Approximate methods for solving nonlinear coupled systems of partial differential equations

Mohamed Meabed Bayomi Khadr

In the last few decades, considerable work has been invested in developing new methods for approximate solutions of linear and non-linear di erential equations which arise in many scientic and engineering problems. Moreover, there has been a great need fore active algorithms to avoid the onerous work required by traditional methods. So, many approximate and numerical methods, such as, the grid points techniques [17], perturbation techniques [30], spline solutions, and others, have been developed. However, each of these methods suers from one or more limitations. The grid points techniques dene the solution at grid points only. The spline solution requires restrictions on boundary points. The perturbation method su ers from a high computational workload specially when the degree of nonlinearity increases. In the following four dierent methods for solving NPDEs are introduced. 1.1 Variational iteration method. In this section, we introduce the analysis of one of the recent methods which has more implementation for solving wide range of linear and non-linear dierential equations. This method is called VIM ([1], [4], [31]), which is proposed by He [29] as a modiﬁcation of a general Lagrange multiplier method. Also, VIM is based on the use of the restricted variations and correction functionals which has found a wide application for the solution of NPDEs [32]. This method does not require the presence of small parameters in the dierential equation, and does not require that the nonlinearities be dierentiable withrespect to the dependent variable and its derivatives. This technique provides a sequence of functions which may converge to the exact solution of the problem. Also, it has been shown that this procedure is a powerful tool for solving various kinds of problems. He [29] has introduced VIM to solve e ectively, easily, and accurately a large class of nonlinear problems with approximations converging rapidly to accurate solutions. 1- In 2006, -He and Xu-Hong Wu [31], applied this method to construct the solitary and compact like solutions. Also, He [32] used VIM for solving the delay dierential equations. He [34] rst applied VIM to solve autonomous ordinary dierential systems. In this method, the general Lagrange multipliers are introduced to construct functional for the systems. The multipliers in the functional can be identiﬁed by the variational theory. The initial approximations can be freely chosen with possible unknown constants, which can be determined by imposing the boundary/initial conditions. Some examples are given toreveal that the method is very eective and convenient. Wazwaz [64] used VIM for analytic treatment for linear and non-linear ordinary dierential equations, he found that this method is capable of reducing the size of calculations and handles both linear and non-linear equations, in a direct manner. However, for concrete problems a huge number of iterations are needed for a reasonable level of accuracy. Wazwaz [65] introduced a framework for obtaining the analytic solutions to linear and non-linear system of partial dierential equations by using VIM. The method reduces the calculations size and overcome the di culty of handling non-linear terms. Numerical examplesare examined to highlight the signi cant features of VIM. Moreover, the method shows improvements over existing numerical techniques. Wazwaz [66] introduced the reliable VIM to determine rational solutions for the KdV,K(2; 2), Burger's, and cubic Boussinesq equations in a straightforward manner. The study highlights the eciency of the method and its dependence on the Lagrange multiplier. Sweilam [54] implemented this method for solving cubic non-linear Schr odinger's equation. Special attention from many authors is given to study a convergence analysis of VIM, one can see for example ([57], [61]). This method is used to solve wide range of problems consisted of ordinary or partial dierential equations. Most authors found that the shortcomingarising in the Adomian decompositions method can be completely eliminated by VIM. The main advantages of VIM are, this technique solves linear or nonlinear problems without discretization of its variables; therefore, it is not aected by computation roundo errors and one is not faced with necessity of large computer memory and time. This method provides the solution of some problems in closed form while the mesh point techniques, such as nite dierence method ([17], [35]) provides the approximation at mesh points only.