On the asymptotic theory of generalized order statistics

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Generalized order statistics (gos), as well as the dual generalized order statistics(dgos), have been introduced as a unified distribution theoretical set-up which containsa variety of models of ordered random variables (rv's). Since Kamps (1995)had introduced the concept of gos as a unification of several models of ascendinglyordered rv's, the use of such concept has been steadily growing along the years. Thisis due to the fact that such concept includes important well-known concepts that havebeen separately treated in statistical literature. Theoretically, many of the models ofordered rv's contained in the gos model, such as ordinary order statistics (oos), orderstatistics with non-integral sample size, sequential order statistics (sos), record values,Pfeifer's record model and progressive type II censored order statistics (pos). Thesemodels can be applied in reliability theory. For instance, the sos model is an extensionof the oos model and serves as a model describing certain dependencies or interactionsamong the system components caused by failures of components, and the pos modelis an important method of obtaining data in lifetime tests. Live units removed earlycan be readily used in other tests, thereby saving cost to the experimenter. Randomvariables that are decreasingly ordered cannot be integrated into the framework ofgos. Therefore, Burkschat et al. in (2003) have introduced the concept of dgos toenable a common approach to desendingly ordered rv's. Each of the concepts of gosand dgos enable a common approach to structural similarities and analogies. Knownresults in submodels can be subsumed, generalized, and integrated within a generalframework.The main aim of this thesis is to study the limit joint distribution function (df) ofany two statistics in wide subclasses of the gos and dgos models, known as m-gos andm-dgos, respectively. These subclasses contain many -important particaular modelsvof gos and dgos, such as oos, order statistics with non -integer sample size, sos andupper and lower record values. The limit df's of lower lower extreme, upper-upperextreme, lower-upper extreme, central-central and lower-lower intermediate m-gos,as well as m-dgos, are obtained. It is revealed that the convergence of the marginalsm-gos, as well as m-dgos, implies the convergence of the bivariate df's. Moreover,the conditions, under which the asymptotic independence between the two marginalsoccurs, are derived. This thesis consists of four chapters:Chapter one. This chapter consists of five sections, the materials of the first foursections are an overview of principle results in and related to the limit theory of oosand the definitions of the gos and dgos models.

Section five contains some auxiliaryresults concerning the limit theory of the univariate extreme, central and intermediatem-gos, as well as m-dgos. All the results in this section concerning m-dgos arenew and are analogues of the results of Barakat (2007a).Chapter two. In this chapter we derive the lower and upper bounds approximationfor bivariate df of (lower,lower), (upper,upper) and (lower,upper) extreme m-gos and m-dgos. By using these inequalities the asymptoticbivariate df of these statistics are derived.Chapter three. In this chapter we derive the limit df's -of bivariate central m-gos,as well as m-dgos. It is revealed that any two central m gos, as well as m-dgos,are asymptotically dependent.Chapter four. In this chapter we derive the limit df's of bivariate intermediatem-gos, as well as m-dgos. The conditions, under which the asymptotic independencebetween the two marginals occurs, are derived.It should be noted that most of the results obtained in this thesis have been onepaper published and two papers submitted for publication.