			Energy levels (MeV)		
	Band	K	I	Exp. *	Present work (RVM)	CSM **
^{174}Hf	g.s.	0	0	0.0	0.0	0.0
			2	0.091	0.091	0.091
			4	0.297	0.294	0.297
			6	0.608	0.587	0.608
			8	1.009	0.963	1.009
			10	1.502	1.412	1.487
	β	0	0	0.828	0.828	0.823
			2	0.900	0.940	0.898
			4	1.062	1.187	1.070
			6	—	1.536	1.332
			8	—	1.962	1.676
	γ	2	2	1.227	1.227	1.253
			4	1.449	1.448	1.434
			6	—	1.780	1.710
^{176}Hf	g.s.	0	0	0.0	0.0	0.0
			2	0.088	0.088	0.088
			4	0.297	0.288	0.290
			6	0.597	0.587	0.597
			8	0.998	0.972	0.998
			10	—	1.435	1.482
	β	0	0	1.150	1.150	1.146
			2	1.227	1.253	1.222
			4	1.390	1.480	1.399
			6	—	1.799	1.669
			8	—	2.383	2.027
	γ	2	2	1.341	1.341	1.351
			4	1.540	1.563	1.531
			6	—	1.914	1.807
^{178}Hf	g.s.	0	0	0.0	0.0	0.0
			2	0.093	0.088	0.093
			4	0.307	0.286	0.307
			6	0.632	0.583	0.632
			8	1.058	0.965	1.058
			10	—	1.425	1.575
	β	0	0	1.199	1.188	1.189
			2	1.277	1.291	1.273
			4	1.451	1.528	1.465
			6	—	1.888	1.760
			8	—	2.357	2.152
	γ	2	2	1.175	1.169	1.194
			4	1.385	1.375	1.387
			6	—	1.683	1.684

* Ref. 115

** abbreviation of the Coherent states model, Ref. 75

Table 6.: (^{232}Th -isotope)

Classification			Energy levels (MeV)		
Band	K	I	Exp. *	Present work (RVM)	CSM**
g.s.	0	0	0.0	0.0	0.0
		2	0.049	0.050	0.048
		4	0.162	0.164	0.160
		6	0.333	0.334	0.329
		8	0.557	0.555	0.551
		10	0.827	0.821	0.820
		12	1.137	1.129	1.131
		14	1.482	1.476	1.477
		16	1.858	1.863	1.856
		18	2.261	2.289	2.262
		20	2.690	2.754	2.694
		22	3.142	3.260	3.149
		24	3.618	3.807	3.624
		26	4.114	4.396	4.118
		28	4.630	5.027	4.628
		30	5.160	5.701	5.154
β	0	0	0.730	0.719	0.729
		2	0.774	0.776	0.773
		4	0.873	0.922	0.871
		6	1.023	1.129	1.023
		8	1.222	1.398	1.223
		10	1.467	1.725	1.469
		12		2.106	1.757
		14		2.539	2.083
		16		3.025	2.449
γ	2	2	0.785	0.786	0.784
		4	0.890	0.893	0.883
		6	1.051	1.065	1.036
		8	1.260	1.288	1.239
		10	1.512	1.556	1.488
		12	1.800	1.866	1.780
		14		2.218	2.112

* Ref. 115

** Ref. 75

experimental data¹¹⁵⁾ of the energy levels of $^{174-178}\text{Hf}$ - isotopes and ^{232}Th - isotope, respectively. From these tables one may notice that both models give quite the same good agreement with the experimental values of the energy levels of the γ - band and the g.s.-band. In case of the g.s.-band we mean the energy levels $I \leq 4$ and $I \leq 20$ of the Hf- isotopes and the Th- isotope, respectively. On the other hand the coherent model⁷⁵⁾ shows a better agreement with experiment¹¹⁵⁾, in β -band, than the rotation - vibration model.

IV-2. The $B(E2)$ - values and $B(E2)$ - branching ratios :-

We stress however that a reliable identification of the validity of the rotation - vibration model should not only be based on the analysis of the energy spectrum, but also involve the transition properties of the band's members of the different bands.

Considering the homogeneous charge distribution (HCD) which associated with RVM, the model dependent deformation parameter β_0 has been extracted from the measured $E2$ - transition probability of the first rotational state, i.e. $B(E2 ; 0_g^+ \rightarrow 2_g^+)$. This has been done as follows. Using the ELEC-programe, with the input parameters : E_{rot} , E_γ , E_β and β_0 , the reduced matrix element of the intraband $(0_g^+ \rightarrow 2_g^+)$ transition has been calculated. We repeat this programe several times, with a constant values for the E_{rot} , E_γ and E_β parameters and a different values for the parameter β_0 , until we get a good

fit with the experimental $B(E2, 0_g^+ \rightarrow 2_g^+)$ value. One should notice that the values of the parameters : E_{rot} , E_γ and E_β are those tabulated in table 1. The theoretical and experimental values of the β_0 - parameter are tabulated in table 7. It is clear that there is a quite good agreement between the theoretical and the experimental values of this parameter.

The δ -parameter, p-n deformation difference are tabulated in table 8. In our choice of the values of this parameter we have tried to get a noticeable improvement for the $B(E2)$ -values, especially, those associated with the interband ($g \rightarrow \beta$) and ($g \rightarrow \gamma$) transitions. At the same time, we have tried to get, by this choice, a right signs and a reasonable values for the mixing ratios.

The calculated absolute $B(E2)$ - values and $B(E2)$ - branching ratios of the ^{150}Nd - isotope, in the frame work of RVM and the 2nd version of Greiner's model, are compared with the corresponding experimental values and the IBA- results in table 9-a and 9-b, respectively. This comparison shows that the results of the Greiner's version agree with experimental values better than the IBA- and the RVM- results, especially in the interband ($g \rightarrow \beta$) and ($g \rightarrow \gamma$) transitions. Also from these tables one may notice that the $B(E2)$ - values and the $B(E2)$ - branching ratios, which associated with the IBA- results, are in general lower than the corresponding experimental values.

Tables 10-a and 11, show a comparison between the calculated $B(E2)$ - values, using the RVM and the 2nd version of

Table (7) :.

Isotope	β_o	
	Exp.	Th.
^{150}Nd	—	0.2530
^{152}Sm	0.286 ± 0.006 <u>a</u>	0.2700
^{154}Sm	0.315 ± 0.007 ; 0.305 <u>a</u> 0.301 ; 0.294 <u>b</u>	0.3010
^{154}Gd	0.290 <u>c</u>	0.2770
^{156}Gd	0.310	0.3010
^{158}Gd	0.330 ± 0.010 <u>a</u> 0.320 <u>c</u>	0.3110
^{160}Gd	0.340 ± 0.010 <u>a</u> 0.330 <u>c</u>	0.3130
^{160}Dy	—	0.2940
^{166}Er	—	0.3970
^{172}Yb	0.326 ± 0.009 ; 0.284 ± 0.008 <u>d</u>	0.2830
^{174}Yb	0.317 ± 0.010 ; 0.276 <u>e</u> 0.280 ± 0.009 ; 0.291 <u>d</u>	0.2944
^{174}Hf	—	0.2710
^{176}Hf	—	0.2665
^{178}Hf	0.2250 <u>f</u>	0.2560
^{180}Hf	—	0.2520
^{230}Th	0.227 ± 0.006 ; 0.195 <u>g</u> 0.204 ± 0.005 ; 0.191	0.2250
^{232}Th	0.230 ± 0.010 <u>h</u>	0.2390

a) Ref. 83 b) Ref. 117 c) Ref. 54

b) Ref. 9 e) Ref. 65 f) Ref. 118

g) Ref. 73 h) Ref. 119

Table(8)

Isotope	p-n deformation difference (δ)
^{150}Nd	24°
^{152}Sm	32°
^{154}Sm	17°
^{154}Gd	25°
^{156}Gd	29°
^{158}Gd	14°
^{160}Gd	7°
^{160}Dy	17°
^{166}Er	15°
^{172}Yb	63°
^{174}Yb	48°
^{174}Hf	44°
^{176}Hf	47°
^{178}Hf	51°
^{180}Hf	79°
^{230}Th	25°
^{232}Th	35°

Table (9-a):. The B(E2) values in case of ^{150}Nd -isotope.

Transition	Exp.* (e ² b ²)	RVM	2 nd version of Greiner's model	IBA**
$2_g^+ \longrightarrow 0_g^+$	$0.56 \pm 0.03; 0.54 \pm 0.04$	0.54	0.53	0.59
$4_g^+ \longrightarrow 2_g^+$	$0.82 \pm 0.04; 0.84 \pm 0.11$	0.85	0.81	0.84
$6_g^+ \longrightarrow 4_g^+$	$0.98 \pm 0.09; 0.99 \pm 0.11$	1.06	0.97	0.88
$8_g^+ \longrightarrow 6_g^+$	$1.21 \pm 0.12; 1.03 \pm 0.09$	1.22	1.09	0.85
$2_g^+ \longrightarrow 2_\beta^+$	0.030 ± 0.008	0.073	0.033	0.0006
$4_g^+ \longrightarrow 2_\beta^+$	0.052 ± 0.015	0.112	0.065	0.0002
$2_g^+ \longrightarrow 2_\gamma^+$	0.030 ± 0.005	0.045	0.024	0.0003
$2_\beta^+ \longrightarrow 0_g^+$	0.003 ± 0.0006	0.0217	0.007	0.0030
$2_\gamma^+ \longrightarrow 0_g^+$	0.015 ± 0.001	0.028	0.011	0.0002

Table (9-b):. The B(E2) branching ratios in case of ^{150}Nd -isotope.

Transition ratio	Exp.*	RVM	2 nd version of Greiner's model	TBA**
$4_g^+ \longrightarrow 2_g^+ / 2_g^+ \longrightarrow 0_g^+$	1.46 ± 0.15	1.57	1.53	1.39
$6_g^+ \longrightarrow 4_g^+ / 2_g^+ \longrightarrow 0_g^+$	1.75 ± 0.25	1.96	1.83	1.46
$8_g^+ \longrightarrow 6_g^+ / 2_g^+ \longrightarrow 0_g^+$	2.16 ± 0.33	2.26	2.07	1.41
$6_g^+ \longrightarrow 4_g^+ / 4_g^+ \longrightarrow 2_g^+$	1.20 ± 0.17	1.25	1.20	1.05
$3_g^+ \longrightarrow 6_g^+ / 6_g^+ \longrightarrow 4_g^+$	1.24 ± 0.24	1.15	1.12	0.97
$2_\beta^+ \longrightarrow 0_g^+ / 2_\beta^+ \longrightarrow 2_g^+$	0.100 ± 0.047	0.297	0.207	0.63
$2_\gamma^+ \longrightarrow 0_g^+ / 2_\gamma^+ \longrightarrow 2_g^+$	0.507 ± 0.118	0.622	0.468	0.660

*Ref's 41 , 120 , and 121 . **Ref.39

Table(10-a):. The B(E2) values in case of ^{152}Sm -isotope.

Transition	Exp.* (e ² b ²)	RVM	2 nd version of Greiner's model
$2^+_g \rightarrow 0^+_g$	0.670 ± 0.015	0.671	0.659
$4^+_g \rightarrow 2^+_g$	$0.989 \pm 0.035; 1.017 \pm 0.014$	1.052	0.985
$6^+_g \rightarrow 4^+_g$	$1.20 \pm 0.06; 1.179 \pm 0.033$	1.30	1.16
$8^+_g \rightarrow 6^+_g$	$1.39 \pm 0.14; 1.45 \pm 0.22$	1.50	1.28
$10^+_g \rightarrow 8^+_g$	1.70 ; 1.55	1.65	1.38
$0^+_g \rightarrow 2^+_g$	3.40 ± 0.15	3.36	3.300
$0^+_g \rightarrow 2^+_B$	0.0228 ± 0.0017	0.1392	0.0213
$0^+_g \rightarrow 2^+_Y$	0.0813 ± 0.0057	0.1517	0.0359

Table(10-b):. The B(E2) branching ratios in case of ^{152}Sm -isotope.

Transition ratio.	Exp.*	RVM	2 nd version of Greiner's model	Alaga**	IBA**
$4^+_g \rightarrow 2^+_g / 2^+_g \rightarrow 0^+_g$	1.62 ± 0.13	1.57	1.50	—	—
$6^+_g \rightarrow 4^+_g / 2^+_g \rightarrow 0^+_g$	1.68 ± 0.18	1.94	1.75	—	—
$2^+_B \rightarrow 0^+_g / 2^+_B \rightarrow 2^+_g$	0.20(7); 0.165(3)	0.337	0.179	0.70	0.04
$4^+_B \rightarrow 2^+_g / 4^+_B \rightarrow 4^+_g$	0.113(5); 0.131(8)	0.166	0.028	1.10	0.03
$2^+_Y \rightarrow 0^+_g / 2^+_Y \rightarrow 2^+_g$	>0.25; 0.378(2)	0.591	0.378	0.70	0.93
$4^+_Y \rightarrow 2^+_g / 4^+_Y \rightarrow 4^+_g$	0.10(4); 0.096(6)	0.472	0.107	0.34	0.33

*Ref's. 122 , 52 , 123 , and 124 . **Ref. 125

Table(11):. The B(E2) values in case of ^{154}Sm -isotope.

Transition	Exp* (e ² b ²)	RVM	2 nd version of Greiner's model
$2^+_g \rightarrow 0^+_g$	0.84; 1.222 ± 0.008	0.84	0.84
$4^+_g \rightarrow 2^+_g$	1.20	1.24	1.23
$6^+_g \rightarrow 4^+_g$	1.37	1.43	1.40
$8^+_g \rightarrow 6^+_g$	1.49; 1.54; 1.50	1.58	1.53
$10^+_g \rightarrow 8^+_g$	1.49	1.71	1.64
$2^+_Y \rightarrow 0^+_g$	0.014 ± 0.002	0.020	0.012

*Ref's. 126 , 127 , and 20 .

Greiner's model, and the corresponding experimental values for $^{152-154}\text{Sm}$ - isotopes, respectively. From these tables, it seems that Greiner's version agree with experiment better than the RVM, especially for the $B(E2)$ - values of the interband ($g \rightarrow \beta$) transition in case of ^{152}Sm - isotope. Also from table 10-b, which associated with the $B(E2)$ - branching ratios of the ^{152}Sm - isotope, the preferable results are those which related to the Greiner's version. In fig.(11) there is a pictorial comparison between the results of the RVM, Greiner's version, Alaga⁾, IBA⁾, and the corresponding experimental values of a four $B(E2)$ - branching ratios associated with ^{152}Sm - isotope. From this figure one may notice that the results, which associated with the Greiner's version, agree with the experimental values better than the others.

In tables 12-a, 13-a, 14-a and 15-a, the theoretical - results of the RVM, 2nd version of Greiner's model and other theoretical results are compared with the corresponding experimental data of $^{154-160}\text{Gd}$ - isotopes, respectively. For the ^{154}Gd isotope, the results of Greiner's version, van Isacker et al⁾ and DPPQ seems to agree with the experimental $B(E2)$ - values of the interband ($\beta \rightarrow g$) transitions, better than the others. Also for the $^{156-158}\text{Gd}$ - isotopes, one may notice that the results of IBA of the $B(E2)$ - values of the interband ($\beta \rightarrow g$) transitions agree with the experimental values better than the others. Relative to the $B(E2)$ - values of the interband ($\gamma \rightarrow g$) transitions, the IBA- results, in case of ^{156}Gd - isotope, agree with the experimental values better than those obtained by RVM

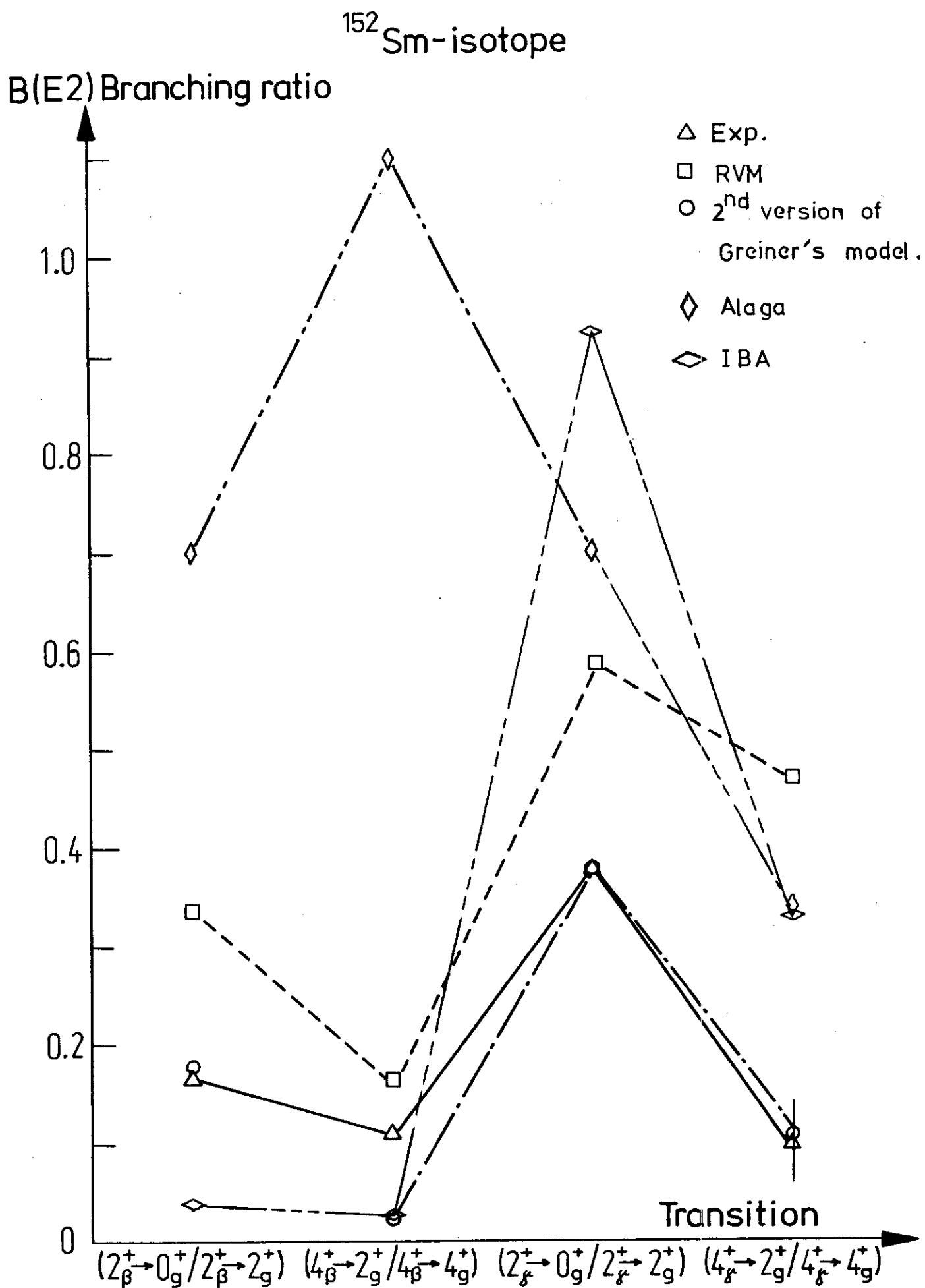


Fig. 11

and Greiner's version. At the same time the superiority of the results of the Greiner's version over the RVM- results of $B(E2)$ - values is remarkable, generally, for the $^{154-156}\text{Gd}$ - isotopes more than those which is observed for the ^{158}Gd - isotope. This behaviour may due to the choice of the value of the δ - parameter (p.n deformation difference). The δ -parameters of $^{154-156}\text{Gd}$ - isotopes are 25° and 29° , respectively, while the δ - parameters of ^{158}Gd - isotope is 14° .

The calculated $B(E2)$ - branching ratios of $^{154-160}\text{Gd}$ - isotopes, using the RVM and the 2nd version of Greiner's model, are compared with the corresponding experimental data and other theoretical results in tables 12-b, 13-b, 14-b and 15-b, respectively. Considering the $B(E2)$ - branching ratios of the interband ($g \rightarrow g$) transition ratios of ^{154}Gd - isotope, both of Greiner's version and RVM agree with experiment better than the others. In case of the ^{154}Gd - isotope, the results of DPPQ¹, Greiner's version and Girit et al.'s version gives the same level of agreement with the corresponding experimental data of the $B(E2)$ - branching ratios of the interband ($\beta \rightarrow g$) transition ratios. While for the interband ($\gamma \rightarrow g$) transition ratios, IBA together with DPPQ and Greiner's version gives, respectively, a good agreement with the corresponding experimental data. For the $B(E2)$ - branching ratios of the interband ($\beta \rightarrow g$) and ($\gamma \rightarrow g$) transition ratios, in case of the $^{156-158}\text{Gd}$ - isotopes, one may notice that the projection model and IBA agree with experiment better than the RVM and Greiner's version.

Table (12-a): The B(E2) values in case of ^{154}Gd -isotope.

Transition	Exp. (e^2b^2)	RVM	2nd version of Greiner's model	IBA				DPPQ $\bar{d}$	DNSB $\bar{d}$	Projection model $\bar{e}$	
				V. Isacker $\bar{a}$ et al.	C. Girit $\bar{a}$ et al.	P. O. Lipas $\bar{b}$ et al.	$\bar{c}$			Isotropic	Anisotropic
$2_1^+ \rightarrow 0_1^+$	0.75; 0.78	0.77	0.76	0.77	0.77	0.77	—	—	—	—	—
$4_1^+ \rightarrow 2_1^+$	1.20; 1.18	1.20	1.15	1.10	1.09	1.15	1.16	1.09	—	—	—
$6_1^+ \rightarrow 4_1^+$	1.40; 1.38	1.47	1.36	1.19	1.16	1.30	1.34	1.38	—	—	—
$8_1^+ \rightarrow 6_1^+$	1.60; 1.53	1.70	1.52	1.21	1.16	1.34	—	—	—	—	—
$10_1^+ \rightarrow 8_1^+$	1.72; 1.70	1.87	1.64	1.18	1.12	1.33	—	—	—	—	—
$0_2^+ \rightarrow 2_1^+$	3.85 ± 0.08	3.84	3.79	—	—	—	3.86	3.32	3.85	3.85	—
$2_2^+ \rightarrow 4_1^+$	1.97	2.16	2.06	—	—	—	—	—	—	—	—
$0_3^+ \rightarrow 2_2^+$	0.015 ± 0.004	0.150	0.047	—	—	—	0.019	0.020	0.118	0.096	—
$0_4^+ \rightarrow 2_2^+$	0.143 ± 0.011	0.183	0.071	—	—	—	0.139	0.132	0.248	0.439	—
$2_3^+ \rightarrow 0_1^+$	0.004 ± 0.001	0.030	0.009	0.017	—	0.004	0.004	—	—	—	—
$2_3^+ \rightarrow 2_1^+$	0.033 ± 0.008	0.096	0.042	0.025	0.006	0.106	0.033	—	—	—	—
$2_3^+ \rightarrow 4_1^+$	0.090 ± 0.023	0.259	0.148	0.087	0.017	0.037	0.087	—	—	—	—
$4_3^+ \rightarrow 2_1^+$	0.003 ± 0.008	0.016	0.003	0.016	0.003	0.001	0.006	—	—	—	—
$4_3^+ \rightarrow 4_1^+$	0.027 ± 0.007	0.128	0.057	0.015	0.016	0.086	0.034	—	—	—	—
$4_3^+ \rightarrow 6_1^+$	0.071 ± 0.002	0.171	0.096	0.089	0.002	0.024	0.086	—	—	—	—
$2_4^+ \rightarrow 0_1^+$	0.0286 ± 0.022	0.0366	0.0141	0.0144	—	0.0214	—	—	—	—	—
$2_4^+ \rightarrow 2_1^+$	0.059	0.058	0.030	—	—	—	—	—	—	—	—

* Ref.'s 192, 52, 72, 128

a) Ref. 53 b) Ref. 45

c) Ref. 47 d) Ref. 36

e) Ref. 55

Table (12-b) The B(E2) branching ratios in case of ^{154}Gd -isotope.

Transition ratio	Exp.*	RVM	2 nd version of Greiner's model	IEA		DPPQ	DNSB	Projection model $\bar{d}$	
				V_L Ifacker's $\bar{d}$	$C_{\bar{d}}^{\text{Gd}} \bar{d}$			Isotropic	Anisotropic
$4_2^+ \rightarrow 2_2^+ / 2_2^+ \rightarrow 0_2^+$	1.52(10)	1.56	1.51	1.42	1.41	1.51	1.64	—	—
$6_2^+ \rightarrow 4_2^+ / 4_2^+ \rightarrow 2_2^+$	1.17(8)	1.23	1.19	1.08	1.07	1.16	1.27	—	—
$8_2^+ \rightarrow 6_2^+ / 6_2^+ \rightarrow 4_2^+$	1.10(6)	1.16	1.12	1.01	1.00	—	—	—	—
$10_2^+ \rightarrow 8_2^+ / 8_2^+ \rightarrow 6_2^+$	1.13(15)	1.10	1.08	0.98	0.96	—	—	—	—
$2_2^+ \rightarrow 0_2^+ / 2_2^+ \rightarrow 2_2^+$	0.123(8)	0.313	0.222	0.660	0.520	0.110	—	0.580	0.600
$2_2^+ \rightarrow 4_2^+ / 2_2^+ \rightarrow 2_2^+$	2.76 ± 0.19	2.70	3.52	3.44	2.85	2.61	—	2.69	2.54
$4_2^+ \rightarrow 2_2^+ / 4_2^+ \rightarrow 4_2^+$	0.086(3)	0.125	0.054	0.180	0.170	0.180	—	0.730	0.780
$4_2^+ \rightarrow 6_2^+ / 4_2^+ \rightarrow 4_2^+$	2.63(87) 2.38(18)	1.33	1.67	5.81	3.12	2.54	—	—	—
$2_2^+ \rightarrow 0_2^+ / 2_2^+ \rightarrow 0_2^+$	0.008(3)	0.038	0.012	0.033	0.005	4.9	—	—	—
$4_2^+ \rightarrow 2_2^+ / 4_2^+ \rightarrow 2_2^+$	0.0025(3)	0.0144	0.0030	0.021	0.003	5.0	—	—	—
$2_2^+ \rightarrow 0_2^+ / 2_2^+ \rightarrow 2_2^+$	0.468(70)	0.631	0.474	0.520	0.640	0.560	—	0.470	0.460
$2_2^+ \rightarrow 0_2^+ / 2_2^+ \rightarrow 4_2^+$	3.25(13)	2.65	1.67	3.30	4.50	6.30	—	—	—
$2_2^+ \rightarrow 4_2^+ / 2_2^+ \rightarrow 2_2^+$	0.144(22)	0.238	0.283	0.380	0.143	0.090	—	0.030	0.040
$4_2^+ \rightarrow 2_2^+ / 4_2^+ \rightarrow 4_2^+$	0.148(81)	0.731	0.305	0.185	0.290	0.320	—	0.120	0.109
$4_2^+ \rightarrow 6_2^+ / 4_2^+ \rightarrow 4_2^+$	0.27(4)	2.33	2.58	0.081	0.360	0.370	—	—	—
$2_2^+ \rightarrow 0_2^+ / 2_2^+ \rightarrow 0_2^+$	0.0052(9)	0.0390	0.0123	—	—	—	—	—	—
$2_2^+ \rightarrow 0_2^+ / 2_2^+ \rightarrow 0_2^+$	0.034(13)	0.048	0.019	—	—	—	—	—	—
$2_2^+ \rightarrow 2_2^+ / 2_2^+ \rightarrow 2_2^+$	1.00(2)	0.66	1.16	1.30	18.5	1.47	—	—	—

*Ref. 5 45, 52, 117
a) Ref. 53 b) Ref. 45
c) Ref. 36 d) Ref. 55

Table (13-a): The B(E2) values in case of ^{156}Gd -isotope

Transition	Exp.*(e ² b ²)	RVM	2 nd version of Greiner's model	IBA**
$2_g^+ \longrightarrow 0_g^+$	0.93; 0.92	0.91	0.91	0.91
$4_g^+ \longrightarrow 2_g^+$	1.39	1.35	1.33	1.29
$6_g^+ \longrightarrow 4_g^+$	1.63; 1.47	1.57	1.51	1.38
$8_g^+ \longrightarrow 6_g^+$	1.50; 1.60	1.75	1.63	1.38
$10_g^+ \longrightarrow 8_g^+$	1.60; 1.53	1.91	1.74	1.34
$0_g^+ \longrightarrow 2_g^+$	4.57(0.05)	4.57	4.55	—
$2_g^+ \longrightarrow 4_g^+$	2.36	2.43	2.39	—
$0_g^+ \longrightarrow 2_\beta^+$	0.013(4)	0.011	0.002	—
$0_g^+ \longrightarrow 2_\gamma^+$	0.120(4)	0.266	0.080	—
$2_\beta^+ \longrightarrow 0_g^+$	0.00316(18)	0.00210	0.00050	0.00400
$2_\beta^+ \longrightarrow 2_g^+$	0.0164(16)	0.1049	0.0390	0.0061
$2_\beta^+ \longrightarrow 4_g^+$	0.0181(17)	0.0770	0.0342	0.0148
$4_\beta^+ \longrightarrow 2_g^+$	0.0061(7)	0.0369	0.0070	0.0047
$4_\beta^+ \longrightarrow 4_g^+$	0.0140(13)	0.0008	0.0001	0.0048
$4_\beta^+ \longrightarrow 6_g^+$	0.0091(14)	0.1375	0.0684	0.0148
$2_\gamma^+ \longrightarrow 0_g^+$	0.0222(11)	0.0532	0.0159	0.0212
$2_\gamma^+ \longrightarrow 2_g^+$	0.0355(19)	0.0027	0.0015	0.0333
$2_\gamma^+ \longrightarrow 4_g^+$	0.0032(3)	0.0785	0.0370	0.0023
$4_\gamma^+ \longrightarrow 2_g^+$	0.0078(9)	0.0008	0.0002	0.0108
$4_\gamma^+ \longrightarrow 4_g^+$	0.046(5)	0.113	0.043	0.0385

*Ref. 53 , 127 , and 129.

**Ref. 53

Table (13-b): The B(E2) branching ratios in case of ^{156}Gd -isotope

Transition ratio	Exp.*	RVM	2 nd version of Greiner's model	Projection model**		IBA**
				Isotropic	Anisotropic	
$2^+_{\beta} \rightarrow 0^+_{\text{g}}/2^+_{\beta} \rightarrow 2^+_{\text{g}}$	0.17(2)	0.02	0.012	0.63	0.65	0.82
$2^+_{\beta} \rightarrow 4^+_{\text{g}}/2^+_{\beta} \rightarrow 2^+_{\text{g}}$	1.09(14)	0.73	0.970	2.32	2.16	2.86
$4^+_{\beta} \rightarrow 2^+_{\text{g}}/4^+_{\beta} \rightarrow 4^+_{\text{g}}$	0.20(8)	49.14	86.26	0.85	0.92	2.60
$2^+_{\gamma} \rightarrow 0^+_{\text{g}}/2^+_{\gamma} \rightarrow 0^+_{\text{g}}$	0.015(5)	0.058	0.018	—	—	—
$2^+_{\gamma} \rightarrow 0^+_{\text{g}}/2^+_{\gamma} \rightarrow 2^+_{\text{g}}$	0.67(4)	19.70	10.39	0.56	0.62	0.61
$2^+_{\gamma} \rightarrow 4^+_{\text{g}}/2^+_{\gamma} \rightarrow 2^+_{\text{g}}$	0.087(10)	29.68	24.08	0.04	0.04	0.06
$4^+_{\gamma} \rightarrow 4^+_{\text{g}}/4^+_{\gamma} \rightarrow 2^+_{\text{g}}$	6.21(53)	141.25	210.83	5.38	4.06	4.18

*Ref. 51 , 130 and 55

**Ref. 55

Table (14-b): The B(E2) branching ratios in case of ^{158}Gd -isotope

Transition ratio	Exp.*	RVM	2 nd version of Greiner's model	Projection model**		IBA**
				Isotropic	Anisotropic	
$2_g^+ \rightarrow 0_g^+ / 2_g^+ \rightarrow 2_g^+$	0.65(8)	0.71	0.68	0.65	0.66	0.55
$2_g^+ \rightarrow 4_g^+ / 2_g^+ \rightarrow 2_g^+$	0.67(9)	3.04	3.29	2.17	2.08	1.96
$4_g^+ \rightarrow 2_g^+ / 4_g^+ \rightarrow 4_g^+$	0.80(12)	1.81	1.30	0.91	0.95	0.50
$4_g^+ \rightarrow 6_g^+ / 4_g^+ \rightarrow 4_g^+$	2.1(4)	5.24	5.88	—	—	—
$2_\gamma^+ \rightarrow 0_g^+ / 2_\gamma^+ \rightarrow 2_g^+$	0.59(8)	0.45	0.37	0.60	0.65	0.64
$2_\gamma^+ \rightarrow 4_g^+ / 2_\gamma^+ \rightarrow 2_g^+$	0.046(8)	0.033	0.039	0.050	0.60	0.070
$4_\gamma^+ \rightarrow 2_g^+ / 4_\gamma^+ \rightarrow 4_g^+$	0.15(2)	0.044	0.035	0.222	0.285	0.279
$4_\gamma^+ \rightarrow 6_g^+ / 4_\gamma^+ \rightarrow 4_g^+$	0.25(4)	0.014	0.017	—	—	—

*Ref. 133 **Ref. 55

Table (15-a): The B(E2) values in case of ^{158}Gd -isotope

Transition	Exp.* (e^2b^2)	RVM	2 nd version of Greiner's model	IBA
$0_g^+ \rightarrow 2_g^+$	5.15(6)	5.16	5.14	—
$2_g^+ \rightarrow 4_g^+$	2.68	2.74	2.72	—
$0_g^+ \rightarrow 2_g^+$	0.002(2)	0.206	0.170	—
$0_g^+ \rightarrow 2_\gamma^+$	0.101(3)	0.104	0.087	—
$2_\gamma^+ \rightarrow 0_g^+$	0.0176(9)	0.0207	0.0174	0.0176
$2_\gamma^+ \rightarrow 2_g^+$	0.0298(18)	0.0770	0.0661	0.0275
$2_\gamma^+ \rightarrow 4_g^+$	0.0016(1)	0.0003	0.0003	0.0018

*Ref. 54 , and 72 . **Ref. 57

Table (15-b): The B(E2) branching ratios in case of ^{158}Gd -isotope

Transition ratio	Exp.*	RVM	2 nd version of Greiner's model
$2_\gamma^+ \rightarrow 2_g^+ / 2_\gamma^+ \rightarrow 0_g^+$	1.6 ± 0.3	3.70	3.81
$2_\gamma^+ \rightarrow 0_g^+ / 2_\gamma^+ \rightarrow 0_g^+$	0.014 ± 0.001	0.020	0.017

*Ref. 20

As a pictorial comparison for the $^{154-158}\text{Gd}$ - isotopes, one can see fig. 12. In this figure the theoretical results, using the RVM and Greiner's version, are compared with the experimental data of the $B(E2)$ - branching ratio which associated with $(0_g^+ \rightarrow 2_g^+ / 0_g^+ \rightarrow 2_g^+)$ transition ratio. It is clear that the Greiner's version agree, qualitatively, with experiment better than the RVM and vice versa for the quantitative agreement with experiment.

An instructive comparison of the results of the RVM and Greiner's version with the experimental results can be made in the Michailov plot. In this plot the quantity $\sqrt{B(E2; I_i \rightarrow I_f) / (I_i K_i 2 K_f - K_i I_f K_f)}$ is plotted as a function of $I_f (I_f + 1) - I_i (I_i + 1)$, where $I_i(I_f)$ is the angular momentum of the initial (final) state and $K_i(K_f)$ is the projection on the intrinsic symmetry axis of the initial (final) state. In figures 13 - 15 we show such Michailov plots for the $E2$ - transitions of the β - and γ - bands to the ground state band in $^{154-158}\text{Gd}$ - isotopes, respectively. Concerning these figures, one may notice that the agreement of the theoretical results with the experimental data, in case of the interband ($\beta \rightarrow g$) transitions, is better than those which associated with the interband ($\gamma \rightarrow g$) transitions. In addition the results of the 2nd version of Greiner's model agree quantitatively with the experimental data better than the results of the RVM.

Table 16-a represents a comparison between a different theoretical results of the $B(E2)$ - values of ^{160}Dy and the

Gd-isotopes

B(E2) Branching ratio ($0_g^+ \rightarrow 2_g^+ / 0_g^+ \rightarrow 2_\delta^+$)

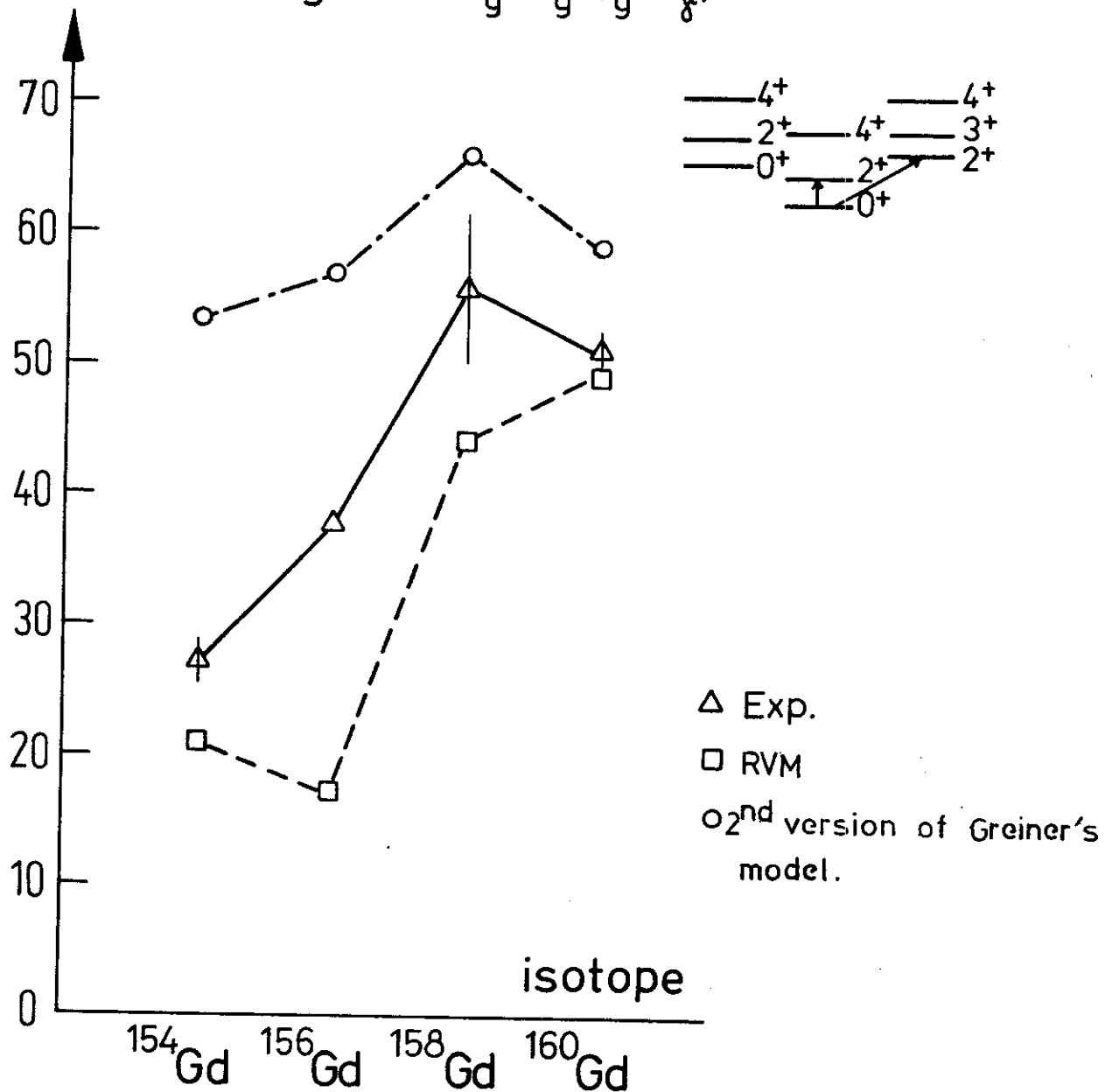


Fig. 12

^{154}Gd - isotope

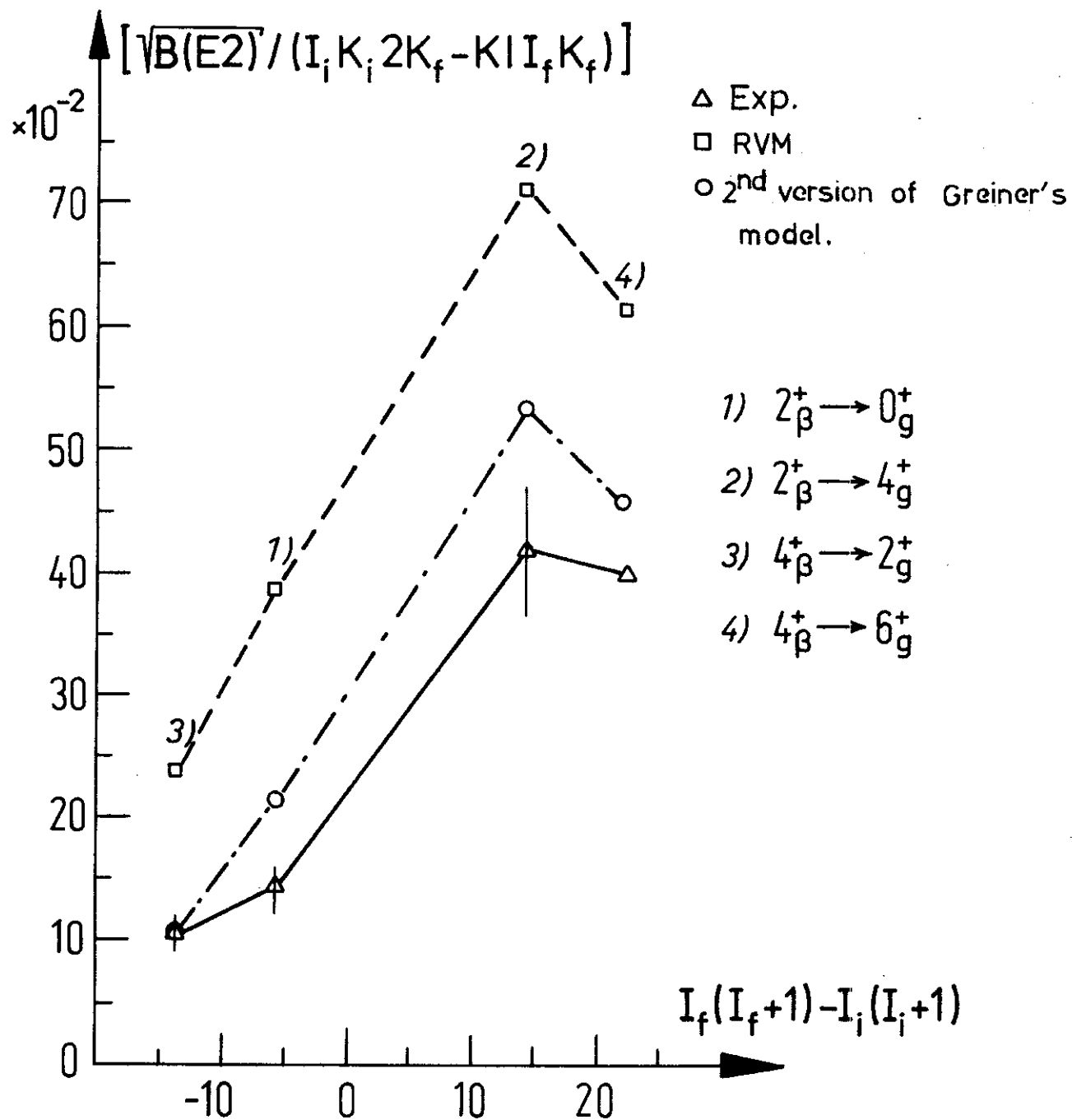


Fig. 13

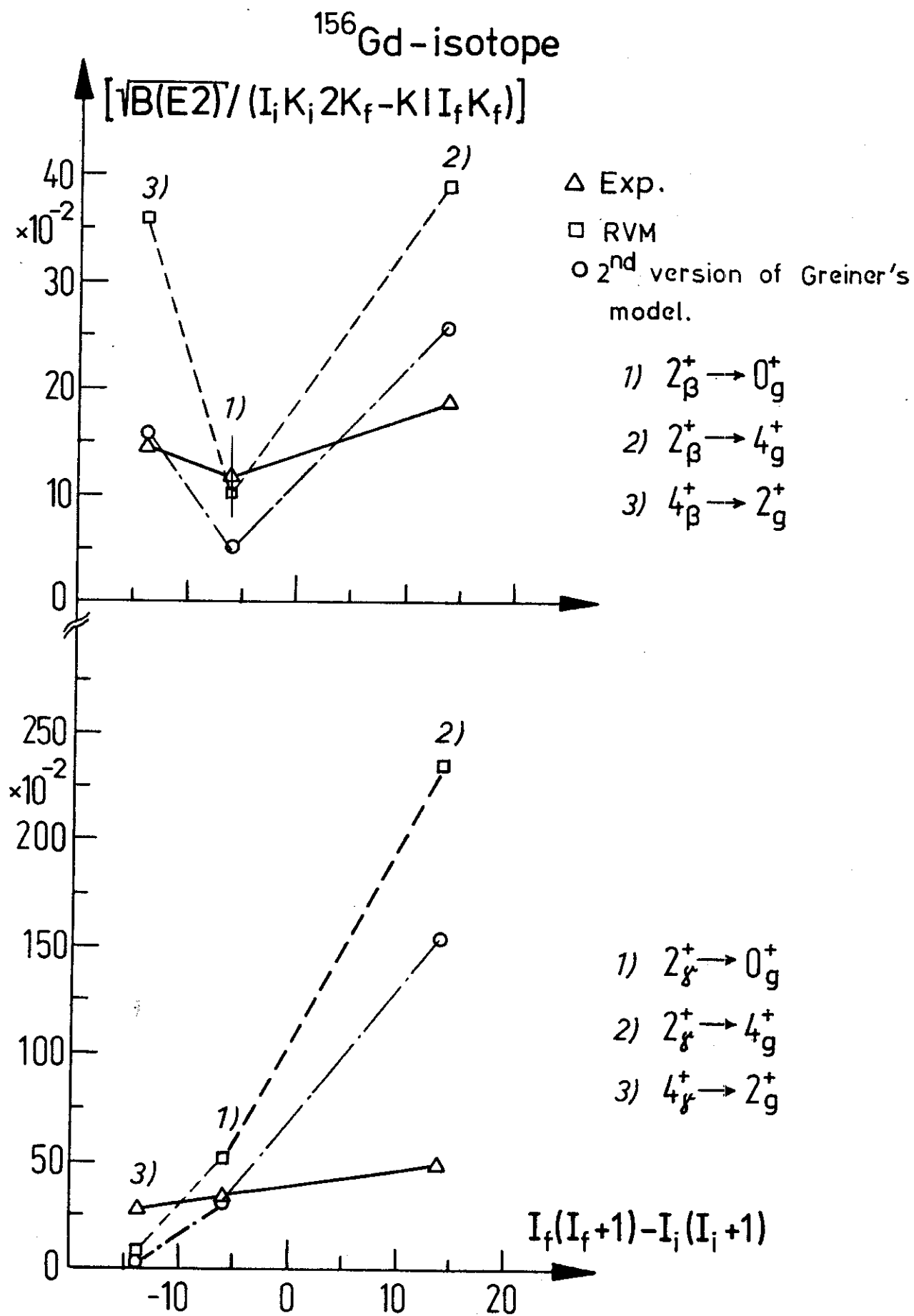


Fig. 14

^{158}Gd -isotope

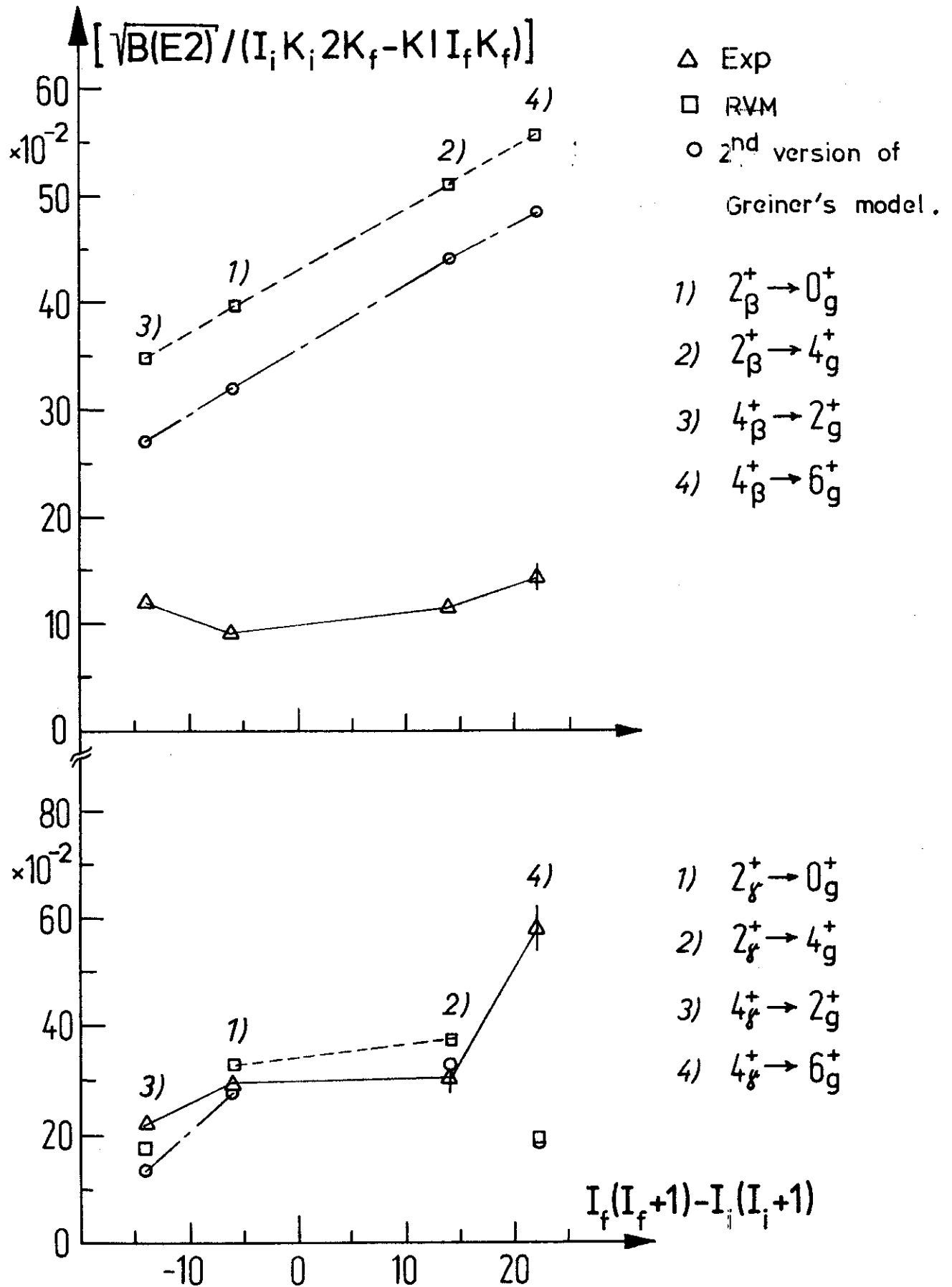


Fig. 15

Table (16-a): The B(E2) values in case of ^{160}Dy isotope

Transition	Exp.* e^2b^2	RVM	2 nd version of Greiner's model	IBA**
$2_g^+ \rightarrow 0_g^+$	1.00	0.95	0.95	—
$4_g^+ \rightarrow 2_g^+$	1.50	1.40	1.39	—
$6_g^+ \rightarrow 4_g^+$	—	1.61	1.58	—
$8_g^+ \rightarrow 6_g^+$	1.86	1.78	1.73	—
$10_g^+ \rightarrow 8_g^+$	1.70	1.93	1.86	—
$2_\gamma^+ \rightarrow 0_g^+$	0.0244(13)	0.0324	0.0190	0.0244
$2_\gamma^+ \rightarrow 2_g^+$	0.0445(28)	0.0776	0.0515	0.0375
$2_\gamma^+ \rightarrow 4_g^+$	0.0030(3)	0.0062	0.0048	0.0023
$2_\beta \rightarrow 0_g^+$	0.0037(3)	0.0231	0.0162	0.0010
$2_\beta \rightarrow 2_g^+$	0.0037(19)	0.0462	0.0283	0.0016
$2_\beta \rightarrow 4_g^+$	0.0089(9)	0.1263	0.0860	0.0036

*Ref. 134 , 127 , and 57 . **Ref. 57

Table (16-b): The B(E) branching ratios in case of ^{160}Dy -isotope

Transition ratio	Exp.*	RVM	2 nd version of Greiner's model	IBA **	
				IBA(a)	IBA(b)
$2_\gamma^+ \rightarrow 0_g^+ / 2_g^+ \rightarrow 0_g^+$	0.034(1)	0.034	0.020	—	—
$2_\gamma^+ \rightarrow 0_g^+ / 2_\gamma^+ \rightarrow 2_g^+$	0.523(12)	0.418	0.369	0.649	0.610
$2_\gamma^+ \rightarrow 0_g^+ / 2_\gamma^+ \rightarrow 4_g^+$	8.21(28) 7.35(3)	5.23	3.97	10.64	7.19
$2_\gamma^+ \rightarrow 2_g^+ / 2_g^+ \rightarrow 0_g^+$	0.055(3)	0.082	0.054	—	—
$2_\gamma^+ \rightarrow 2_g^+ / 2_\gamma^+ \rightarrow 2_g^+$	1.61(7) 1.85(9)	2.40	2.71	1.54	1.64
$2_\gamma^+ \rightarrow 2_g^+ / 2_\gamma^+ \rightarrow 4_g^+$	15.7(5) 13.60(8)	12.52	10.78	16.38	11.80
$4_\gamma^+ \rightarrow 2_g^+ / 4_\gamma^+ \rightarrow 4_g^+$	0.134(14)	0.070	0.440	0.290	0.260

*Ref. 20 and 135 . **Ref. 58

corresponding experimental data. From this table, it seems that the theoretical results of IBA give a good agreement with the corresponding experimental $B(E2)$ -values better than the other theoretical results, especially, for the interband ($\gamma \rightarrow g$) transitions. At the same time, the predictions of the Greiner's version are better than those which associated with RVM. From table 16-b, which associated with the $B(E2)$ - branching ratios of the interband ($\gamma \rightarrow g$) transition ratios, one may notice that the theoretical results of IBA, RVM and Greiner's version agree, respectively, with the corresponding data. In other word one can say that the results of RVM agree with the experimental values better than those which associated with Greiner's version. In fact, this behaviour can be understood if one put in his mind the rate of reduction of the $B(E2)$ - values, as a result of inserting the δ - parameter. For example the rate of reduction of the $B(E2)$ - values of the interband ($2_Y^+ \rightarrow 0_g^+$), ($2_Y^+ \rightarrow 2_g^+$) and ($2_Y^+ \rightarrow 4_g^+$) transitions are given by 40.36 %, 33.63 % and 22.58 %, respectively. Thus it is acceptable that some values of $B(E2)$ - branching ratios will decrease, as shown for the ($2_Y^+ \rightarrow 0_g^+$ / $2_Y^+ \rightarrow 4_g^+$) transition ratio, while the values of the other $B(2)$ - branching ratios will increase, as shown for the ($2_Y^+ \rightarrow 2_g^+$ / $2_Y^+ \rightarrow 0_g^+$) transition ratio. For the sake of completeness of this point, we shall give here the rate of reduction of the $B(E2)$ - values of the interband ($\beta \rightarrow g$) transitions. The rate of reduction of the interband ($2_\beta^+ \rightarrow 0_g^+$), ($2_\beta^+ \rightarrow 2_g^+$) and ($2_\beta^+ \rightarrow 4_g^+$) transitions are given by 42.35 %, 38.77 % and 31.91 %, respectively. Considering the reduction rates of the $B(E2)$ - values

for the interband ($\gamma \rightarrow g$) and ($\beta \rightarrow g$) transitions, one can say that the transition probability from a higher - spin members to a lower one is more sensitive for the p-n deformation difference than those which associated with a transition from a lower - spin members to a higher one. In addition the rates of reduction in case of the interband ($\beta \rightarrow g$) transitions are larger than the corresponding reduction rates of the interband ($\gamma \rightarrow g$) transitions. From the above discussion one may conclude that the interband ($\beta \rightarrow g$) transitions are more sensitive to the p-n deformation difference than those which associated with the interband ($\gamma \rightarrow g$) transitions. Furthermore, this interpretation may be obtained for others considered nuclei.

Table 17-a contains the experimental $B(E2)$ - values of the ^{166}Er isotope beside the results of RVM, 2nd version of Greiner's model and IBA. Both of the calculated results by RVM and Greiner's version give, respectively, a reasonable agreement with the corresponding experimental $B(E2)$ - values of the interband ($\beta \rightarrow g$) and ($\gamma \rightarrow g$) transitions. Concerning table (17-b), one can observe that the results of RVM and the 2nd version of Greiner's model agree in the same level with the corresponding experimental values of the $B(E2)$ - branching ratios of the interband ($\gamma \rightarrow g$) transition ratios.

The available experimental $B(E2)$ - values, in case of ^{172}Yb isotope, together with the results of RVM, 2nd version, IBA and Alaga are tabulated in table (18-a). From this table one may notice that all the theoretical results agree in the

Table (17-a): The values in case of ^{166}Er -isotope .

Transition	Exp.*(e^2b^2)	RVM	2 $\frac{\text{nd}}{\text{ei}}$ version of Greiner's model	IBA **
$2_g^+ \longrightarrow 0_g^+$	1.10	1.16	1.16	_____
$4_g^+ \longrightarrow 2_g^+$	1.70	1.70	1.69	_____
$6_g^+ \longrightarrow 4_g^+$	1.60	1.94	1.92	_____
$8_g^+ \longrightarrow 6_g^+$	1.98 , 1.75	2.13	2.09	_____
$10_g^+ \longrightarrow 8_g^+$	2.17, 2.00	2.31	2.24	_____
$2_\gamma^+ \longrightarrow 0_g^+$	0.0280(16)	0.0439	0.0280	0.0280
$2_\gamma^+ \longrightarrow 2_g^+$	0.0539(29)	0.1106	0.0787	0.0427
$2_\gamma^+ \longrightarrow 4_g^+$	0.0042(12)	0.0111	0.0090	0.0027
$2_\beta^+ \longrightarrow 0_g^+$	0.00084(12)	0.02871	0.01818	0.00084
$2_\beta^+ \longrightarrow 2_g^+$	0.00096(20)	0.04710	0.03662	0.00132
$2_\beta^+ \longrightarrow 4_g^+$	_____	0.1173	0.0834	0.0029
$2_\beta^+ \longrightarrow 2_\gamma^+$	0.0045(9)	0.0002	0.0004	0.0275

* Ref. 131 , 123 , 63 and 127 . ** Ref. 63.

Table (17-b) :. The B (E2) branching ratios in case of ^{166}Er -isotope

Transition ratio	Exp. *	RVM	2 $\frac{\text{nd}}{\text{ei}}$ version of Greiner's model
$4_g^+ \longrightarrow 2_g^+/2_g^+ \longrightarrow 0_g^+$	1.50(7)	1.47	1.46
$2_\gamma^+ \longrightarrow 0_g^+/2_g^+ \longrightarrow 0_g^+$	0.052(4)	0.038	0.024
$2_\gamma^+ \longrightarrow 2_g^+/2_g^+ \longrightarrow 0_g^+$	0.089(6)	0.096	0.068
$2_\gamma^+ \longrightarrow 2_g^+/2_g^+ \longrightarrow 0_g^+$	1.71(21)	2.52	2.85

* Ref. 20 and 136 .

Table (18-a) : The B(E2) - values in case of ^{172}Yb - isotope.

Transition	Exp. * e ² b ²	RVM	2 nd version of Greiner's model	IBA **	Alaga **
$2_g^+ \rightarrow 0_g^+$	1.087(2)	1.086	1.076	1.087	1.087
$4_g^+ \rightarrow 2_g^+$	1.80(13)	1.59	1.53	1.54	1.55
$6_g^+ \rightarrow 4_g^+$	1.76(9)	1.84	1.68	1.67	1.71
$8_g^+ \rightarrow 6_g^+$	1.95(9)	2.03	1.74	1.71	1.79
$10_g^+ \rightarrow 8_g^+$	2.24(22)	2.20	1.78	1.71	1.84
$0_g^+ \rightarrow 2_\beta^+$	0.0066(3)	0.1563	0.0070	0.0605	—
$0_g^+ \rightarrow 2_\beta^+$	0.395(16)	0.1226	0.0019	0.0395	—
$0_g^+ \rightarrow 2_{\beta\beta}^+$	0.0021(4)	0.00004	0.00014	0.0000	—
$0_g^+ \rightarrow 2_{\gamma-\gamma}^+$	0.0040(21)	0.0010	0.0000	0.0020	—
$0_g^+ \rightarrow 2_{\beta\gamma}^+$	0.0101(7)	0.0000	0.0000	0.0035	—
$0_\beta^+ \rightarrow 2_g^+$	0.0162(49)	0.2504	0.0003	0.0685	—

* Ref. 137 , 123 , 138 , 127 , 139 , 2 , and 66 . ** Ref. 67 .

Table (18-b) : The B(E2) branching ratios in case of ^{172}Yb - isotope

Transition ratio	Exp. *	RVM	2 nd version of Greiner's model	IBA **	Alaga **
$4_g^+ \rightarrow 2_g^+ / 2_g^+ \rightarrow 0_g^+$	1.50(11)	1.46	1.42	—	—
$6_g^+ \rightarrow 4_g^+ / 2_g^+ \rightarrow 0_g^+$	1.46(9)	1.69	1.56	—	—
$8_g^+ \rightarrow 6_g^+ / 2_g^+ \rightarrow 0_g^+$	1.24(32)	1.87	1.62	—	—
$8_g^+ \rightarrow 6_g^+ / 6_g^+ \rightarrow 4_g^+$	0.86(22)	1.10	1.04	—	—
$2_\beta^+ \rightarrow 4_g^+ / 2_\beta^+ \rightarrow 0_g^+$	7.55(96)	5.05	2.21	3.12	2.57
$2_\beta^+ \rightarrow 4_g^+ / 2_\beta^+ \rightarrow 2_g^+$	2.54(+29 , -22)	2.65	9.86	2.09	1.80
$4_\beta^+ \rightarrow 6_g^+ / 4_\beta^+ \rightarrow 4_g^+$	4.60(+13, -8)	2.62	22.54	2.26	1.75
$6_\beta^+ \rightarrow 8_g^+ / 6_\beta^+ \rightarrow 6_g^+$	3.40(+15, -11)	2.23	29.54	2.48	1.69
$8_\beta^+ \rightarrow 10_g^+ / 8_\beta^+ \rightarrow 8_g^+$	≈ 45	1.66	27.47	2.76	1.65
$2_\gamma^+ \rightarrow 2_g^+ / 2_\gamma^+ \rightarrow 0_g^+$	1.69(11)	1.70	0.018	2.28	1.43
$2_\gamma^+ \rightarrow 4_g^+ / 2_\gamma^+ \rightarrow 0_g^+$	0.169(17)	0.161	0.458	0.228	0.071
$2_\gamma^+ \rightarrow 4_g^+ / 2_\gamma^+ \rightarrow 2_g^+$	0.0977(13)	0.0943	25.00	0.100	0.050
$4_\gamma^+ \rightarrow 4_g^+ / 4_\gamma^+ \rightarrow 2_g^+$	3.24(+14, -10)	3.91	0.0041	10.70	2.95

* Ref. 140 , 141 , 66 , 3 , 1 and 67 . ** Ref. 67 .

same level with the corresponding experimental data of the intraband ($g \rightarrow g$) transitions, especially, the ($2_g^+ \rightarrow 0_g^+$), ($4_g^+ \rightarrow 2_g^+$) and ($6_g^+ \rightarrow 4_g^+$) transitions. While for the other interband transitions, only the IBA and Greiner's version agrees, respectively, with the experiment. Concerning table (18-b), which associated with the $B(E2)$ - branching ratio, one may observe that the results of RVM, IBA and Alaga give a reasonable agreement with the corresponding experimental data of the $B(E2)$ - branching ratios of the interband ($\beta \rightarrow g$) and ($\gamma \rightarrow g$) transition ratios. At the same time the agreement, which associated with Greiner's version, is not so good as those associated with the other theoretical calculations. The large value of the p-n deformation difference of the ^{172}Yb - isotope may be responsible for this discrepancy. On the other hand this large value for δ - parameter allowed us to get both the right signs for the mixing ratios $\delta(E2 / M1)$ of this isotope and a reasonable values of $\delta(E2 / M1)$, which agree with the correspond experimental data. The comparison with experiment for the $B(E2)$ - values and $B(E2)$ - branching ratios of the ^{174}Yb - isotope in tables 19 - a and 19 - b, respectively, show that Greiner's version agree, generally, with experiment better than RVM.

In tables 20 and 21, the theoretical calculations obtained by RVM and the 2nd version of Greiner's model are compared with the available experimental $B(E2)$ - values of $^{174-176}\text{Hf}$ - isotopes, respectively. It is evident that the results of both RVM and Greiner's version agree, in the same

Table (19-a): The B(E2) values in case of ^{174}Yb -isotope

Transition	Exp.* e^2b^2	RVM	2 nd version of Greiner's model
$2_g^+ \longrightarrow 0_g^+$	1.2	1.20	1.19
$4_g^+ \longrightarrow 2_g^+$	1.61	1.74	1.71
$6_g^+ \longrightarrow 4_g^+$	1.80	1.97	1.89
$8_g^+ \longrightarrow 6_g^+$	2.10	2.13	2.00
$10_g^+ \longrightarrow 8_g^+$	2.06	2.28	2.07

*Ref.142,52,142 and 144 .

Table (19-b): The B(E2) branching ratios in case of ^{174}Yb -isotope.

Transition ratio	Exp.*	RVM	2 nd version of Greiner's model
$4_g^+ \longrightarrow 2_g^+ / 2_g^+ \longrightarrow 0_g^+$	1.48(11)	1.45	1.43
$6_g^+ \longrightarrow 4_g^+ / 2_g^+ \longrightarrow 0_g^+$	1.53(10)	1.64	1.59
$8_g^+ \longrightarrow 6_g^+ / 2_g^+ \longrightarrow 0_g^+$	1.43(35)	1.78	1.67
$8_g^+ \longrightarrow 6_g^+ / 6_g^+ \longrightarrow 4_g^+$	0.89(22)	1.09	1.05

*Ref. 123

standard, with the experimental values. The calculated $B(E2)$ - values and the $B(E2)$ - branching ratios, of ^{178}Hf - isotope, are compared with the corresponding experimental data in tables (22-a) and (22-b), respectively. The comparison shows that the calculated results of the IBA-2 version, RVM and the 2nd version of Greiner's model gives, respectively, an agreement with the experimental $B(E2)$ - values of the interband ($g \rightarrow g$) transitions. At the same time the IBA-2 results gives an agreement with the experimental data of the $B(E2)$ - branching ratios, of the interband ($\beta \rightarrow g$) and ($\gamma \rightarrow g$) transitions ratios, better than the other theoretical results. A different theoretical $B(E2)$ - values of the ^{180}Hf - isotope and a corresponding experimental values are presented in table 23. From this table, one may notice that the 2nd version of Greiner's model behaves with experiment better than RVM. A pictorial comparison for $^{174-178}\text{Hf}$ - isotopes, associated with the $B(E2)$ - branching ratio of an interband ($0_g^+ \rightarrow 2_\gamma^+ / 0_g^+ \rightarrow 2_\beta^+$) transition ratio, is given in fig. 16. From this figure, one can say that the $B(E2)$ - branching ratio is reproduced quantitatively well by the 2nd version of Greiner's model, while the results of RVM seems to be small, relatively, comparative to the experimental values.

A different theoretical results are compared with the experimental $B(E2)$ - values of $^{230-232}\text{Th}$ - isotopes in tables 24-a and 25-a, respectively. Concerning table 24-a, it is evident that the overall agreement is those associated with

Table (20) :: The B(E2) values in case of ^{174}Hf - isotope .

Transition	Exp. * e^2b^2	RVM	2 nd version of Greiner's model
$0_g^+ \longrightarrow 2_g^+$	5.35(35)	5.35	5.29
$0_g^+ \longrightarrow 2_\beta^+$	0.062(1)	0.192	0.006
$0_g^+ \longrightarrow 2_\gamma^+$	0.138(2)	0.172	0.012
$2_\gamma^+ \longrightarrow 2_g^+$	0.042	0.060	0.009

* Ref. 128 and 68

Table (21) :: The B (E2) values in case of ^{176}Hf - isotope .

Transition	Exp. * e^2b^2	RVM	2 nd version of Greiner's model
$0_g^+ \longrightarrow 2_g^+$	5.19(6)	5.20	5.17
$0_g^+ \longrightarrow 2_\beta^+$	0.031(30) 0.025(5)	0.134	0.003
$0_g^+ \longrightarrow 2_\gamma^+$	0.119(80) 0.075(6)	0.171	0.007
$2_\gamma^+ \longrightarrow 2_g^+$	0.032	0.048	0.005

* Ref. 72 , 69 ,and 128 .

Table (22 - a) :: The B (E2) values in case of ^{178}Hf - isotope .

Transition	Exp. * e^2b^2	RVM	2 nd version of Greiner's model	IBA - 2 ^{**}
$2_g^+ \longrightarrow 0_g^+$	0.99	0.97	0.96	0.87
$4_g^+ \longrightarrow 2_g^+$	—	1.42	1.38	1.24
$6_g^+ \longrightarrow 4_g^+$	1.32	1.64	1.52	1.34
$8_g^+ \longrightarrow 6_g^+$	1.39	1.82	1.61	1.38
$10_g^+ \longrightarrow 8_g^+$	1.57	1.98	1.67	—
$0_g^+ \longrightarrow 2_g^+$	4.86(5)	4.83	4.80	—
$0_g^+ \longrightarrow 2_\gamma^+$	0.115(4)	0.140	0.002	—
$0_g^+ \longrightarrow 2_\beta^+$	0.018(7)	0.159	0.001	—
$2_\gamma^+ \longrightarrow 2_g^+$	0.0390	0.710	0.0046	—

* Ref. 145 , 146 , 144 , 72 ,and 128 . ** Ref. 34 .

Table (22 - b) :: The B(E2) branching ratios in case of ^{178}Hf - isotope .

Transition ratio	Exp. *	RVM	2 nd version of Greiner's model	IBA - 2 **
$2^+_g \rightarrow 2^+_g / 2^+_g \rightarrow 0^+_g$	1.45	2.53	9.60	2.198
$2^+_g \rightarrow 4^+_g / 2^+_g \rightarrow 0^+_g$	0.0934	0.0932	134.14	0.117
$4^+_g \rightarrow 2^+_g / 4^+_g \rightarrow 2^+_g$	0.00821	0.02509	0.00031	0.02711
$4^+_g \rightarrow 4^+_g / 4^+_g \rightarrow 2^+_g$	0.0456	0.0190	0.004	0.1200
$6^+_g \rightarrow 4^+_g / 6^+_g \rightarrow 4^+_g$	0.00521	0.01730	0.00152	0.00346
$6^+_g \rightarrow 6^+_g / 6^+_g \rightarrow 4^+_g$	0.0345	0.0095	0.00173	0.0454
$2^+_g \rightarrow 0^+_g / 2^+_g \rightarrow 4^+_g$	0.170	10.734	0.548	0.166
$4^+_g \rightarrow 2^+_g / 4^+_g \rightarrow 6^+_g$	0.0630	2.6992	0.216	0.0513
$4^+_g \rightarrow 2^+_g / 4^+_g \rightarrow 6^+_g$	270	338.09	670.79	270

* Ref. 34 ** Ref. 34

Table (23) :: The B(E2) values in case of ^{180}Hf - isotope .

Transition ratio	Exp. * e^2b^2	RVM	2 nd version of Greiner's model
$2^+_g \rightarrow 0^+_g$	0.96	0.95	0.93
$4^+_g \rightarrow 2^+_g$	1.30	1.40	1.30
$6^+_g \rightarrow 4^+_g$	1.34	1.63	1.39
$8^+_g \rightarrow 6^+_g$	1.54	1.82	1.41
$0^+_g \rightarrow 2^+_g$	4.73(5)	4.73	4.65
$2^+_g \rightarrow 4^+_g$	2.10	2.52	2.34
$0^+_g \rightarrow 2^+_g$	0.114(7)	0.031	0.005
$2^+_g \rightarrow 2^+_g$	0.032	0.112	0.010

* Ref. 138 , 145 , 144 , 128 , 147 and 72 .

Hf-isotopes

B(E2) Branching ratio ($0_g^+ \rightarrow 2_g^+ / 0_g^+ \rightarrow 2_\beta^+$)

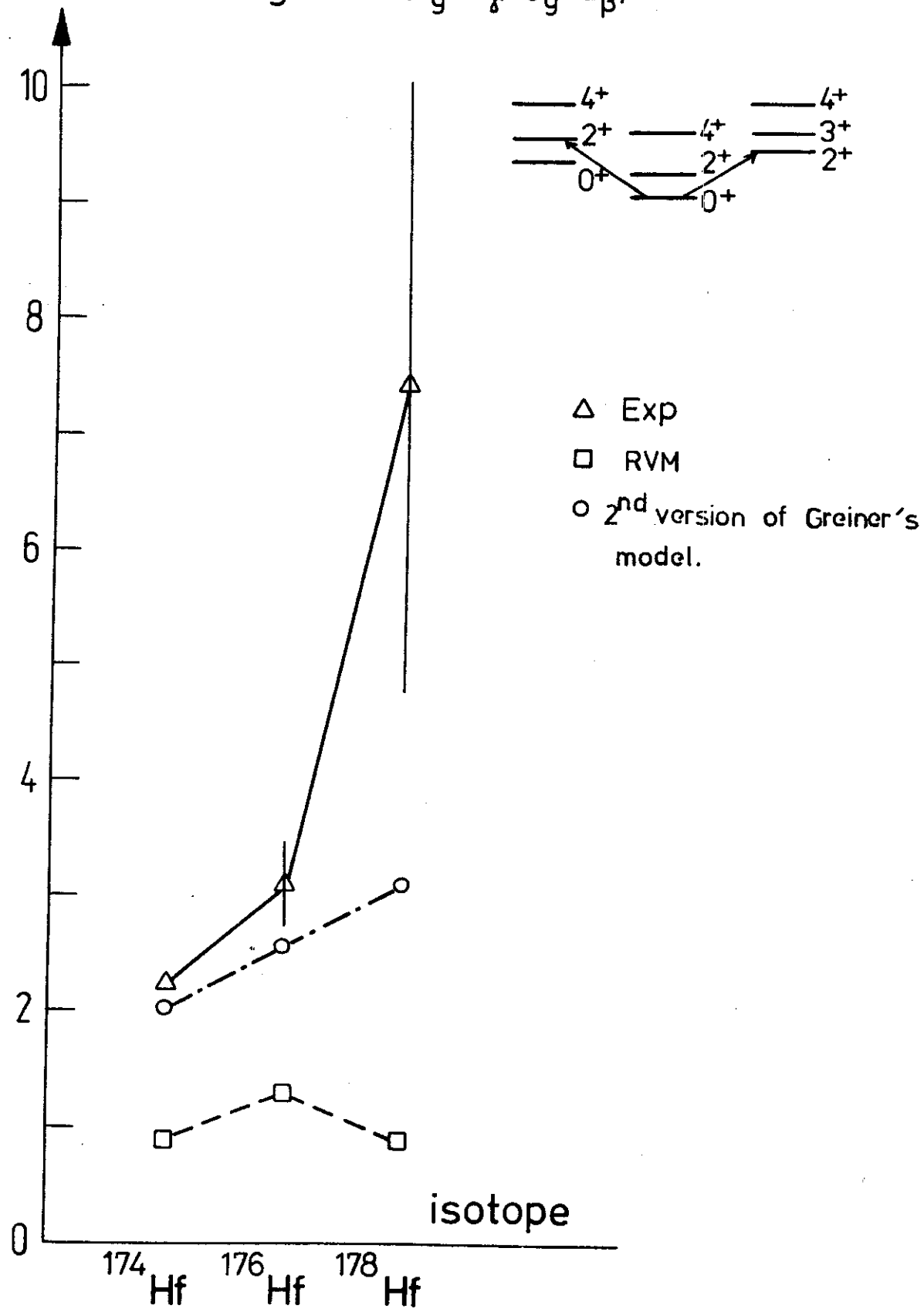


Fig. 16

the results of the 2nd version of Greiner's model. On the other hand and as shown in table 25-a, both of the 2nd version of Greiner's model and RVM may reproduced the experimental values better than the other theoretical calculations. In tables 24-b and 25-b, there is a comparison between a different theoretical calculations and the experimental values of the B(E2)- branching ratios of the ²³⁰⁻²³²Th- isotopes, respectively. Concerning the $(2_Y^+ \rightarrow 2_g^+ / 2_g^+ \rightarrow 0_g^+)$, $(2_Y^+ \rightarrow 0_g^+ / 2_g^+ \rightarrow 0_g^+)$ and $(2_Y^+ \rightarrow 2_g^+ / 2_g^+ \rightarrow 0_g^+)$ transition ratios, it seems that the Davydov and Rostovsky's estimations give a quite good agreement with the experimental data. On the other hand, the rotational model of Bohr and Mottelson agree with the experimental values for the other transition ratios very well. This may be due to fact that the band mixing effect had been considered during the formulation of this model, especially, for the calculation of the probability transitions.

A pictorial comparison associated with the B(E2)- branching ratio, of the $(0_g^+ \rightarrow 2_g^+ / 0_g^+ \rightarrow 0_\beta^+)$ transition ratio, is given in fig. 17 for ¹⁵²Sm, ¹⁵⁴Gd- and ¹⁷²Yb- isotopes. It seems that the 2nd version of Greiner's model agree with the experimental values better than those associated with RVM. In addition, the last one seems to be too low comparative to the experimental values. Other pictorial comparison associated with the B(E2)- branching ratio, of the $(2_g^+ \rightarrow 0_g^+ / 2_Y^+ \rightarrow 0_g^+)$ transition ratio, is given in fig. 18 for the ¹⁵⁰Nd-, ¹⁵⁴Sm-, ¹⁵⁴⁻¹⁵⁸Gd-, ¹⁶⁰Dy-, ¹⁶⁶Er- and ²³⁰Th- isotopes. It is clear, from this pictorial comparison, that the B(E2)- branching ratio

Table (24 - a) :. The (E2) values in case of ^{230}Th - isotope .

Transition	Exp.* e^2b^2	RVM	2 nd version of Greiner's model	DR *	DF *
$2_g^+ \longrightarrow 0_g^+$	1.6(4)	1.61	1.61	1.29	1.57
$2_\gamma^+ \longrightarrow 0_g^+$	0.0246	0.057	0.0226	0.0586	0.0490
$2_\gamma^+ \longrightarrow 2_g^+$	0.066	0.090	0.042	0.1091	0.097

* Ref. 128

Table (24 - b) :. The B(E2) branching ratios in case of ^{230}Th - isotope

Transition ratio	Exp. *	RVM	2 nd version of Greiner's model	Alaga	Bohr Mottelson	DR*	DF*
$2_\gamma^+ \longrightarrow 2_g^+/2_g^+ \longrightarrow 0_g^+$	0.074(29)	0.056	0.026	—	—	—	—
$2_\gamma^+ \longrightarrow 0_g^+/2_g^+ \longrightarrow 0_g^+$	0.052(20)	0.036	0.014	—	—	0.045	0.030
$2_\gamma^+ \longrightarrow 2_g^+/2_g^+ \longrightarrow 0_g^+$	2.1(2)	1.60	1.90	1.40	2.10	1.85	1.90
$6_\gamma^+ \longrightarrow 6_g^+/6_g^+ \longrightarrow 4_g^+$	8.1(1.2)	2.20	3.70	2.90	8.30	—	—
$6_\gamma^+ \longrightarrow 6_g^+/6_g^+ \longrightarrow 4_g^+$	> 20	0.93	2.2.38	3.70	28.30	—	—

* Ref. 76 , and 20 .

Table (25 - a) :. The B (E2) values in case of ^{232}Th - isotope .

Transition	Exp. * e^2b^2	RVM	2 nd version of Greiner's model	DR *	DF *
$0_g^+ \longrightarrow 2_g^+$	9.21(9)	9.26	9.23	—	—
$0_g^+ \longrightarrow 2_\beta^+$	0.10(4)	0.07	0.010	—	—
$0_g^+ \longrightarrow 2_\gamma^+$	0.20(3);0.122(8)	0.46	0.085	—	—
$2_g^+ \longrightarrow 0_g^+$	1.850(80)	0.46	0.085	1.57	1.82
$2_\gamma^+ \longrightarrow 0_g^+$	0.0244	0.0920	0.0170	0.0745	0.0520
$2_\gamma^+ \longrightarrow 2_g^+$	0.0550	0.0327	0.010	0.1360	0.0950

* Ref. 128

Table (25-b) :. The B(E2) branching ratios in case of ^{232}Th - isotope .

Transition ratio	Exp. *	RVM	2 nd version of Greiner's model	Alaga*	Bohr* Mottelson	DF*	DR*
$2^+_{\gamma} \rightarrow 2^+_{\text{g}} / 2^+_{\gamma} \rightarrow 0^+_{\text{g}}$	1.37(18)	0.353	0.58	—	—	1.86	1.84
$2^+_{\gamma} \rightarrow 2^+_{\text{g}} / 2^+_{\text{g}} \rightarrow 0^+_{\text{g}}$	0.07(3)	0.018	0.005	—	—	0.052	0.085
$2^+_{\gamma} \rightarrow 0^+_{\text{g}} / 2^+_{\text{g}} \rightarrow 0^+_{\text{g}}$	0.051(20)	0.050	0.009	—	—	0.028	0.046
$2^+_{\gamma} \rightarrow 4^+_{\text{g}} / 2^+_{\gamma} \rightarrow 0^+_{\text{g}}$	0.21(1)	0.84	1.59	0.071	0.21	—	—
$4^+_{\gamma} \rightarrow 6^+_{\text{g}} / 4^+_{\gamma} \rightarrow 2^+_{\text{g}}$	2.16(70)	25.62	90.27	0.25	2.05	—	—
$6^+_{\gamma} \rightarrow 8^+_{\text{g}} / 6^+_{\gamma} \rightarrow 4^+_{\text{g}}$	>10.2	17.52	42.01	0.39	12.2	—	—
$4^+_{\beta} \rightarrow 6^+_{\text{g}} / 4^+_{\beta} \rightarrow 2^+_{\text{g}}$	10.1(3.2)	3.07	12.37	1.59	7.30	—	—
$6^+_{\beta} \rightarrow 8^+_{\text{g}} / 6^+_{\beta} \rightarrow 4^+_{\text{g}}$	12.7(1.8)	4.85	54.78	1.37	14.20	—	—
$8^+_{\beta} \rightarrow 10^+_{\text{g}} / 8^+_{\beta} \rightarrow 6^+_{\text{g}}$	> 32.8	6.37	263.04	1.27	37.00	—	—

Ref. 148 and 20

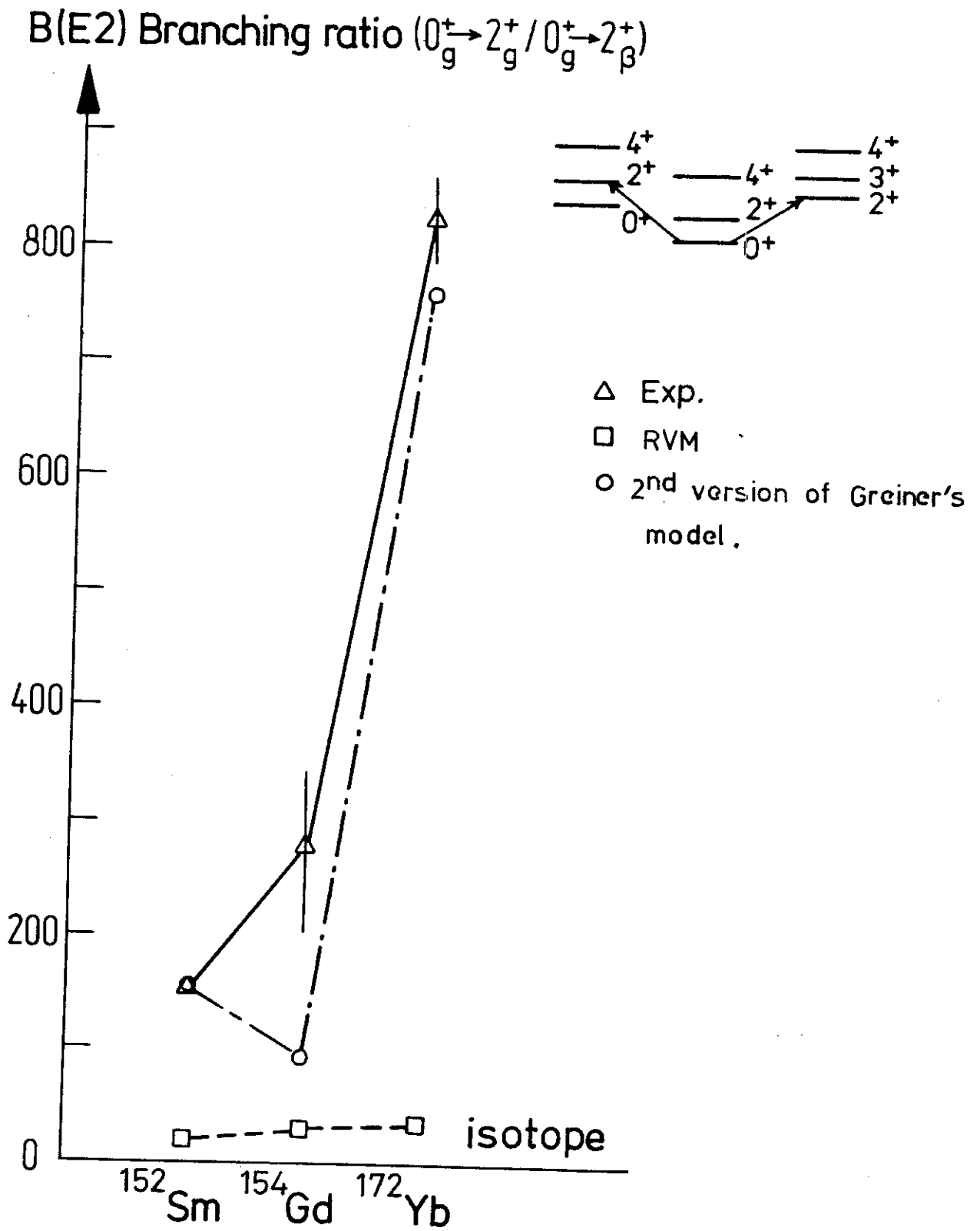
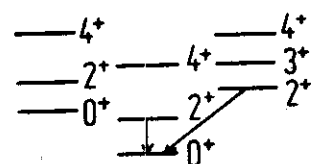


Fig. 17

B(E2) Branching ratio ($2_g^+ \rightarrow 0_g^+ / 2_g^+ \rightarrow 0_g^+$)

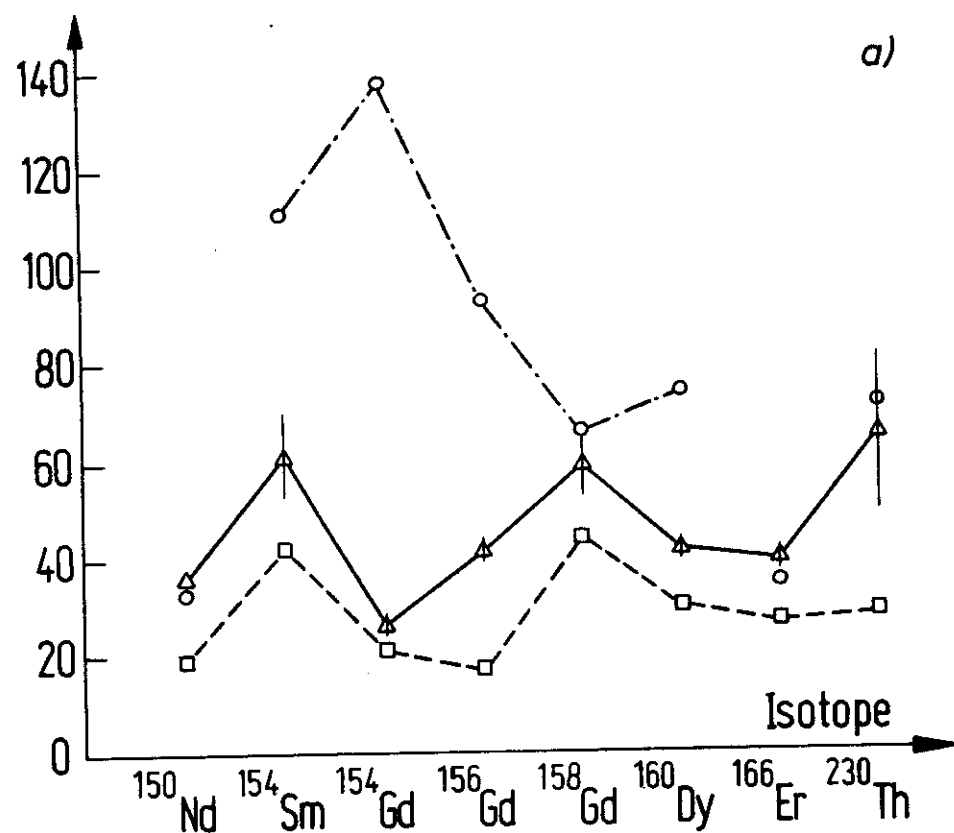
b)



Δ Exp

$\square$ RVM

$\circ$ 2nd version of Greiner's model.



Isotope	p-n deformation difference (δ)	
	a	b
^{150}Nd	17°	24°
^{154}Sm	26°	17°
^{154}Gd	37°	25°
^{156}Gd	35°	29°
^{158}Gd	14°	14°
^{160}Dy	25°	17°
^{166}Er	10°	15°
^{230}Th	25°	25°

Fig. 18

is sensitive to the p-n deformation difference parameter, δ .

IV-3 . The mixing ratio and g_R -factor:-

Although a large number of models and submodels have been developed and applied to the study of the energy levels, the $B(E2)$ - values, the quadrupole moments and the magnetic moments, only a few of them have been employed for predicting the mixing ratios. This is understandable because until recently the systematics of the experimental mixing ratios has been comparatively incomplete (especially keeping track of the various signs).

Now we shall discuss our calculations. From table 26, one can show that there is a reasonable agreement between the experimental data and the RVM- results of the transition energies. But at the same time there are some exceptions associated with the interband transitions. For example, in case of ^{154}Gd - and ^{176}Hf - isotope, as one goes from the lower - spin members to the higher - spin one the discrepancy with experiment becomes more remarkable. This confirms, the general conclusion of Cresswell et al.¹⁰⁾ that the band mixing effect becomes, increasingly, pronounced as one goes to higher spin members.

In table 27, the theoretical calculations of the mixing ratios, present work, are compared with the corresponding experimental values. Concerning this table, some general remarks can be made. First one is that a systematic behaviour of the

Table (26) :.

Isotope	Transition	Transition energy (Mev)	
		Exp. *	Th.
^{152}Sm	$2^+_{\gamma} \longrightarrow 2^+_g$	0.9641	0.9603
	$4^+_{\gamma} \longrightarrow 4^+_g$	1.0053	1.0041
	$2^+_{\beta} \longrightarrow 2^+_g$	0.6887	0.7188
	$4^+_{\beta} \longrightarrow 4^+_g$	0.6565	0.7901
^{154}Gd	$2^+_{\gamma} \longrightarrow 2^+_g$	0.8730	0.8765
	$4^+_{\gamma} \longrightarrow 4^+_g$	0.8930	0.9237
	$2^+_{\beta} \longrightarrow 2^+_g$	0.6930	0.7057
	$4^+_{\beta} \longrightarrow 4^+_g$	0.6770	0.7647
	$8^+_{\beta} \longrightarrow 8^+_g$	0.6122	1.2070
^{156}Gd	$2^+_{\gamma} \longrightarrow 2^+_g$	1.0651	1.0656
	$4^+_{\gamma} \longrightarrow 4^+_g$	1.0672	1.0546
	$2^+_{\beta} \longrightarrow 2^+_g$	1.040	1.0506
^{158}Gd	$2^+_{\gamma} \longrightarrow 2^+_g$	1.1076	1.1075
^{160}Dy	$2^+_{\gamma} \longrightarrow 2^+_g$	0.8794	0.8513
^{166}Er	$2^+_{\gamma} \longrightarrow 2^+_g$	0.7053	0.6794
	$4^+_{\gamma} \longrightarrow 4^+_g$	0.6912	0.6976
	$6^+_{\gamma} \longrightarrow 6^+_g$	0.6705	0.7217
	$8^+_{\gamma} \longrightarrow 8^+_g$	0.6445	0.7483

* Ref. 115 .

Isotope	Transition	Transition energy (Mev)	
		Exp.*	Th.
^{172}Yb	$2^+_{\gamma} \longrightarrow 2^+_g$	1.3870	1.3860
	$4^+_{\gamma} \longrightarrow 4^+_g$	1.3976	1.3964
	$2^+_{\beta} \longrightarrow 2^+_g$	1.0391	1.0522
	$4^+_{\beta} \longrightarrow 4^+_g$	1.0262	1.0804
	$6^+_{\beta} \longrightarrow 6^+_g$	0.9977	1.1199
^{174}Hf	$2^+_{\beta} \longrightarrow 2^+_g$	0.8094	0.8494
	$4^+_{\beta} \longrightarrow 4^+_g$	0.7650	0.8946
	$6^+_{\beta} \longrightarrow 6^+_g$	0.6991	0.9487
^{176}Hf	$2^+_{\beta} \longrightarrow 2^+_g$	1.184	1.165
	$4^+_{\beta} \longrightarrow 4^+_g$	1.100	1.192
	$6^+_{\beta} \longrightarrow 6^+_g$	0.9977	1.2123
	$8^+_{\beta} \longrightarrow 8^+_g$	0.9421	1.4110
	$10^+_{\beta} \longrightarrow 10^+_g$	0.8427	1.5200
^{178}Hf	$2^+_{\gamma} \longrightarrow 2^+_g$	1.0820	1.0812
	$2^+_{\beta} \longrightarrow 2^+_g$	1.1840	1.2032
^{180}Hf	$2^+_{\gamma} \longrightarrow 2^+_g$	1.1090	1.1090
^{232}Th	$2^+_{\gamma} \longrightarrow 2^+_g$	0.7357	0.7360
	$2^+_{\beta} \longrightarrow 2^+_g$	0.7247	0.7255

Table (27) ::

Isotope	Transition	Exp. value of S (E2/M1)	Th. value of S (E2/M1)	Reference
^{152}Sm	$2^+_{\gamma} \longrightarrow 2^+_g$	-9.6(3)	-11.89	Ref. 5
	$4^+_{\gamma} \longrightarrow 4^+_g$	$-2.8^{+2.3}_{-0.9}$; $ S \geq 10$	- 5.96	Ref. 5
	$2^+_{\beta} \longrightarrow 2^+_g$	$+8^{+8}_{-3}$; 8^{+6}_{-3}	-11.32	Ref. 5
	$4^+_{\beta} \longrightarrow 4^+_g$	$+8^{+\infty}_{-7}$; 2.1(3)	- 6.13	Ref. 5
^{154}Gd	$2^+_{\gamma} \longrightarrow 2^+_g$	-9.7(5)	-13.44	Ref. 5
	$4^+_{\gamma} \longrightarrow 4^+_g$	-4.1(4)	- 6.54	Ref. 5
	$2^+_{\beta} \longrightarrow 2^+_g$	$+8.3^{+1.5}_{-1.1}$	-14.51	Ref. 5
	$4^+_{\beta} \longrightarrow 4^+_g$	$+1.4 \leq S \leq 6.3$; $2.9^{+2.1}_{-0.9}$	- 7.58	Ref. 5
	$8^+_{\beta} \longrightarrow 8^+_g$	$+1.2^{+0.4}_{-0.3}$	+ 1.94	Ref. 5
^{156}Gd	$2^+_{\gamma} \longrightarrow 2^+_g$	-10^{+45}_{-5} ; $ S \geq 4$	-10.11	Ref. 5
	$4^+_{\gamma} \longrightarrow 4^+_g$	$-4.0^{+1.6}_{-0.9}$ - $17.2^{+2.2}_{-1.7}$	- 9.21	Ref. 5
	$2^+_{\beta} \longrightarrow 2^+_g$	$-14^{+\infty}_{-7}$	-18.78	Ref. 5
^{158}Gd	$2^+_{\gamma} \longrightarrow 2^+_g$	$-23^{+\infty}_{-14}$	-23.34	Ref. 5
^{160}Dy	$2^+_{\gamma} \longrightarrow 2^+_g$	-15(2) ; 12.8(15)	-16.99	Ref. 5
^{166}Er	$2^+_{\gamma} \longrightarrow 2^+_g$	-16^{+13}_{-5} ; $S \leq -25$	-15.45	Ref. 5
	$4^+_{\gamma} \longrightarrow 4^+_g$	$+7.5^{+\infty}_{-1.5}$; -10^{+27}_{-4}	- 8.32	Ref. 5
	$6^+_{\gamma} \longrightarrow 6^+_g$	-20^{+90}_{-9} + $6.3^{+}_{-2.9}$	- 6.03	Ref. 5
	$8^+_{\gamma} \longrightarrow 8^+_g$	$S \leq -6$	- 4.94	Ref. 5

Table (27) :. continued

Isotope	Transition	Exp. value of $S(E2/M1)$	Th. value of $S(E2/M1)$	Reference
^{172}Yb	$2^+_{\gamma} \rightarrow 2^+_g$	$-2.4^{+0.8}_{-1.5}$	-0.46	Ref. 67
	$4^+_{\gamma} \rightarrow 4^+_g$	$-1.1^{+0.23}_{-0.46}$	-0.21	Ref. 67
	$2^+_{\beta} \rightarrow 2^+_g$	$+5^{+2.5}_{-1.6}$	+2.95	Ref. 67
	$4^+_{\beta} \rightarrow 4^+_g$	$+0.85^{+0.13}_{-0.13}$	+1.46	Ref. 67
	$6^+_{\beta} \rightarrow 6^+_g$	$+0.63^{+0.07}_{-0.07}$	+0.92	Ref. 67
^{174}Hf	$2^+_{\beta} \rightarrow 2^+_g$	$-11^{+\infty}_{-7}$	-9.94	Ref. 5
	$4^+_{\beta} \rightarrow 4^+_g$	$-2.5^{+1.3}_{-0.7}$	-5.34	Ref. 5
	$6^+_{\beta} \rightarrow 6^+_g$	-0.9(2)	-3.73	Ref. 5
^{176}Hf	$2^+_{\beta} \rightarrow 2^+_g$	$1S1 = 2.43 ; 1S1 \geq 4$	+10.85	Ref. 70
	$4^+_{\beta} \rightarrow 4^+_g$	$1S1 = 1.56$	+5.61	Ref. 70
	$6^+_{\beta} \rightarrow 6^+_g$	$1S1 = 1.13$	+3.77	Ref. 70
	$8^+_{\beta} \rightarrow 8^+_g$	$1S1 = 0.85$	-2.22	Ref. 70
	$10^+_{\beta} \rightarrow 10^+_g$	$1S1 = 0.60$	-1.21	Ref. 70
^{178}Hf	$2^+_{\gamma} \rightarrow 2^+_g$	$1S1 \geq 11 ; 1S1 \geq 2$	+7.21	Ref. 5
	$2^+_{\beta} \rightarrow 2^+_g$	$+0.41^{+0.09}_{-0.07} ; +0.410(36)$	-7.27	Ref. 5
^{180}Hf	$2^+_{\gamma} \rightarrow 2^+_g$	$+9.6^{+22}_{-5.8}$	+9.12	Ref. 5
^{232}Th	$2^+_{\gamma} \rightarrow 2^+_g$	+23(10)	-12.70	Ref. 5
	$2^+_{\beta} \rightarrow 2^+_g$	$-1.5^{+2.8}_{-0.7}$	-16.11	Ref. 5

phase of the mixing ratio is apparent from an inspection of the table. With a minor exceptions, transitions from the γ -band have a negative phase, while the majority of the transitions from the β -band seems to show the opposite phase. The sign discrepancies, in case of ^{152}Sm - and ^{154}Gd - isotope, may be due to the fact that both of them are located in the beginning of transition region of the rare earth nuclei, for which the isotopes have an abrupt onset of deformation. On the other hand, there are some experimental data of a same transition, has a +ve and a -ve signs, as in case of the ^{166}Er - isotope. For example, every one of the following interband transitions:- $4^+_{\gamma} \rightarrow 4^+_g$ and $6^+_{\gamma} \rightarrow 6^+_g$, has a +ve and a -ve signs at the same time. This may means that the signs discrepancies, in our calculations, is not a serious problem. The second remark is that the value of the mixing ratios for the β -bands are slightly lower than those which associated with the γ -bands. This may give an indication that the M1- transition rates of the interband $\beta \rightarrow g$ transition are stronger, for the deformed nuclei, than the E2- transition rates. The third remark is that the mixing ratios are, generally, decreases as one goes to the higher spin members. This may indicate that the E2- transition rates decrease faster than the M1- transition rates as one goes to the higher spin members. The fourth remark is that our theoretical predictions, at least, agree qualitatively with experiment. It is noticeable to mention, before the end of this paragraph, that in our calculation we have calculated the M1- transition probability in the framework

of the 1st version of Greiner's model and we have calculated the E2- transition probability in the framework of the 2nd version of Greiner's model, i.e. using a nonhomogeneous charge distribution for both E2- and M1- transitions.

In fig. 18 there is a pictorial discription for the behaviour of the mixing - ratios $\delta(E2 / M1)$ with the p-n deformation difference δ - parameter , associated with two interband transitions : $2_{\beta}^{+} \longrightarrow 2_g^{+}$ and $2_{\gamma}^{+} \longrightarrow 2_g^{+}$ in case of ^{172}Yb - isotope. This figure shows that the mixing ratios of these two interband transitions have the same sign for a large number of p-n deformation differences, while they have an opposite signs for a small number of p-n deformation differences. On the other hand, it is clear that for δ -parameter $\leq 60^{\circ}$ the mixing ratios of these two interband transitions have a -ve signs and for δ -parameter $\geq 64^{\circ}$ they have +ve signs. While for the interval :- $61^{\circ} \leq \delta$ - parameter $\leq 63^{\circ}$, the mixing ratios of these two interband transitions have an apposite signs :- a -ve sign associated with the interband $2_{\gamma}^{+} \longrightarrow 2_g^{+}$ transition and a +ve sign associated with the interband $2_{\beta}^{+} \longrightarrow 2_g^{+}$ transition. Thus one can observe that the p-n deformation difference has an influence on the phase of the mixing ratio.

The present calculations and other theoretical calculations associated with the mixing ratios of ^{152}Sm -, ^{156}Gd - ^{160}Dy - and ^{166}Er - isotopes are compared with the corresponding experimental values in table 28. In spite of the signs discrepancies for some of our calculations, as in case of $2_{\beta}^{+} \longrightarrow 2_g^{+}$

^{172}Yb - isotope

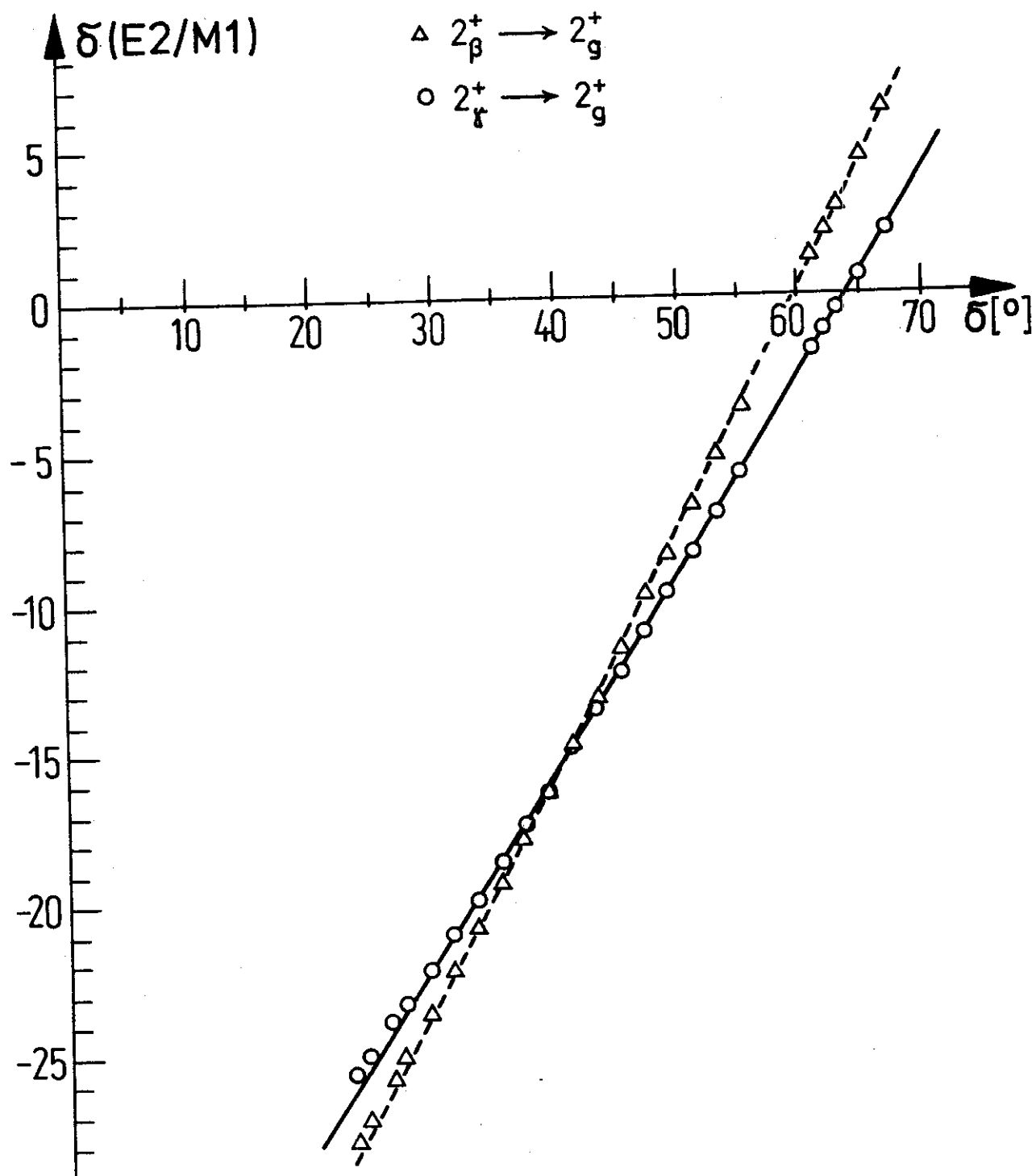


Fig. 19

Table (28) :.

Isotope	Transition	$S(E2/M1)$					
		Exp.*	Present work	PPQ _{a,b}	IBA _c	Baker et al. _d	Greiner _f
^{152}Sm	$2^+_{\gamma} \rightarrow 2^+_g$	-9.6(3)	-11.89	-24.00	- 6.00	_____	_____
	$4^+_{\gamma} \rightarrow 4^+_g$	$-2.8^{+2.3}_{-0.9}$	- 5.96	-10.00	_____	_____	_____
	$2^+_{\beta} \rightarrow 2^+_g$	$+ 8^{+8}_{-3}$	-11.32	-11.00	+22.00	_____	_____
	$4^+_{\beta} \rightarrow 4^+_g$	$+ 8^{+\infty}_{-7}$	- 6.13	+ 4.00	_____	_____	_____
^{156}Gd	$2^+_{\gamma} \rightarrow 2^+_g$	$-17.2^{+2.2}_{-1.7}$	-10.11	-41.00	_____	_____	_____
	$4^+_{\gamma} \rightarrow 4^+_g$	$- 4^{+1.6}_{-1.4}$	- 9.21	-14.00	_____	_____	_____
	$2^+_{\beta} \rightarrow 2^+_g$	$-14^{+\infty}_{-7}$	-18.78	+21.00	_____	_____	_____
^{160}Dy	$2^+_{\gamma} \rightarrow 2^+_g$	-15(2)	-16.99	_____	_____	-12.9	- 9.6
^{166}Er	$2^+_{\gamma} \rightarrow 2^+_g$	-16^{+3}_{-5}	-15.45	_____	_____	-21.2	-12.3
	$4^+_{\gamma} \rightarrow 4^+_g$	-10^{+27}_{-4}	- 8.32	_____	_____	-11.4	- 6.6
	$6^+_{\gamma} \rightarrow 6^+_g$	-20^{+90}_{-9}	- 6.03	_____	_____	- 8.0	- 4.6
		$+6.3^{+\infty}_{-2.9}$					

* These experimental data had been taken from table 27 .

a) Ref. 5 , b) Ref. 5 ,c) Ref. 5 ,d) Ref. 149 , e) Ref. 149 ,

and $4_{\beta}^{+} \rightarrow 4_g^{+}$ transitions in ^{154}Gd - isotope, one may notice that the present calculations reproduce the experimental values better than those associated with DPPQ, in case of ^{152}Sm - and ^{154}Gd - isotopes. For the ^{160}Dy - and ^{166}Er - isotopes, one can say that the theoretical calculations of present work, Baker et al and Greiner agree, in the same standard, with the experimental values. It is important to mention, here, that for the Greiner's results the M1-transition probability had been calculated in the framework of the 1st manipulation of Greiner's model, i.e. with non- HCD assumption, while E2-transition probability has been calculated in the framework of RVM, i.e. with HCD assumption. The IBA- calculations gives a right signs but, at the same time, one can see that it is smaller than experimental value in case of the interband $2_{\gamma}^{+} \rightarrow 2_g^{+}$ transition and it is higher than the experimental value in case of $2_{\beta}^{+} \rightarrow 2_g^{+}$ transition.

The theoretical calculations in the present work, dynamic pairing - plus-quadrupole (DPPQ), dynamic Nilsson Struttinsky and Belyaev (DNSB) and Girit et al. are compared with the experimental mixing ratios of ^{154}Gd - isotope in table 29. From this table and for the interband ($2_{\gamma}^{+} \rightarrow 2_g^{+}$) and ($4_{\gamma}^{+} \rightarrow 4_g^{+}$) transitions, Girit et al. results agree with experimental values better than those obtained in the present work, DPPQ and DNSB. At the same time and for the other interband transitions, DPPQ give an agreement with experiment better than our calculations and DNSB's calculations.

Table (29) :.

Isotope	Transition	S(E2/M1)				
		Exp.*	Present work	DPPQ _a	DNSB _a	Girit et al. _b
¹⁵⁴ Gd	$2^+_{\gamma} \rightarrow 2^+_g$	-9.7(5)	-13.44	-41.0	-16.3	-6.6 ± 0.4
	$4^+_{\gamma} \rightarrow 4^+_g$	-4.1(4)	- 6.54	-12.4	- 6.2	-3.5 ± 0.2
	$2^+_{\beta} \rightarrow 2^+_g$	+8.3 ^{+1.5} _{-1.1}	-14.51	+ 4.9	-10.1	————
	$4^+_{\beta} \rightarrow 4^+_g$	+1.4 ≤ S exp. ≤ 6.3	-7.58	+2.1	+69.6	————

* The experimental data had been taken from table .27

a) Ref. 36 b) Ref. 45

The reduced mixing ratio is defined as $[\delta(E2 / M1)] / E_\gamma$, where E_γ is the transition energy. Concerning table 30, the theoretical results obtained by present work, DPPQ, IBA-2, Bes et al. and Greiner are compared with the experimental data of the reduced mixing ratios of ^{159}Sm -, $^{154-156}\text{Gd}$ -, ^{160}Dy -, ^{166}Er -, ^{172}Yb - and $^{174-178}\text{Hf}$ - isotopes. From an inspection of table 30 one can observe that the overall agreement with experiment is the calculation in the present work, especially, when we consider the absolute magnitude of the reduced mixing ratio.

Concerning tables 31 and 32, a different theoretical results are compared with the corresponding experimental values of the reduced mixing ratios for ^{154}Gd - isotope and ^{172}Yb - isotope, respectively. It is clear from the comparison, in case of ^{154}Gd - isotope, that the calculations obtained by both Warner and IBA-1 agree with experiment better than those obtained in the present work and DPPQ. Contrariwise we observe that our calculations agree with experiment better than IBA- calculations in case of ^{172}Yb - isotope.

As a general observation on the mixing ratio, the measured $\delta(E2 / M1)$ of some isotopes had been done for one kind of interband transitions : $\gamma \rightarrow g$ in case of ^{166}Er and $\beta \rightarrow g$ in case of $^{174-176}\text{Hf}$ - isotopes. The reason of this behaviour can be understood as follows. In case of ^{166}Er - isotope, the calculated $B(E2)$ - values of the interband $2_\gamma^+ \rightarrow 2_g^+$, $2_\beta^+ \rightarrow 2_g^+$, $4_\gamma^+ \rightarrow 4_g^+$, $4_\beta^+ \rightarrow 4_g^+$; $6_\gamma^+ \rightarrow 6_g^+$, $6_\beta^+ \rightarrow 6_g^+$; $8_\beta^+ \rightarrow 8_g^+$ and $8_\beta^+ \rightarrow 8_g^+$ are equal to 0.079, 0.031; 0.096, 0.027; 0.103,

Table (30) :.

Isotope	Transition	$S(E2/M1)/E_\gamma$ (MeV)					
		Exp.	Present work	DPPQ _{a)}	IBA-2 _{b)}	Bes et al. _{c)}	Greiner _{c)}
^{152}Sm	$2^+_{\gamma} \longrightarrow 2^+_g$	- 9.96	-12.38	-25.1	-1.2 ; -2.9 -13.0	8.5	12.5
	$4^+_{\gamma} \longrightarrow 4^+_g$	- 2.79	- 5.94	_____	_____	5.1	6.5
	$2^+_{\beta} \longrightarrow 2^+_g$	+11.62	-15.75	+15.7	+4.0 ; +8.9 +16.0	_____	6.6
	$4^+_{\beta} \longrightarrow 4^+_g$	-12.19	- 7.76	_____	_____	_____	3.4
^{154}Gd	$2^+_{\gamma} \longrightarrow 2^+_g$	-11.11	-15.33	-24.3	_____	7.3	12.4
	$4^+_{\gamma} \longrightarrow 4^+_g$	- 4.59	- 7.08	_____	_____	4.4	6.4
	$2^+_{\beta} \longrightarrow 2^+_g$	+11.98	-20.56	+46.5	_____	_____	6.4
	$4^+_{\beta} \longrightarrow 4^+_g$	+ 4.28	- 9.91	_____	_____	_____	3.4
^{156}Gd	$2^+_{\gamma} \longrightarrow 2^+_g$	- 9.39	- 9.49	- 7.7	_____	9.5	14.1
	$2^+_{\beta} \longrightarrow 2^+_g$	-13.46	-17.88	_____	_____	_____	7.3
^{160}Dy	$2^+_{\gamma} \longrightarrow 2^+_g$	-17.06	-19.96	_____	_____	8.5	13.6
^{166}Er	$2^+_{\gamma} \longrightarrow 2^+_g$	-22.69	-22.74	_____	_____	9.0	15.7
	$4^+_{\gamma} \longrightarrow 4^+_g$	-14.47	-11.93	_____	_____	5.4	8.1
^{172}Yb	$2^+_{\gamma} \longrightarrow 2^+_g$	-1.73	- 0.33	_____	_____	28.0	16.5
^{174}Hf	$2^+_{\beta} \longrightarrow 2^+_g$	-13.59	-11.70	_____	_____	_____	8.2
	$4^+_{\gamma} \longrightarrow 2^+_g$	- 3.27	- 5.97	_____	_____	_____	4.2
^{178}Hf	$2^+_{\gamma} \longrightarrow 2^+_g$	≥ 1.85	+ 6.67	_____	_____	3.0	14.3

* The experimental data had been calculated by using the experimental which is found in tables 26 and 27

a) Ref. 150 b) Ref. 31 c) Ref. 151 .

Table (31) ::

Isotope	Transition	S(E2/M1)/E _γ				
		Exp. *	Present work	DPPQ	Warner _{b,c}	IBA-1 _b
¹⁵⁴ Gd	2 _γ ⁺ → 2 _g ⁺	-11.11	-15.33	-46.76	-7.93	-11.11
	4 _γ ⁺ → 4 _g ⁺	- 4.59	- 7.08	-13.86	-4.18	- 5.93
	2 _β ⁺ → 2 _g ⁺	+11.98	-20.56	+ 7.10	+12.02	+12.11
	4 _β ⁺ → 4 _g ⁺	+ 4.28	- 9.91	+ 3.09	+ 6.26	+ 5.18
	8 _β ⁺ → 8 _g ⁺	+ 1.96	+ 1.61	—	+ 3.26	+ 3.67

* Experimental data had been calculated by using the experimental data which is found in tables 26 and 27 .

a) Ref. 36 , b) Ref. 46 ; c) Ref. 152 .

Table (32) ::

Isotope	Transition	S(E2/M1)E _γ		
		Exp. *	Present work	IBA _a
¹⁷² Yb	2 _γ ⁺ → 2 _g ⁺	-1.73	-0.33	-3.69
	4 _γ ⁺ → 4 _g ⁺	-0.79	-0.15	-1.84
	2 _β ⁺ → 2 _g ⁺	+4.81	+2.81	+4.84
	4 _β ⁺ → 4 _g ⁺	+0.83	+1.35	+2.41
	6 _β ⁺ → 6 _g ⁺	+0.64	+0.82	+1.52

* Experimental data had been calculated by using the experimental data which is found in tables 26 and 27 ..

a) Ref. 67

0.029 ; 0.111, 0.012, respectively. This may give an indication that the transition probabilities of the interband $\gamma \rightarrow g$ transitions are more preferable than those associated with the interband $\beta \rightarrow g$ transitions. Hence our calculation confirm the general conclusion of McGowan et al. that one simply can not find $K = 0$ states with an appreciable $E2$ - strength which is true for the $^{166-170}\text{Er}$ isotopes.

In table 33, the calculated g_R - factor of 2_g^+ are compared with available experimental data. In fact the theoretical values of g_R - factor are calculated in the framework of the 1st manipulation of Greiner's model. The comparison show a good agreement between the theoretical and the experimental values.

Now from our discussion of the calculated, $B(E2)$ - values, $B(E2)$ - branching ratios and $\delta(E2 / M1)$, one may notice, generally, that there is a considerable improvement for the calculated values of this nuclear phenomena as a result of inserting the p-n deformation difference in the formulation of the expression of $B(E2)$ - values. In addition the $B(E2)$ values could be improved more and more by accepting a wrong sign in some $\delta(E2 / M1)$ value.

At the end of our discussion one can suppose that a study should be made in such away that the band - mixing effect should be taken into account during the formulation of the expression of $B(E2)$ - values.

Table (33) :

Isotope	$g(2^+_{g-s})$ -factor	
	Exp.	Th.
^{150}Nd	0.310 ± 0.021 ^a	0.3050
^{152}Sm	0.350 ± 0.030 ; 0.310 ± 0.060 ^a 0.270 ± 0.070	0.3120
^{154}Sm	0.288 ± 0.029 ; 0.317 ± 0.028 ^a 0.290 ± 0.030 ; 0.310 ± 0.060 ^b	0.3070
^{154}Gd	0.360 ± 0.060 ^c 0.370 ± 0.040 ^d	0.0320
^{156}Gd	0.307 ± 0.019 ; 0.310 ± 0.020 ^c 0.330 ± 0.030 ^e	0.3140
^{158}Gd	0.315 ± 0.025 ; 0.327 ± 0.018 ^c 0.385 ± 0.022 ^e	0.3090
^{160}Gd	0.306 ± 0.026 ; 0.323 ± 0.015 ^c 0.360	0.3060
^{160}Dy	0.356 ± 0.017 ; 0.364 ± 0.011 ^c 0.370 ± 0.040 ; 0.360 ^f	0.3160
^{166}Er	0.305 ; 0.312 ^d 0.329	0.3130
^{172}Yb	0.278 ± 0.014 ; 0.333 ± 0.008 ^d	0.3110
^{174}Yb	0.247 ± 0.013 ; 0.337 ± 0.007 ^d 0.338 ± 0.015	0.3070
^{174}Hf	—	0.3170
^{176}Hf	0.266 ± 0.02 ^c	0.3130
^{178}Hf	0.300 ± 0.035 ^d	0.3100
^{180}Hf	0.317 ± 0.035 ^d	0.305
^{230}Th	—	0.297
^{232}Th	—	0.294

a) Ref.153 b) Ref.117 c) Ref. 154

d) Ref. 155 e) Ref. 74 f) Ref. 156

Figure Captions

Figure (1) Illustration of how the initial values of the four parameters of the rotation - vibration model, which are taken from experiment ξ, β_0, E_γ and E_β , are determined.

Figure (2) The energy level's scheme of the lowest bands for the $^{150}_{60}\text{Nd}_{90}$ -isotope.

Figure (3) a The energy level's scheme of the lowest bands for the $^{152}_{62}\text{Sm}_{90}$ -isotope.

b The energy level's scheme of the lowest bands for the $^{154}_{62}\text{Sm}_{92}$ -isotope.

Figure (4) a The energy level's scheme of the lowest bands for the $^{154}_{64}\text{Gd}_{90}$ - isotope.

b The energy level's scheme of the lowest bands for the $^{156}_{64}\text{Gd}_{92}$ - isotope.

c The energy level's scheme of the lowest bands for the $^{158}_{64}\text{Gd}_{94}$ - isotope.

d The energy level's scheme of the lowest bands for the $^{160}_{64}\text{Gd}_{96}$ - isotope.

Figure (5) The energy level's scheme of the lowest bands for the $^{160}_{66}\text{Dy}_{94}$ - isotope.

Figure (6) The energy level's scheme of the lowest bands for the $^{166}_{68}\text{Er}_{98}$ - isotope.

Figure (7) a The energy level's scheme of the lowest bands for the $^{172}_{70}\text{Yb}_{102}$ - isotope.

b The energy level's scheme of the lowest bands for the $^{174}_{70}\text{Yb}_{104}$ - isotope

Figure (8) a The energy level's scheme of the lowest bands for the $^{174}_{72}\text{Hf}_{102}$ - isotope.

b The energy level's scheme of the lowest bands of the $^{176}_{72}\text{Hf}_{104}$ - isotope.

c The energy level's scheme of the lowest bands for the $^{178}_{72}\text{Hf}_{106}$ - isotope.

d The energy level's scheme of the lowest bands for the $^{180}_{72}\text{Hf}_{108}$ - isotope.

Figure (9) a The energy level's scheme of the lowest bands for the $^{230}_{90}\text{Th}_{140}$ -isotope.

b The energy level's scheme of the lowest bands for the $^{232}_{90}\text{Th}_{142}$ - isotope.

Figure(10) A comparison between the theoretical calculations of the energy levels of the lowest bands in case of ^{154}Gd -isotope and the corresponding experimental data.

Figure(11) A pictorial comparison between a different theoretical results for four $B(E2)$ -branching ratios associated with ^{152}Sm -isotope and the corresponding experimental data.

Figure(12) A comparison between the theoretical results of the $B(E2)$ -branching ratio associated with $(0_g^+ \rightarrow 2_g^+ / 0_g^+ \rightarrow 2_{\gamma}^+)$ transition ratio, in case of Gd-isotopes, and the corresponding experimental data.

Figure(13) A Michailov plot for the E2-transitions of the β -band to the ground state band in case of ^{154}Gd -isotopes.

Figure (14) A Michailov plot for the E2-transitions of the β - and γ -bands to the ground state band in case of ^{156}Gd -isotope.

Figure (15) A Michailov plot for the E2-transitions of the β - and γ -bands to the ground state band in case of ^{158}Gd -isotope.

Figure (16) A comparison between the theoretical results of the B(E2)-branching ratio associated with $(0_g^+ \rightarrow 2_\gamma^+ / 0_g^+ \rightarrow 2_\beta^+)$ transition ratio in case of Hf-isotopes, and the corresponding experimental data.

Figure (17) A comparison between the theoretical results of the B(E2)-branching ratio associated with $(0_g^+ \rightarrow 2_g^+ / 0_g^+ \rightarrow 2_\beta^+)$ transition ratio, in case of ^{152}Sm ^{154}Gd and ^{172}Yb - isotopes, and the corresponding experimental data.

Figure (18) A pictorial illustration for the effect of changing of the p-n deformation difference on the fitting of the theoretical results of the B(E2)-branching ratio . $(2_g^+ \rightarrow 0_g^+ / 2_\gamma^+ \rightarrow 0_g^+)$ transition ratio in case of ^{150}Nd ^{154}Sm , $^{154-158}\text{Gd}$, ^{160}Dy , ^{166}Er and ^{230}Th -isotopes, and the corresponding experimental data.

Figure (19) A pictorial discription for the behaviour of the mixing - ratios $\{\lambda(E2/ M1)\}$ with the p-n deformation difference $\{\delta\text{-parameter}\}$, associated with two interband transitions: $2_\beta^+ \rightarrow 2_g^+$ and $2_\gamma^+ \rightarrow 2_g^+$ in case of ^{172}Yb -isotope.