Chapter one

"INTRODUCTION"

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I.1- The Single-Particle Shell Model:-

The single-particle shell model describes non-interacting particles, moving in the common potential well, which is formed by all particles of the nucleus. The energy orbits in the potential well form group, i.e., shells, divided by considerable energy intervals. The single-particle shell model (i.e., the model of independent particles) is too crude to describe the nuclear structure accurately. It gives, however, a basis for the treatment of nuclear correlations caused by the residual interaction. (1)

The solutions of the Hartee-Fock equations, based on the nucleon-nucleon interaction, are available only for a few light and magic nuclei. Therefore, the average field potential is usually chosen empirically. It is assumed that the behavior of the average field potential (as a function of the radius) and the nuclear density distribution are correlated. Further, the potential should correctly reproduce the magic numbers. Finally, details of the average field potential are determined from the large amount of experimental data.

The square well and harmonic oscillator potentials were originally used as the nuclear average field. The experimental evidence from nucleon scattering on nuclei suggests that the harmonic oscillator potential is more suitable for light nuclei, while the square well potential is more suitable for the heavier nuclei. The real nuclear potential must be finite, must have finite surface thickness (similar to the nuclear density) and the radial dependence should be intermediate between the square well and harmonic oscillator. (1)

It was known that the sequence of levels in the infinite, spherical harmonic oscillator well. Having the following potential :

$$V(r) = \frac{1}{2} m\omega_0^2 r^2 \qquad \dots\dots\dots (1.1)$$

where m is the nucleon mass and ω_0 is the classical oscillator frequency. The Schrödinger equation

$$\left[\nabla^2 + \frac{2m}{\hbar^2}(E - V(r))\right]\psi = 0 \qquad \dots\dots\dots (1.2)$$

has the solutions

$$\psi_{nlm} = \frac{u_{nl}(r)}{r} Y_{lm}(\theta,\varphi). \qquad \dots\dots\dots (1.3)$$

Where the spherical functions $Y_{lm}(\theta,\varphi)$ are eigenfunctions of the l^2 operator (l is the orbital angular momentum) and l_z is the projection of l on the z-axis (m_z) . The radial part of the wave function $u_{nl}(r)$ satisfies the equation

$$\left\{-\frac{1}{2m}\frac{d^2}{dr^2} + V(r) + \frac{1}{2m}\frac{l(l+1)}{r^2} - E\right\}(u_{nl}) = 0. \qquad \dots\dots\dots (1.4)$$

the corresponding eigenvalues equal

$$E_{nl} = \left(2n + l + \frac{3}{2}\right)\omega_0, n = 0,1,2,.... \qquad \dots\dots\dots (1.5)$$

Where $N=2n+l$, n is the number of nodes in the radial function, N = $0, 1, 2, \dots$ Each of the eigenvalues E_N is degenerate, corresponding to several l values. If N is an even interger, l has the values 0, 2, 4, …, if N is an odd interger, l = 1, 3, …, N. The maximum number of particles in the degenerate N state equals

$$n_N = \sum_l 2(2l+1) = (N+1)(N+2) \qquad \text{......... (1.6)}$$

The total number of particles, which fill states from $N=0$ up to $N=N_0$, equals

$$\sum_N n_N = \frac{1}{3}(N_0+1)(N_0+2)(N_0+3) \qquad \text{......... (1.7)}$$

The harmonic oscillator levels are usually denoted by a pair of integer number *(n,l)*. *n* means that corresponding *l* value appears *n*th time in the level sequence. The *l* is often denoted by a letter instead of a number, namely

l= 0, 1, 2, 3, 4, 5, 6, 7, 8,
s, p, d, f, g, h, i, j, k. .... (1.8)

so, for example, the sequence begins with *1s, 1p; 2s, 1d; 2p, 1f, etc.*[1]

Even when more realistic radial dependence of the average field potential is used, it is still impossible to reproduce correctly the numbers corresponding to the filled shells. Therefore it is necessary to introduce a new interaction which breaks the degeneracy of the harmonic oscillator. When the nuclear shell model was developed, it was suggested that sufficiently strong spin-orbital interaction exists and might fulfill such a purpose. The corresponding spin-orbit potential has the form

$$V_{ls} = -v\,(r)\,\vec{l}\,.\,\vec{s}, \qquad \text{......... (1.9)}$$

where $\vec{l} = \vec{r} \times \vec{p}$, and s is the nuclear spin,

$$V_{ls} \approx \frac{1}{r}\frac{dV\ (r)}{dr}, \text{ always} \quad +ve \qquad \text{......... (1.10)}$$

The spin-orbit potential breaks the degeneracy of the single-particle levels with respect to the total angular momentum j. using the relation

$$j^2=\left(\vec{l}+\vec{s}\right)^2=l^2+s^2+2\left(\vec{l}.\vec{s}\right), \qquad \text{......... (1.11)}$$

Where

$$\vec{l}.\vec{s}=\frac{1}{2}\{j(j+1)-l(l+1)-s(s+1)\}=\begin{cases}\frac{1}{2}l & \text{for } j=l+\frac{1}{2}\\ -\frac{1}{2}(l+1) & \text{for } j=l-\frac{1}{2}\end{cases} \qquad \text{......... (1.12)}$$

The spin-orbit forces do not cause large changes of the radial wave functions. Therefore, the main effect is the following: The level $j=l+½$ is lowered by

$$\tfrac{1}{2}l\left\langle V_{ls}(r)\right\rangle_{nl}, \qquad \text{......... (1.13)}$$

This means that the energy of a state with a given l having two values depending on the mutual orientation of the spin $\vec{s}$ and the orbital angular momentum $\vec{l}$ of the nucleon, the parallel orientation corresponding to the lower energy value (i.e. higher interaction energy).

while the level $j = l-½$ is raised by

$$\tfrac{1}{2}(l+1)\left\langle V_{ls}(r)\right\rangle_{nl}. \qquad \text{......... (1.14)}$$

The splitting of the two levels is therefore equal to

$$\tfrac{1}{2}(2l+1)\left\langle V_{ls}(r)\right\rangle_{nl}. \qquad \text{......... (1.15)}$$

The splitting increases when l increases, while $<V_{ls}(r)>_{nl}$, i.e., the average value of the $V_{ls}(r)$ in the state (nl), depends weakly on l.

The introduction of the spin-orbit potential is not sufficiently theoretically based. However, many experimental facts confirm the existence of the relatively strong spin-orbit part in the average field potential. Among such facts belongs the splitting of the $j = l+½$ levels, particularly in nuclei with closed shells plus (or minus) one nucleon. Other evidence comes from the observed polarization effects in the interaction between nucleons and nuclei. These polarization effects confirm the accepted sign and strength of the spin-orbit coupling.

The term shell will be further used for a set of states between two magic numbers; the term sub-shell will be used for the degenerate states characterized by the quantum numbers n, l, j.

I.2- Single-Particle Model and the Deformed Nucleus:-

The nucleus in the shell model was assumed to have a spherical shape. Therefore, particles moved in a spherically symmetric potential. There are, however, convincing arguments that nuclei with the neutron and proton numbers sufficiently far from the magic numbers have non-spherical, axially symmetric ellipsoidal shapes.

The classification of the single-particle states depends on the symmetry of the average potential. The single-particle states in spherical nuclei are characterized by their energy, parity, total angular momentum j, and its projection m; states with different m-values are degenerate, i.e., have the same energy. In deformed, axially symmetric ellipsoidal nuclei, the single-particle states are characterized by their energy and parity and by the projection K of the total angular momentum on the nuclear symmetry axis; the total angular momentum j is not a good quantum number. In nuclei without axial symmetry both j and K lose their meaning as good quantum numbers.