New exact travelling wave solutions for some nanlinear evolution equation

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1.1 IntroductionRecently, considerable attention has been directed towards exact solutions includingsolitary solutions, periodic solutions, doubly periodic solutions in terms of elliptic functionsand doubly periodic solutions in terms of Weierstrass elliptic function for nonlinearequations. Many powerful methods have been presented. Among these are Hirota’s bilinearmethods [1], the inverse scattering transform [1], painlev´e expansions [2], homogeneousbalance method [3, 4], tanh method [5–8], extended tanh method [9–13], modifiedextended tanh method [14], sine-cosine method [15, 16], the factorization method [17],F-expansion method [18–22]. F-expansion method was later extended in different manners[23–25], the Exp-method [26], the Weierstrass elliptic function expansion method[27, 28].This chapter is organized as follows: In Section 1.2, we present new solution of ODE(□02(□) = b0 + b2□2(□) + b3□3(□) + b4□4(□)) and used them to find some new exact travellingwave solutions to the Kawahara equation. In Section 1.3, we expressed the generalform of the solution of NLEEs in terms of the Weierstrass elliptic function and applyingthis method to the higher-order nonlinear Schrodinger equation and (3+1)-dimensionalKadomtsev-Petviashvili equation. In Section 1.4, we find travelling wave solutions of someNLEEs by assuming the solution of reduced nonlinear ODEs in the general form4u(□) = a0 +Pni=1 aiwi(□) + biw−i(□) + ciwi−1(□)w0(□) + diw0(□)w−i(□),where w(□) is the solution of (w02(□) = Pw4(□) + Qw2(□) + R) in terms of Weierstrasselliptic function.1.2 The novel solutions of auxiliary equation and theirapplication to Kawahara equation1The Kawahara equation (KE) given byut + a u ux + b uxxx + c uxxxxx = 0, (1.1)in which a, b and c are arbitrary constants, occurs in the theory of magneto-acoustic wavesin a plasma [29], capillary-gravity water waves [30] and in the theory of shallow waterwaves with surface tension [31].A lot of marvellous works have been published to obtain exact and explicit solutionsfor KE (see, for example, Refs. [32–34]).In [32] Wazwaz obtained analytic solutions for KE by using the tanh method and theextended tanh method . Jang [33] used the auxiliary equation method to find analyticsolutions for the KE. In [34] Fu et al. applied the Jacobi elliptic function expansionmethod to construct the exact periodic solutions to KE.In the present section, the key idea of this method is to introduce the following auxiliaryODE □d□d□□2= b0 + b2 □2(□) + b3 □3(□) + b4 □4(□), (1.2)where b0, b2, b3, b4 are real parameters, and use the following solutions of Eq. (1.2)□(□) =8>>>>>>>>>>>>>>:b23 b3(1 − 3 tanh2(12 □p−b2)),b23 b3(1 − 3 coth2(12 □p−b2)),−b212 b3(2 + 3 tanh2(14 □p−b2) + 3 coth2(14 □p−b2)),−2 b2b3(−23 + coth2(□p−b2) + coth(□p−b2) csch(□p−b2)),-1.31This part published in International Journal of Applied Mathematics 22 (3) (2009) 407-417.5where b4 = 0, b0 = −4 b3227 b23, >>>>>>>>>>:r−b22 b4pμ2 − 1 sech(□p−2 b2) + tanh(□p−2 b2)μ sech(□p−2 b2 b2) + 1,r−b22 b4pμ2 + 1 csch(□p−2 b2) + coth(□p−2 b2)μ csch(□p−2 b2) + 0 and μ is arbitrary constant.By using these 1,-1.4where b3 = 0, b0 = b224 b4, b2 solutions of Eq. (1.2), we indeed successfully find some new exacttravelling wave solutions to the KE.1.2.1 Exact travelling wave solutions for Kawahara equationIn this subsection, we use the solution of equation (1.2) to develop travelling wavesolutions to the KE (1.1) using the wave variable u = u(□), □ = k x − w t and integratingwith respect to □, Eq. (1.1) becomes−□ − w u +12a k u2 + b k3 u00 + c k5 u(4) = 0, (1.5)where □ is an integral constant. Let us choose a solution of (1.5) in the formu(□) =Xni=0ai□i, (1.6)where ai (i = 0, 1, 2, 3, 4) are constants to be determined later. To determine the parametern, we define a polynomial degree function as D(u) = n, thus we haveD(up (dsud□s)q) = np + q(n + s). Therefore, n = 4 from balancing the highest order linearterm u(4) with nonlinear terms u2.u(□) =Xni=0ai□i = a0 + a1 □(□) + a2 □2(□) + a3 □3(□) + a4 □4(□), (1.7)Substituting (1.7) into (1.5) with the aid of (1.2), collecting the coefficients of the powers□ and equating these coefficients to zero, solving these system gives6Case 1.a4 = 0, a3 = 0, b4 = 0, b0 =−4 b2327 b32 ,a2 =−105 c k4 b32a, a1 =−70 k2 (b + 13 c k2 b2) b313 a,a0 =31 b2 k + 507 cw − 910 b c k3 b2 + 1183 c2 k5 b22507 a c k,b2 =b13 c k2 , □ =1296 b4 k2c2 − 28561w257122 a k.-1.8Case 2.a4 =−1680 c k4 b42a, a3 = 0, b3 = 0, b0 =b224 b4,a2 =−280 k2 (b + 52 c k2 b2) b413 a, a1 = 0, b2 =b26 c k2 ,a0 =31 b2 k + 507 cw − 3640 b c k3 b2 + 18928 c2 k5 b22 − 340704 c2 k5 b0 b4507 a c k,□ =648 b4 k28561 c2 − w22 ka.-1.9Substituting Eq. (1.8) and Eq. (1.9) in Eq. (1.7), we get the following formulas for thesolutions of KE:u =1507 a c k(−32 b2 k + 507 cw − 1365 c k3 b3 □(□)4 b + 39 c k2 b3 □(□)□),u =1169 a c k(−69 b2 k + 169 cw − 10920 c k3 b4 □(□)2 b + 26 c k2 b4 □(□)2□).-1.1This in turn gives the following travelling wave solutionsu1(□) =1169 a c k[−69 b2 k + 169 cw + 210 b2 k tanh2(q−c k2□□2p13)− 105 b2 k tanh4(q−c k2□□2p13)],-1.11u2(□) =1169 a c k[−69 b2 k + 169 cw + 210 b2 k coth2(q−c k2□□2p13)− 105 b2 k coth4(q−c k2□□2p13)],-1.127u3(□) =12704 a c k[−54 b2 k + 2704 cw + 420 b2 k coth2(q−c k2□□4p13)− 105 b2 k coth4(q−c k2□□4p13) + 420 b2 k tanh2(q−c k2□□4p13)− 105 b2 k tanh4(q−c k2□□4p13)],-1.13u4(□) =11352 a c k[csch4(q−(bc k2) □2p13) (−732 b2 k + 507 cw− 4 (36 b2 k + 169 cw) cosh(q−(bc k2) □p13)+(36 b2 k + 169 cw) cosh(2q−(bc k2) □p13))],-1.14u5(□) =−69 b2 k + 169 cw169 a c k+1338 a c k (μ + cosh(q−(bc k2) □p13))4×[105 b2 k (p−1 + μ2 + sinh(q−(bc k2) □p13))2 (5 + 2 μ2+ 8 μ cosh(q−(bc k2) □p13) + cosh(2q−(bc k2) □p13)− 4p−1 + μ2 sinh(q−(bc k2) □p13))],-1.15u6(□) =−69 b2 k + 169 cw169 a c k−1338a c k (μ + sinh(q−(bc k2) □p13))4×[105 b2 k (p1 + μ2 + cosh(q−(bc k2) □p13))2 (5 − 2 μ24p1 + μ2 cosh(q−(bc k2) □p13)−cosh(2q−(bc k2) □p13) − 8 μ sinh(q−(bc k2) □p13))].-1.16In solution u5 if we put μ = 1, we obtainu11(□) =1169 a c k[36 b2 k + 169 cw − 105 b2 k sech4(q−c k2□□2p13)], (1.17)8and if μ = −1, thenu12(□) =1169 a c k[36 b2 k + 169 cw − 105 b2 k csch4(q−c k2□□2p13)], (1.18)where □ = k x − w t. Also, we plot the solutions u11, u12 for k = 1,w = 0.05 and noticethat the form of solutions depend on the signs of the coefficients b and c in the KE.Fig. 1.1. The solution u11 for a = 1, b = −1, c = 1 and a = −1, b = 1, c = −1.Fig. 1.2. The solution u11 for a = 1, b = 1, c = −1 and a = −1, b = −1, c = 1.9Fig. 1.3. The solution u12 for a = 1, b = −1, c = 1 and a = −1, b = 1, c = −1.Fig. 1.4. The solution u12 for a = 1, b = 1, c = −1 and a = −1, b = −1, c = 1.Remark 1.1. All solution obtained by Wazwaz [32] is in agreement with some result inthis section if we put = 0.Remark 1.2. If we put = 0 in solution u11 , we have k = −169 cw36b2 so we can rewrite u11asu11() =−105 b2169 a csech4(q−13 bc (x − wk t)26). (1.19)Let wk = ¯c) ¯c = −36 b2169c sou11() =35¯c12:00:00 صsech4(q−13 bc (x − ¯c t)26), (1.20)10which agreement with solution (3.8) in Ref. [35] and if we put a = 1, b = p, c = −q, ¯c = c,then all solutions in [35] are obtained.